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Annalisa Fabretti, Tommy Gärling, Stefano Herzel and Martin Holmen

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Annalisa Fabretti[†], Tommy Gärling[‡], Stefano Herzel[§], Martin Holmen[¶]

Abstract

This paper uses agent-based simulation to analyze how financial markets are affected by market participants with convex incentives, e.g. option-like compensation. We document that convex incentives are associated with (i) higher prices, (ii) larger variations of prices, and (iii) larger bid-ask spreads. We conclude that convex incentives may lead to decreased stability of financial markets. Our analysis suggests that the decreased stability is driven by the fact that convex incentives pushes agents towards more extreme decisions. Furthermore, while risk preferences affect agent behavior if they have linear incentives, the effect of risk preferences vanishes with convex incentives.

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Keywords: incentives, market instability, agent-based simulations.

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[†]University of Rome, Tor Vergata, Department of Economics and Finance, Via Columbia 2, 00133 Rome, Italy. E-mail: Annalisa.Fabretti@uniroma2.it.

[‡]University of Gothenburg, Department of Psychology, Centre for Finance, Haraldsgatan 1, 41314 Gothenburg, Sweden. E-mail: tommy.garling@psy.gu.se.

[§]Corresponding author. University of Rome, Tor Vergata, Department of Economics and Finance, Via Columbia 2, 00133 Rome, Italy. Phone: +39 06 7259 5946, E-mail: stefano.herzel@uniroma2.it.

[¶]University of Gothenburg, Department of Economics, Centre for Finance, Vasagatan 1, 40530 Gothenburg, Sweden. E-mail: martin.holmen@cff.gu.se.

1 Introduction

After the unfolding of the financial crisis in 2007-2008, the role of specific compensation structures of financial market participants became a highly discussed issue (see e.g. Bebchuk et al., 2010, Dewatripont et al., 2010, French et al., 2010, Gennaioli et al., 2010). Rajan (2006) argues that one of the main origins of instability in highly developed financial markets is convex incentives structures. Convex incentive structures are typically used to reduce moral hazard concerns to align the interests of the portfolio manager (agent) and the investor (principal) (see e.g. Allen, 2001; Kritzman, 1987; Goetzmann et al., 2003; Cuoco and Kaniel, 2011).)

In Allen and Gorton's (1993) model of the agency problem, the portfolio manager does not share the losses with the investor but receives a proportion of the profits. They report rational bubbles, as the portfolio manager's convex incentives and limited downside risk make it rational for her to trade at prices above fundamental value. This is similar to the risk-shifting problem between shareholders and bondholders (Jensen and Meckling, 1976). In a similar vein Malamud and Petrov (2014) and Sotes-Paladino and Zapatero (2014) model how convex incentives may lead to mispricing and bubbles.

Holmen et al (2014) (henceforth HKK) and Kleinlercher et al (2014) investigate price formation in experimental markets under convex incentives documenting that convex incentives induce significantly higher market prices than linear incentives. Other market variables such as volatility and volume are not different in the convex treatment compared to the linear treatment. Linear treatment resembles the incentive structure if the trader invest her own money.

This paper uses agent-based simulations of asset markets to explore how convex incentive structures affect prices, volatility, turnover, and bid-ask spreads.[1] It allows us to investigate the market consequences of convex incentives observed by HKK for a known type of market regime (continuous double-auction markets), varying the number of traders and making different assumptions about their utility functions (risk preference). In this way we hope to be able to generalize the experimental results of HKK to actual asset markets. Accordingly, we first attempt to replicate the experimental results under as identical conditions as possible, then we expand these conditions to

[1] Agent-based modeling has been used to investigate asset market behavior (see Samanidou et al., (2007) and Hommes (2006) for reviews)

become more similar to actual asset markets. The model developed would be possible to use for simulations of the influence of still other factors than those we investigate.

We start with developing a theoretical model that is applied to the experimental set-up in HKK. The model implies that the traders' demand functions may be dis-continuous, i.e., at a certain price the agents switch from a positive demand for the asset to a negative demand (supply). Such a discontinuity is more likely and stronger in the presence of convex incentives. In contrast, with risk-aversion and linear incentives, the demand functions result to be continuous.

We then run agent-based simulations based on the set-up in the HKK experiments varying the number of agents with convex incentives. Our results show that convex incentives are associated with higher market prices but also with higher volatility and larger spreads than linear incentives. When we control for aspects that cannot be controlled for in experiments with humans, such as the agents' risk preferences and decision criteria, we find that convex incentives are associated with both higher market prices and less stable markets.

Finally, we rerun the simulations increasing the number of agents and randomly varying the fraction of agents with convex incentives and the degree of risk aversion. The main results remain the same. Independently of the degree of risk-aversion, increasing the fraction of agents with convex incentives leads to higher prices and volatility as well as larger spreads. Varying the degree of risk-aversion conditional on convex incentives, on the other hand, does not affect market behavior.

Our paper has three main contributions. The first one is to show that convex incentives lead to non-continuous demand functions that result in larger variations of prices. This result is consistent with Rajan's (2006) argument that convex incentives are one of the main reasons for instability in financial markets. The second one is the comparison of the effect of risk preferences and incentives on the decision of the agents. We document that incentives dominate risk preferences in the sense that while there are clear effects of risk preferences with linear incentives, the effect of risk preferences vanishes with convex incentives. Thus, agents with very different risk-preferences make similar decisions when they have convex incentives. The third one is the comparison of market experiments with humans and agent-based simulation of asset markets. We expect to learn from one approach that cannot be learned from the other (Duffy, 2006). Convex incentives lead to higher market prices in both experiments with humans and agent-based simulations. The average market prices in our simulations are also quite similar to the market prices in the experiments with humans. However, other market characteristics vary between the experiments with humans and agent-based simulations. In the convex treatments in HKK, standard deviation of prices as well as spreads are roughly the same as in the linear treatments. In contrast, convex incentives in our agent-based simulations are associated with higher standard deviations and spreads compared to the simulations where the agents have linear incentives. The explanation appears to be related to the non-continuous demand functions among the rational simulated agents.

The question is also raised why price volatility is higher in the simulation with convex contracts than in the simulations with linear contracts. In the HKK experiments there is no significant difference. In a recent experiment similar to HKK (Baghestanian and Walker, 2014), it is shown that an initial price may work as an anchor such that subsequent price volatility is reduced. A possibility is that in the HKK experiments with convex contracts, anchoring on the initial price reduces the volatility associated with discontinuous demand functions. Since no anchor effect is modeled in our agent-based simulation price volatility is higher in the simulations with convex contracts compared to the simulations with linear contracts.

Section 2 presents the experimental set-up and develops the theoretical framework. In section 3 we analyze the demand functions and the equilibrium prices. The comparison of the simulations and the human experiments are done in the first part of section 4. In section 4 we also analyze the effect of varying the fraction of agents with convex incentives and the degree of risk-aversion. Section 5 summarizes and concludes.

2 Definitions and model settings

What follows is a description of the experimental asset market in HKK which will be the base for our simulations.

There is a single risky asset paying a dividend X at time T. We assume that X is a binomial random variable defined as

$$X = \begin{cases} X_1 & \text{with probability} \quad p \\ X_2 & \text{with probability} \quad 1-p \end{cases}$$

where X_1 and X_2 are greater than or equal to zero. There are N agents trading the asset, each of them is provided with an initial wealth W_0 and a number of asset ω. Shorting assets and borrowing money are not allowed. Trading is made in a continuous double auction market with open order books.

Each agent i is endowed with a contract function representing the payoff to be received at the end of the contract at time T. The contract is a function of W_i, the final value of the holding of agent i. In particular we consider the following specifications of the contract function

$$f_i(W_i) = \begin{cases} W_i & \text{linear} \\ \phi + \delta \max(W_i - K, 0) & \text{convex} \end{cases} \tag{1}$$

where ϕ, δ and K are constants.

In the experiments with human subjects performed by HKK the number of individuals for each test were $N = 10$, endowed with 40 assets and 2000 units of the experimental currency Taler. The terminal dividends of the risky asset are either 15 Taler or 65 Taler with probabilities of 0.8 and 0.2, respectively. Thus, the expected cash-flow of each risky asset is 25 Taler. Each experiment terminates after 12 rounds of trading.

To achieve comparability between the treatments with linear incentives and convex incentives, the constants in the contract functions have been set so that the expected earnings for the hold strategy are the same for both treatments, see HKK for details. The specific values, which will be also used throughout the present paper are given in Table 1.

[insert TABLE (1) about here]

The final value of agent's portfolio depends on θ_i, the shares of asset exchanged, and on the price P for each share

$$W_i(\theta_i, P) = W_0 + (\omega + \theta_i)X - \theta_i P.$$

A market session is divided into twelve rounds, the traders access to the market one by one in a random order. Agent's strategy is determined by maximizing the expected utility. The utility function u_i is an increasing function of the payoff of the contract function and it may be concave or convex depending on the risk-aversion of the agent. At round t, the trader i with a current position consisting of $m_i(t)$ amount of cash and $w_i(t)$ shares of the asset, determines the optimal amount of units θ_i^* to be exchanged at a price P by maximizing expected utility subject to budget constraints (no short-selling and no money-borrowing),

$$\begin{aligned} &\max_{\theta} E[u_i(f_i(W_i)))] \\ &m_i(t) - \theta P \geq 0 \\ &w_i(t) + \theta \geq 0 \end{aligned} \tag{2}$$

The agents follow a strategy starting from the best quotes available in the market to see if they are interested in placing a buy order or a sell order. Afterwards, they place a competitive limit order, that is an offer that improves the current trading book with a lower bid or a higher ask price.

The implementation of the simulated market experiment proceeds by following these steps:

1. A trader i is randomly selected among those who have not traded in the present round.

2. Any previous limit order by trader i, if still present in the book, is canceled.

3. (Submission of a sell order). Let $P^b(t)$ be the current best bid price. Trader i solves Problem (2) with $P = P^b(t)$. If the optimal solution θ_i^* is a negative value, then the agent places a market order (otherwise the agents proceeds to the next step) . If the corresponding quantity posted in the book is greater (in absolute value) than θ_i^*, the quantity θ_i^* is exchanged, otherwise the agent's demand is only partially satisfied and the next bid in the order book is analyzed.

4. (Submission of buy order) The analogous procedure is repeated with respect to the current best ask price.

5. (Submission of a book order) A random value $\tilde{P}$ is chosen between the current best bid and ask price. The agent solves problem (2) with $P = \tilde{P}$ and post a limit order.

6. If there are still agents who have not traded in this round, go to step 1., otherwise go to the next round

We remark that all agents are rational, and that they have access to the same set of information about the asset. Agents differ from each other with respect to their utility functions and contracts.

3 Agent's demands and market clearing

In this section we analyze the optimal demand function of each agent and how it is related to the price that clears the market.

We begin our analysis from Figure (1) which represents the expected utility as a function of θ of a risk-averse agent for the two types of contract at a given price P. We see that when the agent is endowed with a convex contract, the resulting expected utility is piece-wise concave, while when the contract is linear the expected utility is concave.

[insert FIGURE (1) about here]

To study the solution to problem (2) we consider separately linear and non-linear contract functions, starting from the linear case.

Let $\nu(\theta, P)$ be the first derivative with respect to θ of the expected utility, that is

$$\nu(\theta, P) = \frac{\partial}{\partial \theta} E[u(f(W(\theta, P)))].$$

For any given price P, the function $\nu(\theta, P)$ is increasing with respect to θ when the utility function is concave (risk-averse agent) and decreasing for a convex utility. In the risk-averse case, the optimal demand θ^*, that is the solution to problem (2), either satisfies the First Order Condition (FOC)

$$\nu(\theta, P) = 0,$$

when it belongs to the feasibility interval $[-\omega, \frac{W_0}{P}]$ or it coincides with one of the extremes of the interval. More precisely, it is equal to the left extreme $-\omega$ when the marginal expected utility $\nu(-\omega, P)$ is negative, it is equal to the right extreme $\frac{W_0}{P}$ when $\nu(\frac{W_0}{P}, P)$ is positive, it belongs to the interior of the feasibility interval for all the other cases.

Summarizing, in the case of a concave utility and a linear contract, the optimal demand is the continuous function

$$\theta^*(P) = \begin{cases} \frac{W_0}{P} & \text{if} \quad P \leq P^d \\ \theta_z(P) & \text{if} \quad P \in (P^d, P^u) \\ -\omega & \text{if} \quad P \geq P^u \end{cases} \tag{3}$$

where $\theta_z(P)$ is the solution to the FOC

$$\nu(\theta_z(P), P) = 0,$$

P^d is the solution to

$$\nu(\frac{W_0}{P}, P) = 0$$

and P^u solves

$$\nu(-\omega, P) = 0.$$

When the agent is not risk averse, that is when the utility function is convex, the optimal demand function is equal to

$$\theta^*(P) = \begin{cases} \frac{W_0}{P} & \text{if} \quad P \leq \bar{P} \\ -\omega & \text{if} \quad P \geq \bar{P} \end{cases} \tag{4}$$

where, by continuity of the expected utility, the switching price $\bar{P}$ can be identified by solving

$$E\left[u(\frac{W_0}{P}, P)\right] = E\left[u(-\omega, P)\right]$$

that is

$$Eu\left[(\omega + W_0/P)X\right] - u((W_0 + \omega P) = 0. \tag{5}$$

As an example, let us consider the case of a linear contract and a Constant Relative Risk Aversion (CRRA) utility, $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$. We have

$$\nu(\theta, P) = E(W_0 + (\omega + \theta)X - \theta P)^{-\gamma}(X - P).$$

In the risk-averse case, that is for γ positive, it is easy to obtain $P^u = E[X]$ and

$$P^d = \frac{E[X^{1-\gamma}]}{E[X^{-\gamma}]}.$$

The demand function is

$$\theta^*(P) = \begin{cases} \frac{W_0}{P} & \text{if} \quad P \leq \frac{E[X^{1-\gamma}]}{E[X^{-\gamma}]} \\ \theta_z(P) & \text{if} \quad P \in (\frac{E[X^{1-\gamma}]}{E[X^{-\gamma}]}, E[X]) \\ -\omega & \text{if} \quad P \geq E[X] \end{cases}$$

When the agent is risk-neutral, i.e. when $\gamma = 0$, we get

$$P^u = P^d = E[X]$$

and the optimal demand function becomes

$$\theta^* = \begin{cases} \frac{W_0}{P} & \text{if} \quad P < E[X] \\ -\omega & \text{if} \quad P > E[X] \end{cases}$$

Note that in the case $P = E[X]$, the agent would be indifferent between buying or selling any amount of the asset, that is Problem (2) is solved by any θ within the feasible set.

When the contract function (1) is convex and the agent is risk-averse, for any given P the expected utility in Problem (2) is only *piecewise* concave in θ, as shown by Figure (1). Since the asset X assumes only two values, the expected utilities has two nodes, corresponding to the two values of θ satisfying the equation

$$W_0 + (\omega + \theta)X - \theta P = K.$$

for $X = X_1$ and $X = X_2$. In this case the optimal demand $\theta^*(P)$ may be on the edges of the feasibility interval. This is what happens in Figure (1), where $\theta^*(P)$ coincides with the left extreme of the feasibility interval and also for all the instances examined in the paper, although it may not be necessarily so in general. We also note that in the case of Figure (1), the expected utility is first decreasing and then increasing when moving from the value $\theta = 0$ to the optimal point. This means that partially executed orders may lead to a decrease of agent's expected utility.

The two plots in Figure 2 provide a graphical representation of the optimal demands θ^*.

[insert FIGURE (2) about here]

Figure (2-a) represents the optimal demand functions in the case of a linear contract. The demand functions is continuous for a risk-averse agent, discontinuous for risk-neutral and risk-seeking agents. The discontinuity point is the maximum price $\bar{P}$ for which the agent is willing to invest all of his wealth in the risky asset. It can be identified by solving Equation (5). The value of $\bar{P}$ depends on the attitude towards risk of the agent; it decreases with the level of risk aversion, which, in this case, is measured by γ.

When the incentive is convex, the optimal demand function is always discontinuous, independently of the risk-aversion of the agent, see Figure (2-b). By comparing the two plots in Figure (2) we see that the effect of the convex incentive is to increase the demand function $\theta^*(P)$ for all values of P and for all the attitudes towards the risk.

The equilibrium price is the price which clears the market, that is the value P that solves the equation

$$\sum_{i=1}^{N} \theta_i^*(P) = 0, \tag{6}$$

where $\theta_i^*(P)$ represents the optimal demand of agent i. The aggregate demand function is a decreasing function of P, with a number of points of discontinuity that is less than or equal than the number of agents. Since the aggregate demand is not continuous, Equation (6) may not have a zero and hence an equilibrium price may not exist. However, we can still identify a unique value $\tilde{P}$ where the aggregated demand changes its sign. The price $\tilde{P}$ represents the value where there would be the highest volume of exchanges between the agents. We call it the "quasi-equilibrium" price.

For the existence of an equilibrium price there must be a sufficient degree of heterogeneity among agents. We assume that agents have the same initial endowment (W_0 units of cash and ω units of asset), therefore, if they also have the same level of risk aversion and the same contract function, they will make the same choices and obviously no price can clear the market (however, also in this case, there would exist a quasi-equilibrium price). When agents' preferences or contracts exhibit enough variation among agents, the market clearing condition (6) may be satisfied. To clarify this point, let us consider a simple example with only two agents whose demand functions are

$$\theta_i^*(P) = \begin{cases} \frac{W_0}{P} & \text{if} \quad P \leq \bar{P}_i \\ -\omega & \text{if} \quad P > \bar{P}_i \end{cases}$$

with $\bar{P}_1 \leq \bar{P}_2$. By aggregating the demands we get

$$\sum_{i=1}^{2} \theta_i^*(P) = \begin{cases} 2\frac{W_0}{P} & \text{if} \quad P \leq \bar{P}_1 \\ \frac{W_0}{P} - \omega & \text{if} \quad \bar{P}_1 < P \leq \bar{P}_2 \\ -2\omega & \text{if} \quad P > \bar{P}_2 \end{cases}$$

The aggregate demand is equal to zero only if $\bar{P}_1 \neq \bar{P}_2$, that is when the two agents have different preferences. In such a case, the equilibrium price P^* exists and is equal to $\frac{W_0}{\omega}$ if and only if $\bar{P}_1 < P^* < \bar{P}_2$, therefore it depends on the utility functions and on the type of the contract through the values $\bar{P}_i$.

It is possible to generalize the previous argument to a set of N agents with discontinuous demand functions given by

$$\theta_i^*(P) = \frac{W_0}{P}\mathbf{1}_{P<\bar{P}_i} - \omega\mathbf{1}_{P>\bar{P}_i} \quad ,i = 1,\ldots,N$$

where $\mathbf{1}_A$ is the indicator function of the set A. Assuming, without loss of generality, that the sequence of nodes $\bar{P}_i$ is increasing, the equilibrium price exists and is given by

$$P^* = \frac{N-\kappa}{\kappa}\frac{W_0}{\omega},$$

for an integer κ between 1 and $N-1$, if and only if

$$\bar{P}_\kappa < P^* < \bar{P}_{\kappa+1}.$$

In general, the equilibrium, or the quasi-equilibrium price, must be determined through a numerical procedure.

4 Simulations

4.1 A replication of the experiment with human subjects

We begin by comparing the results obtained via simulation of the artificial market to the outcomes of the laboratory experiments by HKK, who report the following main results: (i) significantly higher market prices with convex incentives than linear incentives and (ii) convex incentives do not lead to higher volatility, spreads and volumes than linear incentives.

We want to investigate if similar results can be obtained in an artificial market where the ten agents involved in each market obey to the trading rules stated in Section 2 and have the utility function

$$u_i(x) = \frac{x^{1-\gamma_i}}{1-\gamma_i}, \tag{7}$$

where the coefficients $\gamma_i, i = 1,\ldots,10$ were estimated from data obtained from the elicitation of the risk-preferences of the participants in the HKK experiments.[2]We identified participants whose answers in the elicitation of risk-preferences indicated risk-aversion, risk-neutrality, and risk-seeking behavior, respectively, in order to analyze how risk-preferences affect the simulated markets. Probably due to the relatively small amounts at stake, some participants' answers indicated risk-seeking behavior (Holt and Laury, 2002). The coefficients of risk-aversion are reported in Table 2 and used throughout the simulations in this sub-section (while in the next sub-section all agents are risk-averse). The choices of the γ_i make agents $1, 3, 5, 8, 9$ risk neutral, agents $4, 6, 10$ risk-seekers and agents $2, 7$ risk-averse.

[insert TABLE (2) about here]

Each test involved the same ten agents and twelve trading periods and it was repeated five hundred times. Following HKK we consider three cases, the first one, called *Linear* involving only linear contracts, the second one, called *Hybrid* involving five linear contracts assigned to agents 1 to 5 and five convex contracts assigned to agents 6 to 10, and the last one involving only convex contracts, called *Convex*. We remark that the only factor that varies among simulations is the order in which the agents enter into the market.

For each simulation $i_s = 1, \dots, 500$ and each time period $t = 1, \dots, 12$ we analyzed the mean over i_s of the following quantities:

1. the average price $P(i_s, t)$, that is the mean of the prices of all the trades executed in period t;
2. the volume $V(i_s, t)$ that is the sum of all assets exchanged in period t;
3. the volatility $v(i_s, t)$ that is the standard deviation of the prices for all the trades executed in period t;
4. the relative spread $s(i_s, t)$, defined as the mean of the bid-ask spreads divided by the mid prices recorded after each agent completes his trading session.

Table 3 presents some comparative statistics for the mean over i_s of

- the Average Price $\bar{P}(i_s)$ defined as the mean of $P(i_s, t)$ over t.

[2]Using the subjects' answers to the lottery, that HKK implemented according to Dohmen et al (2011), we obtained agent's i certainty equivalent and the relative risk attitude parameter γ_i.

- the Final Price $P(i_s, 12)$.
- the Average Spread and the Average Volatility, defined analogously to the Average Price;
- the Percentage Volume, that is the Average Volume divided by the total number of assets (e.g. 400) and multiplied by 100.

The three columns in Table 3 refer to the *Linear*, *Hybrid*, and *Convex*, respectively. For each of the quantities we report the mean, the standard deviation, the 2.5 and the 97.5 percentile across the five hundred simulations.

[insert TABLE (3) about here]

We note that the mean values of both the Average Prices and the Final Prices increase when convex incentives are introduced into the market. This is as predicted by our analysis in the previous Section, which showed that the introduction of convex incentives increases the demands of the risky asset and it is also in line with what observed in the experiment with human subjects by HKK, who reported *significantly higher market prices with convex incentives than linear incentives*. Also note that the average final prices are close to the Equilibrium prices reported in the last row. The average final prices in *Convex* (37.88) are remarkably close to the average prices in the HKK experiments with humans (37.73). Percentage Volumes do not appear to be substantially affected by the change of treatments, also confirming what observed by HKK. Differently from HKK, we report higher Average Volatilities for *Hybrid* and *Convex* and higher standard deviations of both the Average and the Final Prices for *Hybrid* and *Convex*. These last observations suggest that the introduction of convex contracts increases the variability and the instability of the market. Thus, with convex incentives, a small change in inputs, such as the order in which agents accede to the market, produces a great variability of the outputs.

Figure (3) reports the means over the 500 simulations of $P(i_s, t)$, $V(i_s, t)$ and $s(i_s, t)$ for the three treatments considered. We note that the mean prices for the *Linear* are much lower and regular. Prices tend to be higher at the first trading periods and then tend to settle towards the equilibrium level. This is more evident for the *Convex*, where the much higher initial prices are the cause of the difference between the Average Price and the Equilibrium Price reported in Table 3. Volumes do not differ significantly among treatments:

they are decreasing with respect to time, as agents tend to trade less when prices approach the equilibrium level. In particular, the Average Volumes for *Linear*, where the equilibrium is reached earlier, is almost zero towards the last trading sessions. Relative spreads appear almost constant for each treatment, but the linear one is much smaller than the other two, suggesting that the introduction of convex incentives may also have negative effects on the liquidity of the market.

[insert FIGURE (3) about here]

Figure 4 reports the average holdings of the risky assets for each of the agents. In *Linear*, we see that the shares are concentrated in the hands of the risk-seeking agents (agent 4, 6, and 10), that the risk-averse agents end up without any asset, and that the risk-neutral agents tend to remain with an average of ten assets each. In *Linear* we observe a very clear division across agents according to their attitudes towards risk. In *Convex* the risk-neutral and risk-averse agents tend to increase the quantities hold and, consequently, the risk-seekers must decrease their holdings, with a result of a rather constant mean value among the agents. *Hybrid* shows that all shares tend to be hold by the agents with convex incentives (agents 1 to 5), while the agents with linear incentives finish with almost zero shares. HKK report similar results in their hybrid treatment.

Summarizing the result of this Section, we conclude that the introduction of convex incentives increases prices and market instability, both in terms of price volatility and chaotic behavior of market prices. A small change in input, that is the order in which agents trade, generate very different outcomes in terms of market prices. Convex incentives do not have an effect on volumes exchanged, while they have a negative effect on market liquidity, measured as the bid-ask spread. In terms of portfolio holdings, we see that convex incentives have such a strong effect on the decisions by the agents to make almost indistinguishable the average holdings of agents with very different attitudes towards risk.

4.2 Extending the simulation

A clear advantage of computer simulations over experiment with human subjects is that the number of trials and of subjects per trials can be increased at almost no cost. A second advantage is the possibility of controlling for quantities, such as the level of risk aversion, which are generally hard to measure.

In this sub-section we present two series of tests in which we changed the number of agents, the distribution of risk aversion, the percentage of convex contracts, and the initial endowments. In the first series, we changed randomly only the number of convex contracts and the agents' risk attitudes, while in the second one we randomized all the quantities involved. Overall, the two series of tests confirm the robustness of our findings and also quantify the impact of introducing new convex contract in the market on variables such as the prices and their volatility, the volume of the exchanges, and the relative bid-ask spread.

The first series of tests consist of 100 simulations of a market where 100 agents trade for 12 rounds. All our random draws are extractions from a uniform distribution. For each simulation i, we set randomly the percentage $\mathcal{C}_i$ of convex contracts. The risk aversions of the agents are also assigned randomly as follows: we define an agent as Lowly Risk Averse (LRA) if his parameter of risk aversion is around the reference value of 2, while he is Highly Risk Averse (HRA) when it is around 10. We assume that an agent can be either LRA or HRA and we randomly select the percentage $\mathcal{R}_i$ of HRA agents. The exact value of risk aversion assigned to each agent is then set by a random perturbation of plus or minus 10% around the two reference values. We impose that the same percentage $\mathcal{R}_i$ of HRA agents are assigned to the linear and to the convex incentivized groups. We estimate the linear models

$$y_i = \alpha + \beta_1 \mathcal{C}_i + \beta_2 \mathcal{R}_i + \epsilon_i, \quad i = 1, \ldots, 100$$

where y_i is a place holder for the following quantities

1. the Relative Final Price, that is the ratio of the average of the prices recorded at the last round of the market over the expected value of the asset (that is 25);

2. the Percentage Volume, that is the ratio (multiplied by 100) of the total number of shares exchanged during the 12 rounds over the total number of assets;

3. the Relative Spread, that is the average of the ratios of the bid-ask spread over the mid price;

4. the Volatility, that is the average over the 12 rounds of the standard deviations, computed at each round, of the prices divided by the expected value of the asset.

The results of the regressions are represented in Table 4. We observe that β_1, that is the coefficient of the percentage of convex contracts, is positive and significant (at a 95% level) for all the cases. On the contrary, the coefficient β_2 of the percentage of HRA agents is never significant. We also see a high level of R^2 for all the cases but the Volume, implying that the linear models are good at explaining the data. We are also able to quantify the impact of changing the number of convex contracts: for example, by increasing by one percent the number of convex contract, the Relative Price will increase by an amount of 0.82. Taken together the results shown in Table 4 support the statement that the impact of convex contracts prevails on the risk aversions of the agents.

[insert TABLE 4 about here]

In the second set of tests all the quantities, including the initial endowments, are set randomly. We perform 100 simulations with 100 agents, selecting the percentage $\mathcal{C}_i$ of the convex contracts as before. The difference from the first round is that we assigned the risk aversion coefficients of the agents randomly between 0 and 10, their cash and shares endowments also randomly, but imposing that the total number of outstanding shares is constant among all simulations and equal to 4000 (as in the first round of tests), and that the total cash available is also constant and equal to $2000 \cdot 100$. We then estimated the model

$$y_i = \alpha + \beta_1 \mathcal{C}_i + \epsilon_i, \quad i = 1, \ldots, 100$$

obtaining the results shown in Table 5, which are remarkably close to those of Table 4, providing further evidence that the number of convex contracts is the key driver of the behavior of the simulated markets.

[insert TABLE 5 about here]

5 Summary and Conclusion

In this paper we used agent-based simulations to investigate how convex incentives to investment managers affect the behavior of asset markets. We report that, as argued by Rajan (2006), convex incentives may lead to mispricing and decreased stability of financial markets. Thus, the simulations show that convex incentives are associated with higher prices and standard deviations of prices as well as higher spreads. In the presence of convex contracts small changes in input, i.e. the order in which agents trade, generate very different outcomes in terms of market prices.

We found that the degree of risk-aversion affects the behavior of agents with linear contracts. However, with convex contracts, the degree of risk-aversion does not affect agent and market behavior. Therefore convex incentive structures eliminate the influence of differences in risk preferences on agent behavior.

Our conclusion that convex incentive schemes may induce mispricing and decrease the stability of financial markets have implications for the debate about policy regulations of investment managers' incentive schemes (see e.g. Turner, 2009; Walker, 2009). Yet, additional research is needed before any firm policy recommendations can be made. It is noteworthy that in the HKK experiment with humans, convex incentives did not result in less stable markets. In HKK some factor or set of factors may have acted to decrease the variability among actors. A possibility is that anchoring on the initial price is such a factor. With convex contracts and discontinuous demand functions, the value of the traded asset is more difficult to estimate compared to with a linear contract and anchoring is therefore stronger, reducing volatility. In the anchoring conditions of the experiment by Bagestian and Walker (2014) volatility is lower compared to the condition with no anchor. They were also able to use the Duffy and Unver (2006) agent-based model to closely simulate the anchoring effect. Since no anchoring effect is modeled in our agent-based simulation, price volatility is higher with convex contracts compared to linear contracts.

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Initial endowment	random variable X	convex contract
$W_0 = 2000$ $\omega = 40$	$X_1 = 15$ $X_2 = 65$ $p = 0.8$	$\phi = 8$ $\delta = 0.021875$ $K = 3000$

Table 1: The parameter set chosen for the numerical experiment

agent	1	2	3	4	5	6	7	8	9	10
γ_i	0	0.208	0	-1.409	0	-0.286	0.208	0	0	-0.286

Table 2: The estimated parameters of risk-aversion (factor gamma) for the ten agents of the simulations

	linear	hybrid	convex
Average Price	26.29	40.63	50.21
	(0.70)	(4.69)	(3.99)
	(25.28, 27.99)	(33.51, 51.01)	(43.67, 58.96)
Final Price	25.24	40.25	37.88
	(0.64)	(13.90)	(11.06)
	(24.87, 27.06)	(24.47, 62.43)	(25.33, 62.60)
Pct. Volume	0.06	0.06	0.05
	(0.016)	(0.018)	(0.013)
	(0.03, 0.09)	(0.032, 0.105)	(0.025, 0.088)
Average Spread	0.20	0.63	0.57
	(0.07)	(0.10)	(0.08)
	(0.07, 0.36)	(0.44, 0.83)	(0.41, 0.73)
Average Volatility	2.07	11.57	9.93
	(1.31)	(2.66)	(3.18)
	(0.45, 5.30)	(6.17, 16.39)	(3.60, 14.91)
Equilibrium Price	25	37.49	39.47

Table 3: Statistics for the Average Price, Final Price, Volume, Bid-Ask Spread and Volatility for 500 simulations. For each quantity we report the mean, the standard deviation, the 2.5 and the 97.5 percentile. The first column refers to the case of all linear contract, the second to the case of 50% of linear contracts and 50% of convex contracts, the last one to only convex contracts. The last line reports the corresponding equilibrium prices.

	α	β_1	β_2	R^2
Relative Final Price	0.6432*	0.8182*	-0.0967	0.7592
Percentage Volume	0.1045*	0.0699*	-0.0128	0.3403
Relative Spread	0.0069	0.4910*	0.0202	0.7175
Volatility	-0.0151	0.2880*	0.0477	0.6193

Table 4: Regressions with i) Relative Final Price, ii) Percentage Volume, iii) Relative Spread, and iv) Volatility, respectively, as dependent variable. β_1 is the coefficient of the percentage of convex contracts. β_2 is the coefficient of the percentage of Highly Risk Averse agents (HRA). The results are based on 100 simulations where 100 agents trade for 12 rounds. A star denotes significance at the 95% level

	α	β_1	R^2
Relative Final Price	0.6154*	0.8813*	0.7469
Percentage Volume	0.1120*	0.0823*	0.3321
Relative Spread	0.0429*	0.4910*	0.6789
Volatility	0.0258*	0.2407*	0.6951

Table 5: Fully randomized test: Regressions with i) Relative Final Price, ii) Percentage Volume, iii) Relative Spread, and iv) Volatility, respectively, as dependent variable. β_1 is the coefficient of the percentage of convex contracts. The results are based on 100 simulations where 100 agents trade for 12 rounds. A star denotes significance at the 95% level

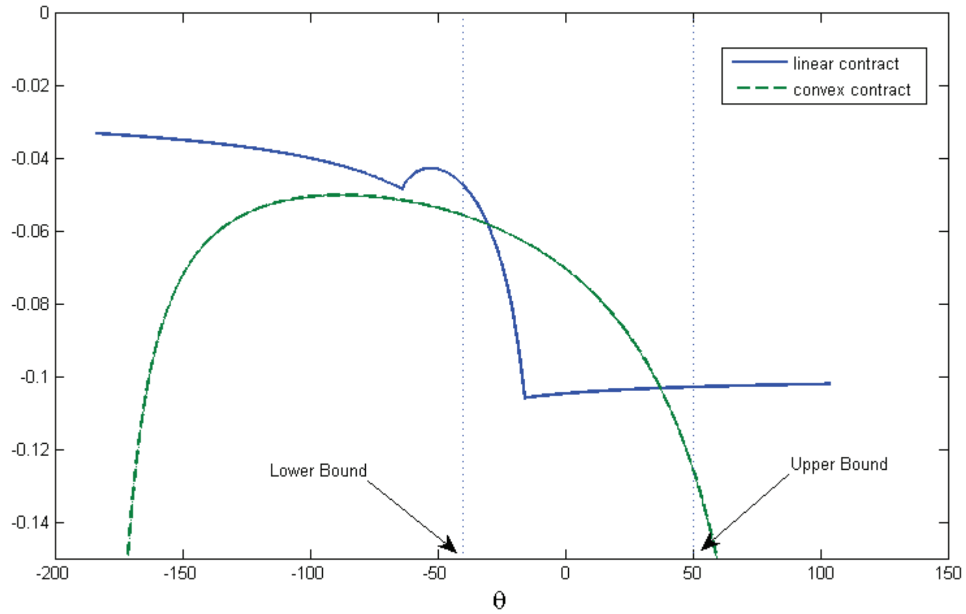
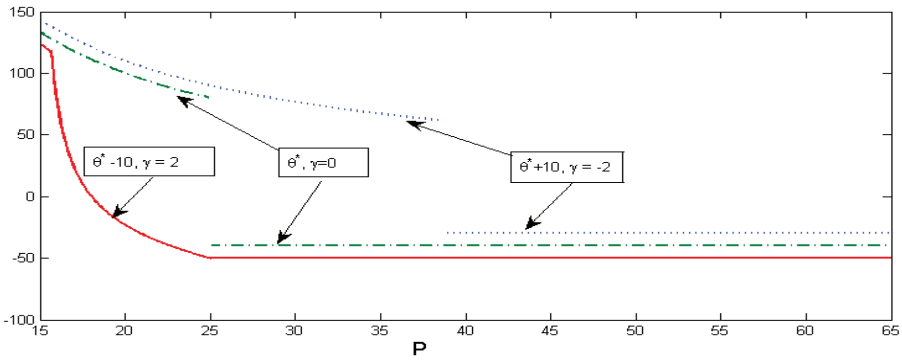
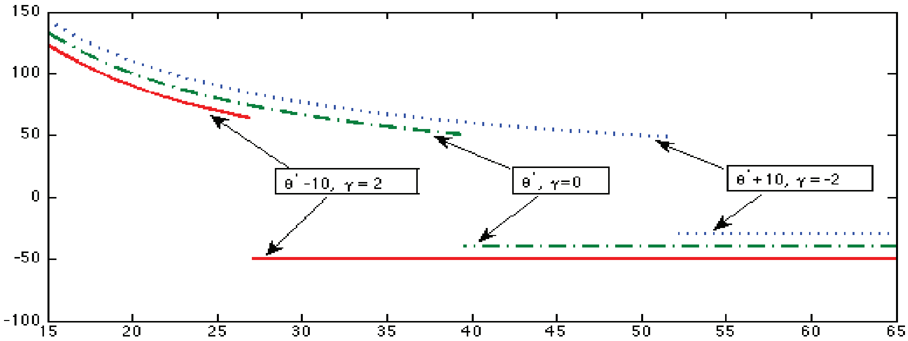


Figure 1: The expected utility for a risk-averse agent (CRRA utility function with $\gamma = 2$) with a linear and a convex contract as a function of the traded quantity θ when the price is $P = 50$ and the other parameters are given in Table (1).

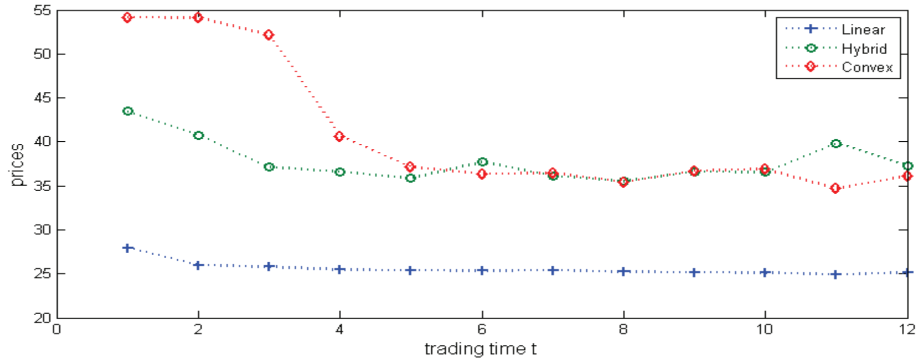


(a) Linear Contract

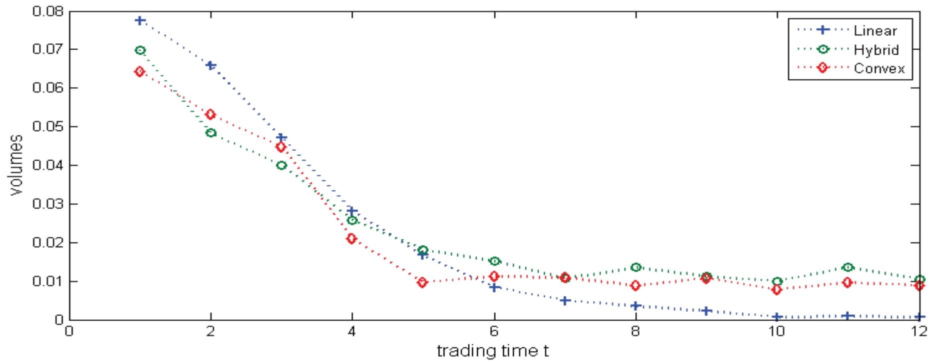


(b) Convex Contract

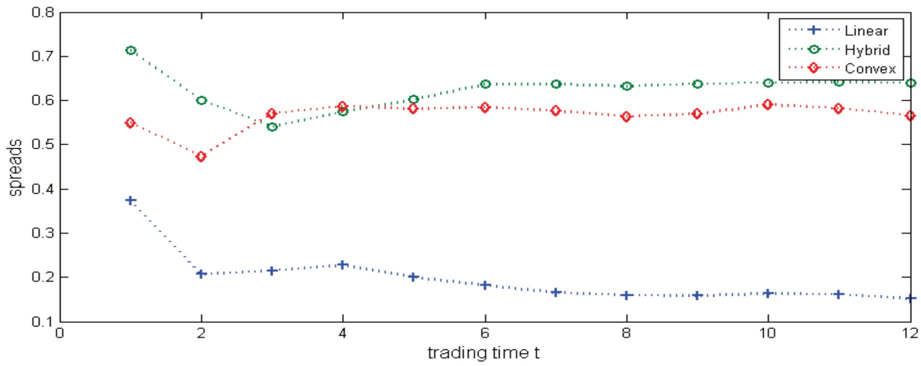
Figure 2: The optimal demand function $\theta^*(P)$ for a linear (top) and a convex (bottom) contract function and risk-averse ($\gamma = 2$), risk-neutral ($\gamma = 0$), and risk-seeking ($\gamma = -2$), CRRA utility agents. The parameters used are those of Table (1). For a clearer picture, the risk-averse demand has been shifted down by 10 units and the risk-seeking one up by 10 units.



(a) Prices



(b) Volumes



(c) Spreads

Figure 3: Mean values in the 12 rounds of prices $P(i_s, t)$ (top), volumes $V(i_s, t)$ (middle) and spreads $s(i_s, t)$ (bottom) for the three treatments over 500 simulations.

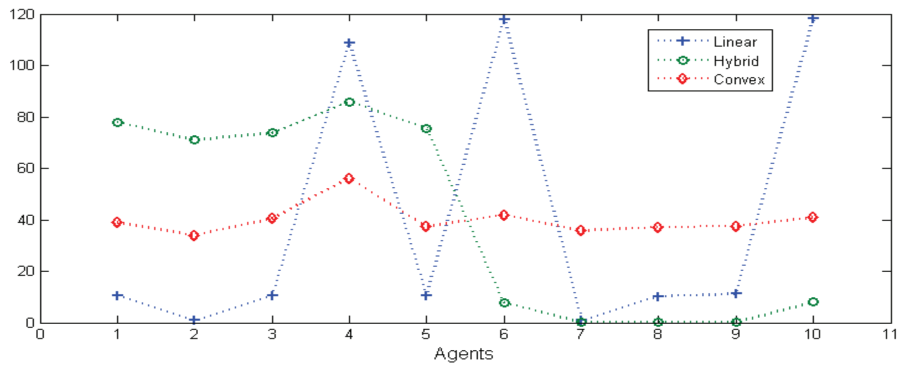


Figure 4: Average share holdings of the ten agents in the three treatments. The means are computed across 500 simulations per treatment. The agents have a CRRA utility function with exponents given by Table 2. In the hybrid treatment the convex contracts are assigned to agents 1 to 5.

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