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Abstract

Gross domestic product (GDP) is the most comprehensive and authoritative measure of economic activity. The macroeconomic literature has focused on nowcasting and forecasting this measure at the monthly frequency, using related high frequency indicators. We address the issue of estimating monthly gross domestic product using a large dimensional set of monthly indicators, by pooling the disaggregate estimates arising from simple and feasible bivariate models that consider one indicator at a time in conjunction to GDP. Our base model handles mixed frequency data and ragged-edge data structure with any pattern of missingness. Our methodology enables to distill the common component of the available economic indicators, so that the monthly GDP estimates arise from the projection of the quarterly figures on the space spanned by the common component. The weights used for the combination reflect the ability to nowcast quarterly GDP and are obtained as a function of the regularized estimator of the high-dimensional covariance matrix of the nowcasting errors. A recursive nowcasting and forecasting experiment illustrates that the optimal weights adapt to the information set available in real time and vary according to the phase of the business cycle.

Keywords: Mixed–Frequency Data; Dynamic Factor Models; State Space Models; Shrinkage.

1 Introduction

This paper proposes a methodology for estimating a monthly indicator of the euro area economic activity, in chained volumes and at market prices, based on the projection of unobserved monthly gross domestic product (GDP) on the information provided by a high dimensional set of monthly economic indicators.

GDP is the most comprehensive and authoritative aggregate measure of economic activity. Its estimation involves processing different sources of information that need to be combined and balanced. The amount of data and the complexity of the statistical process required to compile this aggregate are such that it is currently not possible for statistical agencies to produce GDP estimates in a more timely manner and at an higher observation frequency. Since 2016 Eurostat and the European Union member states have started producing a new flash estimate of quarterly GDP, that is made available 30 days after the end of the quarter. The methodology of the new flash estimate is documented in Kokkinen and Wouters (2016). Despite these important recent developments, it is crucial to provide an assessment of the current macroeconomic situation that goes beyond historical estimation and that is able to "nowcast" current GDP in a timely and accurate way.

Nowcasting deals with predicting the current or recent aggregate state of the economy with the aid of the available indirect information provided by a number of more timely high frequency indicators. See Bansbura et al. (2011), who also point out that now casting is particularly relevant for key macroeconomic variables such as GDP and the national accounts aggregates, which are released with a nonignorable publication lag.

There have been several attempts at nowcasting GDP using related indicators: the class of bridge models, see Baffigi et al. (2004), aims at obtaining predictions of quarterly GDP using monthly indicators; a related approach, based on MIxed frequency DAta Sampling (MIDAS), is considered by Kuzin et al. (2011). Rather than targeting quarterly GDP, it can be thought more relevant for monitoring the current state of the economy to nowcast GDP at the monthly frequency. Temporal disaggregation (distribution and interpolation) using dynamic regression is a well established methoology for this purpose, see Chow and Lin (1971), Fernández (1981), Mitchell et al. (2005), Stram and Wei (1986), Wei and Stram (1990), Proietti (2006), and the references therein. We mention in passing that the flash estimate is an indirect estimate based on this class of methods, and in particular the Chow-Lin regression method applied to annual GDP and quarterly indicators.

Small dimensional dynamic factor models have been used by Mariano and Murasawa (2003), Frale et al. (2011), and Aruoba et al. (2012). Other representative references for temporal disaggregation by unobserved components models are Harvey and Chung (2000) and Moauro and Savio (2005). A conditional regression approach based on the extraction of common factors that distill the information contained in the monthly indicators has been adopted by Angelini et al. (2006). Their approach differs from ours in that the common factors are estimated ex ante and independently of GDP and subsequently temporal regression methods are applied.

For the euro area several high frequency, timely and representative indicators of the state of the economy have been proposed. New Eurocoin is a monthly coincident indicator of economic growth for the euro area Altissimo et al. (2010), published monthly by CEPR (www.cepr.org) and the Bank of Italy. The European Commission (Directorate-General for Economic and Financial Affairs) compiles the Economic Sentiment Indicator (ESI), a survey-based composite coincident indicator for the timely assessment of socio–economic situation in the euro area. Euro–Sting, see Camacho and

Pérez–Quirós (2010), is a monthly indicator of the euro area Gross Domestic Product (GDP), based on a parametric dynamic factor model with mixed frequency data, constructed as an extension of the model described in Mariano and Murasawa (2003). Density forecasting and nowcasting has been considered by Aastveit et al. (2014), Mazzi et al. (2014) and Proietti et al. (2017).

For the U.S. economy, a monthly estimate of GDP growth is obtained by Schorfheide and Song (2015) using a mixed frequency vector autoregressive model estimated with Bayesian methods under a Minnesota-like prior. Monthly GDP estimates are under consideration by the NBER Business Cycle Dating Committee see http://www.nber.org/cycles/recessions faq.html and Markit (2008).

Against this background, the paper proposes an alternative methodology that takes the 'big-data' challenge that is inherent to the task, by pooling the estimates of monthly GDP that arise from estimating all the possible bivariate models of GDP and a monthly indicator at a time, distilling the co-movements between them. While the idea of pooling the estimates is not new, see, for instance, Aastveit et al. (2014), among others, we take a new approach that is partly based on EuroMInd (see Frale et al. (2011)), in that the estimate of monthly GDP satisfies the temporal aggregation constraint, and arises from the projection of the unobserved monthly values on the space generated by the common components of a large set of monthly economic and financial indicators. Rather than performing a purposive selection of the indicators, all the monthly time series can potentially contribute to the monthly estimates, and a composite indicator is distilled, pooling the results of the individual disaggregate estimates, with weights that relate to the relevance of the indicator in nowcasting GDP. Among the benefits of this new approach we stress the possibility of identifying the contribution of the monthly indicators to the nowcasts and forecasts of monthly GDP and of incorporating a large dimensional information set using a simple and effective bivariate dynamic factor model.

The plan of the paper is the following. Section 2 deals with the specification of the bivariate model. The estimation of the model and the alternative pooling strategies are outlined in section 3. Section 4 illustrates the information set available. Section 5 presents the monthly GDP estimates, conditional on the full available sample. The assessment of the alternative pooling strategies is carried out by a recursive experiment aiming at nowcasting and forecasting GDP in real time, described in section 6. The results are presented and commented in section 7. In section 8 we draw some conclusions.

2 Estimation of Monthly GDP: Model Specification

Let $y_t, t = 1, \ldots, n$, denote unobservable GDP for month t, and let $Y_t = y_t + y_{t-1} + y_{t-2}$ be the moving sum of three consecutive monthly values. We observe quarterly GDP, i.e., $Y_{3\tau}$, at times $t = 3\tau, \tau = 1, \ldots, \lfloor n/3 \rfloor$, where $\lfloor \cdot \rfloor$ denotes integer division. Given the availability of N monthly time series, $\{x_{it}, i = 1, \ldots, N, t = 1, \ldots, n\}$, we aim at estimating monthly GDP in real time via its projection on the information carried by the quarterly observed values and the monthly indicators. Hence, our estimation target can be denoted

$$
E_t(y_t) = E(y_t | \mathcal{Y}_t, \mathcal{X}_t), \tag{1}
$$

where $\mathcal{Y}_t = \{Y_{3\tau}, \tau = 1, \ldots, \lfloor (t - \delta)/3 \rfloor\}$ and $\mathcal{X}_{it} = \{x_{i1}, \ldots, x_{i,t-\delta_i}\}$. Here, δ is delay in the release of quarterly GDP and δ_i is that concerning the *i*-th indicator. Some elements of both Y and x may be missing and the set of indicators has a ragged-edge structure, depending on their production schedule.

The estimator of (1) is the conditional mean averaging estimator

$$
\hat{\mathbf{E}}_t(y_t) = \sum_{i=1}^N w_{it} \hat{\mathbf{E}}(y_t | \mathcal{Y}_t, \mathcal{X}_{it}),
$$
\n(2)

where $\hat{E}(y_t|\mathcal{Y}_t, \mathcal{X}_{it}), i = 1, \ldots, N$, arises from a Gaussian bivariate model embodying the common factor structure of the series, presented in section 2, estimated by maximum likelihood with the support of the Kalman filter and associated real time estimation filter. The Gaussianity assumption can be given up, so that (1) are interpreted as optimal linear projections, conditional on the specified model.

The weights $\{w_{it}\}\$ are possibly time-varying and are chosen so as to minimize the mean square nowcast error, as will be discussed in more details in section 3.2.

2.1 Co-movements in economic indicators

We assume that the monthly indicators x_{it} are difference stationary processes with drift m_i ,

$$
x_{it} = x_{i,t-1} + m_i + z_{it}, \quad z_{it} = \chi_{it} + \xi_{it}, \quad i = 1, ..., N,
$$
\n(3)

with an approximate factor structure, see Forni et al. (2000) and Forni and Lippi (2001), so that the changes, z_{it} , are the sum of a common component χ_{it} , which can be written as a dynamic linear combination of common factors, and an idiosyncratic component ξ_{it} . The common factors are pervasive, affecting all the series, whereas the idiosyncratic component is either specific to the i -th series, or weakly correlated across a finite number of time series. The two components are assumed to be orthogonal.

The notion that economic time series are characterized by substantial co-movements which can be ascribed by a small number of common factors has found successful application for the analysis and for the prediction of key macroeconomic variables, see Forni et al. (2009) and Forni et al. (2018).

The key assumption of our methodology is that unobserved monthly GDP lies in the space spanned by the common components χ_{it} , for every t. This space is represented in figure 1 with a filled circle. This is motivated by the fact that GDP arises from the cross-sectional aggregation of many time series, even more than we will consider in our high-dimensional application, so that the idiosyncratic components has been averaged out, and GDP will result from the aggregation of the common component. The generic monthly indicator x_{it} is represented by the solid circle. The portion outside the filled circle is its idiosyncratic component, which is shared only with a small number of 'neighbouring' indicators. The remaining indicators are represented by the dotted circles and ellipses.

We now proceed to formulate the bivariate model of (x_{it}, y_t) that will be used to project quarterly GDP on the space spanned by the common components. By considering all the possible bivariate models, we will obtained N projections that are later aggregated to recover the underlying monthly GDP series.

Figure 1: The space spanned by the common factors is represented by the filled circle. GDP lies in this space. The dotted circles represent the economic indicators. The area outside the filled circle is the idiosyncratic component.

2.2 The Complete Data Monthly Bivariate Model

Let us assume for the moment that we are able to observe GDP at the monthly frequency. The complete data model for the *i*-th monthly indicator, x_{it} , and monthly GDP, y_t , embodies the notion that the two variables display comovements that are due to the presence of common dynamic factors:

$$
\begin{bmatrix}\nx_{it} \\
y_t\n\end{bmatrix} =\n\begin{bmatrix}\nx_{i,t-1} \\
y_{t-1}\n\end{bmatrix} +\n\begin{bmatrix}\nm_i \\
m\n\end{bmatrix} +\n\begin{bmatrix}\n1 \\
\theta\n\end{bmatrix}\n\chi_{it} +\n\begin{bmatrix}\n\xi_{it} \\
\xi_{yt}\n\end{bmatrix},\nt = 1,..., n,\n\chi_{it} = \phi \chi_{i,t-1} + \eta_t - \vartheta \eta_{t-1},\n\xi_{it} = \xi_{jtt}\n\end{bmatrix} \sim \text{IID } N\left(\n\begin{pmatrix}\n0 \\
0\n\end{pmatrix},\n\begin{pmatrix}\n\sigma_i^2 & 0 \\
0 & \sigma_y^2\n\end{pmatrix}\n\right).
$$
\n(4)

In (4) m is the drift in monthly GDP. The comovements between the two series result from sharing the common component of the monthly indicator, χ_{it} , which is modelled according to an ARMA(1,1) process driven by Gaussian random disturbances; we restrict ϕ in the open interval $(0, 1)$ and ϑ in [0, 1]. The coefficient θ is the loading of GDP on the common component of the *i*-th indicator, while ξ_{yt} , captures the variation of the GDP that is not shared by the monthly indicator: in particular, it is the part of GDP related to the remaining indicators and orthogonal to x_{it} , represented in 1 by the area of the big circle external to the x_{it} circle. The assumptions that the ξ_{it} and ξ_{ut} are mutually independent follows from the structure of our model. On the contrary, serial independence is imposed for identifiability of the model. In principle, we can encompass the case when the two series are cointegrated, which arises for $\vartheta = 1$ and $\sigma_i^2 = \sigma_y^2 = 0$; however, we do not expect cointegration, as x_{it} will load on a few, but not all, common factors.

3 Estimation of Monthly GDP: Inference and Model Averaging

3.1 Maximum likelihood estimation and signal extraction

The statistical treatment of the model (4) is based on its state space form, which is presented in appendix A.1. This is modified to take temporal aggregation into account, see appendix A.2, and to process the series sequentially. The Kalman filter applied to the modified state space model computes the one- and multi-step predictions of the series, the prediction errors and their variances, enabling the evaluation of the likelihood. Maximum likelihood estimation of the hyperparameters is carried out by a quasi-Newton algorithm. Estimates of the unobserved components in real time and conditional on the full sample are computed by suitable real time and smoothing filters. Appendix A presents a set of algorithms that are customised to perform estimation and signal extraction, including the estimation of monthly GDP via the conditional mean $\hat{E}(y_t|\mathcal{Y}_t, \mathcal{X}_{it})$ (real time estimation) and $\hat{E}(y_t|\mathcal{Y}_n, \mathcal{X}_{in})$ (historical estimation), taking into account temporal aggregation and ragged–edge data structures. The algorithms rely on the univariate treatment of a multivariate state space model, also referred to as *sequential processing*, see Anderson and Moore (1979). Our treatment is prevalently based on Koopman and Durbin (2000) coupled with the augmentation approach by de Jong (1991) to handle diffuse initial condition and the estimation of fixed effects.

Notice that if $\theta = 0$, then y_t is independent of the *i*-th monthly indicator and has a random walk representation. Hence, the Kalman filter and smoothing algorithms associated with this specification would deliver disaggregate estimates of monthly GDP that coincide with those delivered by the Fernández (1981) method.

3.2 Model averaging: the weighting schemes

All possible bivariate models considering the available monthly indicators x_{it} , $i = 1, \ldots, N$, are estimated. This delivers N potential estimates of monthly GDP, $\hat{E}(y_t|y_s, \mathcal{X}_{is}), i = 1, ..., N$, with s denoting the time of the conditioning information set. These are averaged to form an ensemble estimate, which is our projection of observed GDP onto the space of the common factors. Two alternative weighting schemes are considered.

3.2.1 Deviance-based averaging and screening

The first is based on a two-step procedure rooted in sure independence screening for variable selection, see Fan and Lv (2008) and Chen et al. (2018).

The first step is the initial screening of the variable based on the conditional deviance of GDP. It is adopted to screen out indicators with dominating idiosyncratic component, for which the contribution of χ_{it} is small compared to ξ_{it} . These variable would contribute more to the variance of the estimate than to the reduction of the bias, due to the increase of the coverage of the common component. This approach is also motivated by Boivin and Ng (2006), who point out that when the idiosyncratic component of the x_{it} 's are either the dominant source of variation and/or are cross-correlated, variable selection can improve the forecasting ability of a dynamic factor model. The same may hold for nowcasting monthly GDP.

For the i-th indicator a version of the Kalman filter (based on sequential processing, see appendix

A.3) delivers the deviance measure:

$$
D_i = -2 \sum_{\tau=1}^{\lfloor n/3 \rfloor} \ln f(Y_{3\tau} | \mathcal{Y}_{3\tau-1}, \mathcal{X}_{i,3\tau}), \quad i = 1, \dots, N,
$$
 (5)

where $f(Y_{3\tau} | Y_{3\tau-1}, \mathcal{X}_{i,3\tau})$ is the Gaussian nowcast density of quarterly GDP, conditional on the available past information on GDP, consisting of the quarterly values up to quarter $\tau - 1$, and the monthly values of the *i*-th indicator available up to the current time, $t = 3\tau$.

Letting $D_0 = -2\sum_{\tau=1}^{\lfloor n/3 \rfloor} \ln f(Y_{3\tau} | Y_{3\tau-1})$, which results from fitting the univariate monthly model $y_t = y_{t-1} + m + \xi_{ut}$, to the quarterly GDP series, the screening rule is such that the indicators for which $\exp(D_0 - D_1) > c_N$ are selected. The threshold c_N can be selected by crossvalidation, or it can be set equal to the $1 - \alpha/N$ quantile of a chi-square distribution with 1 degree of freedom. Alternatively, we could select the K best performing models (in the sequel we will consider $K = 10, 30, 50$. We notice in passing that setting w_i proportional to the deviance of the selected indicators would be equivalent to post-selection model averaging according to exponential AIC or BIC weights, see Buckland et al. (1997), Burnham and Anderson (2002) and Claeskens and Hjort (2008), as the number of parameters of the bivariate models remains constant.

3.2.2 High-Dimensional Optimal Nowcast averaging

The optimal weights for combining the nowcasts $\hat{E}(y_t|\mathcal{Y}_t, \mathcal{X}_{it})$ can be derived from the theory of forecast averaging, see Bates and Granger (1969) and the recent work on model averaging marginal conditional mean estimates by Li et al. (2015).

Let $\nu_{i\tau} = Y_{3\tau} - E(Y_{3\tau} | \mathcal{Y}_{3\tau-1}, \mathcal{X}_{i,3\tau}), i = 1, \ldots, N$, denote the nowcasting error that is observable, as it referes to the prediction of quarterly GDP using the information avalable in real time. The aim is that of combining the individual conditional means in such a way that the combined estimator has minimum mean square error. Then, if Ω_N denotes a positive definite estimator of the covariance matrix of the real time prediction error $\nu_{i\tau}$, $i = 1, \ldots, N$. The optimal weights of the linear combination, subject to the constraint $\sum_i w_i = 1$, are given by the elements of the vector

$$
\hat{\mathbf{w}} = \frac{\hat{\mathbf{\Omega}}_N^{-1} \mathbf{i}}{\mathbf{i}' \hat{\mathbf{\Omega}}_N^{-1} \mathbf{i}},\tag{6}
$$

where i is an $N \times 1$ vector of 1's. However, the sample covariance matrix of $\nu_{i\tau}$ is singular, since it is estimated using a number of quarterly observations that is smaller than N (this is even more true if the weights are allowed to vary locally in a rolling forecasting exercise).

For $\hat{\Omega}_N$ we consider the optimal linear shrinkage estimator by Ledoit and Wolf (2004a, 2004b), such that the generic element is obtained as a weighted linear combination of the sample covariance and a shrinkage target covariance, denoted $\tilde{\omega}_{ij}$:

$$
\hat{\Omega}_{ij,N} = \{ (1 - \lambda)\hat{\sigma}_{\nu,ij} + \lambda \tilde{\omega}_{ij}, i, j = 1, \dots N \}.
$$
\n(7)

In the above expression $\hat{\sigma}_{\nu,ij} = \frac{1}{7}$ $\frac{1}{T} \sum_{\tau} (\nu_{i\tau} - \bar{\nu}_{i}) (\nu_{j\tau} - \bar{\nu}_{j})$ is the sample covariance, T being the number of quarterly nowcast error, $\lambda \in [0, 1]$ is the shrinkage intensity, and the shrinkage target is

estimated by assuming a compound symmetry covariance structure:

$$
\tilde{\omega}_{ij} = \begin{cases}\n\hat{\sigma}_{\nu, ii}, & i = j \\
\bar{r}\sqrt{\hat{\sigma}_{\nu, ii}}\sqrt{\hat{\sigma}_{\nu, jj}}, & i \neq j\n\end{cases}
$$

which postulates that the prediction errors have the same variance and are equally correlated with correlation coefficient $\bar{r} = \frac{2}{N-1}$ $\frac{2}{(N-1)N} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \frac{\hat{\sigma}_{ij}}{\sqrt{\hat{\sigma}_{\nu,ii}} \sqrt{\hat{\sigma}_{\nu,jj}}}$. Compound symmetry implies that the N nowcasting errors share a single common factor.

The parameter λ is estimated by minimizing the mean square nowcast error, e.g., performing a grid search over the unit interval or applying the closed form estimator by Ledoit and Wolf (2004a), reported in appendix B.

The choice of the shrinkage target has the following motivation: if λ gets close to one and the accuracy of the individual nowcasts or forecasts is the same, as measured by their prediction error variance, and equal weights averaging is optimal. Otherwise, if the prediction error variance is different, the weights take also into account the precision of the nowcast or forecast, with some shrinkage towards the average. If λ is less than one than the weights take into consideration also the correlation structure of the prediction errors. This prevents overrepresenting in the weighted average class of indicators that are more easily available, such as the soft indicators arising from consumer and business surveys.

4 The information set

The euro area GDP series is available at chained volumes from the National Quarterly Accounts compiled by Eurostat. Its observation frequency is quarterly; as for its publication schedule, it is released with a delay of one month for GDP (preliminary flash estimates) and 45 days (flash estimate) from the end of the reference quarter. These estimates are subsequently revised in the two successive months (respectively 65 and 100 days from the end of the quarter); only the final estimate is included in the published time series.

For estimating a monthly indicator of GDP we have collected $N = 523$ monthly indicators, selected from the Europa database according to the following criteria. The first criterion is a geographical one: we considered time series referring to the eurozone (19 countries) and its four largest economies: France (FR), Germany (DE), Italy (IT) and Spain (ES). Secondly, we aimed at achieving ample coverage of economic sectors (industry, construction, trade), expenditure (sales and consumer sentiment), monetary and financial conditions, the labour market, prices and business sentiment and expectations. The third criterion was timeliness and availability.

The selected monthly indicators are listed in appendix C, grouped according to their measurement domain: Building Permits and Civil Engineering (BP), Consumer Survey based Confidence Indicators (CS), Consumer Price Indices (CPI), Unemployment Rates (UR), Industrial Production (IP), Industry and Construction Surveys (ICS), Monetary and Financial Aggregates (MFA), Producer Price Index (PPI) and Turnover and Retail Sales (RS). Appendix C also reports the publication delay with respect to the reference month. The monetary and financial aggregates are compiled with a publication delay of around 30 days from the closing of the reference month. The time series in the IP and RS measurement domains are released about 45 days after the end of the reference month. Other indicators, such as those for construction and the labour market have a publication delay of about 70 days. The most timely are CS and ICS, the so-called *soft indicators*, which are

made available at the end of the reference month.

Figure 2 displays the distribution of the monthly indicators by country and measurement domain. It is evident that the information set is unbalanced: the Eurozone and Spain are underrepresented, with about 12\% and 14\% of the total number of time series, respectively. The measurement domains that are over represented are the soft indicators (CS and ICS) and industrial production (IP). The dataset has a ragged-edge structure, as the series may start at a later date and the availability of the latest values depends on the publication schedule. In general, most of the series start from January 1996, some from January 1998 (e.g. the Harmonized Unemployment Rate series) a few after January 2000. The quarterly GDP time series is available in chained volumes from the first quarter of 1996 up to the last quarter of 2018. The data were downloaded from the Datastream database on April 30, 2019. The estimate of GDP for the first quarter of 2019 was not yet available, being still preliminary.

Figure 2: Number of monthly indicators by geographical area and measurement domain.

5 Monthly GDP estimates for the euro area

Monthly GDP estimates, conditional on the information provided by each indicator and the available quarterly GDP figures, are produced by the state space methodology applied to the bivariate model (4), as outlined in section 2. All the results were obtained using Ox 7 (64–bit version) (see Doornik, 2009). The N different estimates are combined in this section to produce the historical estimates

$$
\hat{\mathbf{E}}_n(y_t) = \sum_{i=1}^N w_{it} \hat{\mathbf{E}}(y_t | \mathcal{Y}_n, \mathcal{X}_{in}), \ \ t = 1, \dots, n,
$$

using weights proportional to the conditional likelihood of the observed quarterly GDP series, $w_i \propto \exp(D_i/2)$, where D_i is given in (5).

The left panel of figure 3 displays the monthly GDP estimates $\hat{E}_n(y_t)$ (solid line) and the individual estimates $\hat{E}(y_t|\mathcal{Y}_n, \mathcal{X}_{in})$ (dotted lines). Within the available sample, the individual monthly GDP esimates satisfy the temporal aggregation constraints: $Y_{3\tau} = \sum_{j=0}^{2} \hat{E}(y_{3\tau-j} | \mathcal{Y}_n, \mathcal{X}_{in}),$ and this properties extends to their weighted average. The estimates display some variability around their average, which is higher during the great recession (2008-2009) and at the end of the sample. In fact, the last three estimates are nowcasts: the coefficient of variation of the estimate of monthly GDP of March 2019 is about 0.14%.

The right panel displays the estimated monthly series of annual growth rates,

$$
100[\hat{E}(y_t|\mathcal{Y}_n,\mathcal{X}_{in})/\hat{E}(y_{t-12}|\mathcal{Y}_n,\mathcal{X}_{in})-1],
$$

for $i = 1, \ldots, N$, (dots) and their weighted average (solid line). The variability is relatively small: the average standard error within the sample is about 0.03, and at the end of the sample the nowcasting standard deviation is 0.14.

Figure 3: Monthly historical estimates of GDP at market prices - chained volumes 2010 (left) and yearly growth rates (right). The solid line blue line represents the weighted average, where the weights are proportional to the conditional likelihood, $w_i \propto \exp(-0.5D_i)$.

The weights for combining the individual estimates are presented in figure 4. Different colors delimit the measurement domains in which the monthly indicators are divided. A zero weight in the plot reflects estimation failure (the optimization routine failed to converge); it may be due to the nature of the indicator, which may be available for a very short period or it may be heavily contaminated by outliers. It is worth noticing that the indicators belonging to the Industrial Production (green) and Turnover and Retail Sale (orange) sectors receive higher weights.

The selection of the weights for the aggregation of the individual cross-sectional estimates is a fundamental issue. As we will discuss in the next section, weighting according to the ability to explain the observed GDP figures is suitable for historical estimation, i.e., for a static problem involving a batch of time series. This can obscure the role of the available information as it accrues

to the econometrician, over-emphasizing the role of industrial production and turnover. Further insight into the issue can be obtained by performing a real time nowcasting experiment, which is the topic of the next section.

Figure 4: Likehood-based weights for the cross-sectional aggregation of the monthly GDP conditional mean estimates, $w_i \propto \exp(-0.5D_i)$.

6 Description of the pseudo real-time experiment

A recursive nowcasting and forecasting experiment has been conducted that mimics what a forecaster would perform in real time to track the dynamics of GDP. Unfortunately, the vintages of GDP estimates are available only starting from April 2016, which is the time when the new flash estimate was introduced, and thus our exercise can be considered only as a *pseudo* real time one, in the sense that rather than using the GDP and indicators data available at the time we condition on the final estimates compiled by Eurostat. While the revision of the monthly indicators is a minor problem and the size of the GDP revisions are much smaller than in the U.S., it would nevertheless be interesting to perform an analysis similar to Clements and Galvão (2017) for the euro area, which we leave for future research, when a longer record of the GDP vintages will be available.

The design of the recursive nowcasting and forecasting experiment carefully replicates the flow of economic information available in three typical situations, positioned at the end of the three months making up the quarter. The exercise starts at the end of January 2006 and terminates in December 2018. In the test sample, for every month m, $m = 1, 2, 3$, of reference quarter τ we evaluate $\hat{E}(y_{3\tau-j}|\mathcal{Y}_t, \mathcal{X}_{it})$, for $j = 0, 1, 2$, where $t = 3(\tau - 1) + m$, i.e., the estimated monthly GDP for the 3 consecutive months making up the quarter, using the information available at the end of the m-th month of the reference quarter. Hence, we construct the nowcast of quarterly GDP, $\hat{Y}_{\tau} = \sum_{j=0}^{2} \hat{E}(y_{3\tau-j} | \mathcal{Y}_t, \mathcal{X}_{it})$, and compare it to the actual GDP value to get the nowcasting error $\nu_{i\tau}^{(m)}$, $i = 1, \ldots, N$. Furthermore, we evaluate the forecasts of the value of GDP for the three monthls making up the next quarter $\tau + 1$, $\hat{E}(y_{3(\tau+1)-j}|\mathcal{Y}_t, \mathcal{X}_{it})$, for $j = 0, 1, 2$, aggregate them and compute the one-quarter-ahead forecast error referred to quarterly GDP, $\nu_{i,\tau+1}^{(m)*}$. The nowcast and one-quarter-ahead forecast errors constitute the basis for the aggregation weights and for the assessment of the methodology.

More specifically, starting from 2006 up to December 2018, we use the available data in real time to estimate recursively the bivariate models (4) and construct three sets of forecasts within a given quarter, obtained as follows.

- ❼ The first nowcast and forecast are made at the end of the first month of the reference quarter $(m = 1, i.e., January, April, July and October).$ The information available on the GDP of the reference quarter can be labelled as a "Small Information" set, since only the soft indicators (CS and ICS) are available up to the current month, whereas the hard indicators are available according to a ragged edge structure, such that, e.g., industrial production is available up to the last month of the previous quarter. As far as the target series is concerned, the flash estimate of the GDP of quarter $\tau - 1$ has been just published; since this is not available for a large part of the test sample (it becomes available only from April 2016), we replace it by the quarterly GDP figure published by Eurostat. Hence, the information set includes also the GDP series up to the previous quarter $(\tau - 1)$. The status of the information available in the first situation is visualized in figure 5.
- ❼ The second nowcast and forecast are made the last day of the second month of the reference quarter $(m = 2, i.e., February, May, August and November)$. The information set available is illustrated in figure 6; we refer to this as a "Medium Information" setting: along with the information on soft indicators, relative to the first and second month of the reference quarter, producer and consumer prices, and monetary and financial aggregates for the first month of the quarter become available. Moreover, the monthly information concerning quarter $\tau - 1$ is complete, except for building permits and production in construction.
- The third nowcast and forecast are made at the end of the reference quarter $(m = 3, \text{ March},$ June, September and December). A "Large Information" set is available on the reference quarter: not only the information on the previous quarter is complete, but also that concerning the current quarter is relatively rich, featuring the index of industrial production and retail sales for the first month of the quarter. See also figure 7.

Figure 5: Information available at the end of month $m = 1$ of the reference quarter for nowcasting and forecasting GDP of quarters τ and $\tau + 1$, respectively. The coloured squares signify that the information referring to the j-th month of the quarters $\tau - 1$ and τ is available at the time of producing the nowcast and forecast.

Figure 6: Information available at the end of month $m = 2$ of the reference quarter for nowcasting and forecasting GDP of quarter τ and $\tau + 1$, respectively. The coloured squares signify that the information referring to the j-th month of the quarters $\tau - 1$ and τ is available at the time of producing the nowcast and forecast.

Figure 7: Information available at the end of month $m = 3$ of the reference quarter for nowcasting and forecasting GDP of quarter τ and $\tau + 1$, respectively. The coloured squares signify that the information referring to the j-th month of the quarters $\tau - 1$ and τ is available at the time of producing the nowcast and forecast.

7 Empirical results

We evaluate the performance of the individual bivariate models and of the different cross-sectional aggregation scheme by the ability of nowcasting and forecasting one-step-ahead quarterly GDP in the test sample ranging from the beginning of 2006 to the end of 2018. The structure and the output of the recursive scheme are summarized in table 1: the first column is time at which the nowcast and forecast are made; the nature of the information set that is used is described in the second column. The third and fourth column state the estimated target variable, which is quarterly GDP.

Month	Information set	Nowcast	Forecast
$Jan-06$	Small	$Y_{2006:Q1}$	$Y_{2006:Q2}$
$Feb-06$	Medium	$Y_{2006:Q1}$	$Y_{2006:Q2}$
$Mar-06$	Large	$Y_{2006:Q1}$	$Y_{2006:Q2}$
$Apr-06$	Small	$Y_{2006:Q2}$	$Y_{2006:Q3}$
$May-06$	Medium	$Y_{2006:Q2}$	$Y_{2006:Q3}$
$Jun-06$	Large	$Y_{2006:Q2}$	$Y_{2006:Q3}$
Dec-18	Large	$Y_{2018:Q4}$	$Y_{2019:Q1}$

Table 1: Recursive scheme for nowcasting and forecasting quarterly GDP

NOTE: the first column reports the time at which the nowcast and forecast are made; the nature of the information set available is described in the second column. The third and fourth column state the estimated target variable, which is quarterly GDP, indexed by the reference year and quarter.

The N nowcasts and forecasts are averaged according to different schemes.

- 1. Averaging according to the conditional deviance in the training sample: the weights are proportional to $\exp(-0.5D_i)$, where D_i is computed according to 5, computed recursively on the training sample from 1996 to the time of the nowcast and forecast in table 1. Four different selection methods are investigated:
	- (a) DEV_N : all the N time series are considered.
	- (b) DEV_{10} : only the series with the 10 largest weights are considered.
	- (c) DEV30: only the series with the 30 largest weights are considered.
	- (d) DEV $_{50}$: only the series with the 50 largest weights are considered.
- 2. Minimum mean square error weights using the Ledoit and Wolf (LW) estimator of error covariance matrix in the test sample. To produce the nowcast and the forecast the data available at time t are split into an training sample, used for fitting the bivariate models 4 , and a test sample, consisting of the last three years of data before time t , which are used for

estimating the covariance matrix according to 7. The optimal weights are obtained according to 6. The shrinkage intensity is estimated by a grid search and using the closed form estimator by Ledoit and Wolf (2004a).

3. Simple averaging: all the series receive the same weight, 1/N.

The temporally aggregated nowcasts and forecasts of quarterly GDP are then compared using as a benchmark a univariate $AR(p)$ model that is fitted to the quarterly changes of GDP, whose order is selected according to the Bayesian Information Criterion (BIC). Since three years of data are used to estimated the weights for the second averaging scheme, the evaluation sample is 2008:Q1-2018:Q4.

Table 2 reports the root mean square error (RMSE) of the different averaging schemes relative to the AR benchmark. The columns S, M, L, stand for small, medium and large information, respectively, and refer to the size of the information available at the time of the forecast (month $m = 1, 2, 3$, of the reference quarter, see section 6). Numbers less than one imply that the method under consideration outperforms the benchmark.

As far as nowcasting is concerned, all the averaging methods outperform the benchmark. As it is expected, their performance improves systematically as m increases, i.e., as the information set on the reference quarter gets larger. Simple averaging (panel C of table 2) produces about the same results as deviance based averaging considering all the series $(DEV_N,$ first line of panel A). Variable screening according to the smallest conditional deviance yields a sizable reduction in the nowcasting error. Among the first class of averaging schemes the most effective is DEV_{10} , the nowcast based on the best performing 10 bivariate models, according to the conditional deviance in equation (5): when a large information set is available on the reference quarter, the root mean square error is two thirds of the benchmark. The best performing nowcast is obtained with the optimal weights in equation (6) estimated by the Ledoit and Wolf (2004a) methodology. The value of the closed form estimator of the shrinkage parameter, see appendix B, varies according to the recursive sample between 0.3 and 0.4, meaning that the compound symmetry shrinkage target does play a role in determining the cross-sectional aggregation weights. The reduction in the RMSE is quite sizable and statistically significant, when compared to the benchmark.

For forecasting GDP of quarter $\tau + 1$ using the information sets outlined in section 6, which include the GDP of quarter $\tau - 1$, the averaging schemes based on the in-sample performance (deviance, panel A) and equal weights averaging (panel C) do not outperform the benchmark, which is now represented by the two-step-ahead univariate $ARIMA(1,1,0)$ predictor of quarterly GDP. Howerever, when the weights are estimated on the basis of the out-of-sample performance, as for the Ledoit and Wolf estimator reported in panel B, significant improvements arise. The best performance is obtained by adopting the Ledoit and Wolf (2004a) closed form estimator of λ. For $m = 1$ (small information concerning the previous quarter) and $m = 2, 3$ (medium and large information) the RMSE reduction is about one fifth and one fourth, respectively, which is indicative of the fact that large accuracy gains can arise from averaging the N individual forecasts by a suitable weighting scheme.

To get more insights about the relative performance of the different predictors over time, figure 8 displays the 3-year rolling RMSE of the best performing nowcasting (left panels) and forecasting (right panels) methods belonging to the first and second class of averaging methods, for $m = 1, 2, 3$, as well as the rolling RMSE of the AR benchmark. It should be noticed that the first point shown in each graph refers to the RMSE of the years 2008-2010, which include the great recession. The subsequent data points are also affected by the sovereign debt crisis. This explains why there is a

Table 2: RMSE of the nowcasting and forecasting exercise

PANEL B: Ledoit - Wolf Optimal weights

PANEL A: Deviance based averaging

	Nowcasting			Forecasting		
	S $(m=1)$	$M(m=2)$	$L(m=3)$	$S(m=1)$	$M(m=2)$	$L(m=3)$
$\text{OPT}_{LW}^{\lambda=0.1}$	0.771	0.744	0.674	0.880	0.807	0.770 ^{\ddagger}
OPT $\lambda=0.2$	0.7501	0.718 [†]	$0.659+$	0.852	0.785	$0.756 \ddagger$
OPT $\lambda=0.3$	0.7471	0.7101	$0.654\dagger$	0.846	0.7831	0.758 [†]
$\text{OPT}_{LW}^{\lambda=0.4}$	0.7501	0.7091	$0.653+$	0.849	0.7911	0.7661
OPT $\lambda=0.5$	0.7561	0.7131	$0.656\dagger$	0.856	0.804 ₁	0.779 [†]
OPT $\lambda=0.6$	0.7641	0.7201	0.662 ₁	0.868	0.823	$0.797 \ddagger$
OPT $\lambda=0.7$	0.7751	0.7311	$0.671\dagger$	0.885	0.847	0.821
$OPT_{LW}^{\overline{\lambda}=0.8}$	0.7901	0.748 [†]	0.687 [†]	0.911	0.880	0.852
OPT $\lambda=0.9$	0.814 [†]	0.7751	$0.715\dagger$	0.953	0.929	0.899
OPT^{λ}_{I}	0.7301	0.712 [†]	$0.666\dagger$	0.833	0.7771	0.7511

PANEL C: Simple Average

NOTE: The entries in the table are the relative RMSE of each model over the AR benchmark selected recursively by BIC, computed in the test sample going from the first quarter of 2008 to the last quarter of 2018. An entry less than one indicates that the corresponding model outperforms the AR benchmark. The best performance across all models for a given forecast horizon appears in bold. ‡(†) indicates a p-value smaller than 0.1 (0.05), for the Diebold-Mariano test of the null hypothesis of equal forecasting accuracy with respect to the AR benchmark.

step in the RMSE in all the plots. It is evident from the plot that the accuracy gains associated with LW averaging over DEV averaging are larger during the the great recession and the the sovereign debt crises. Also, the former outperforms the benchmark almost uniformly. When it comes to forecasting, the best DEV averaging scheme shows the worst performance during the years 2008-2010.

Hence, it remains to explain why combining the same ingredients in a different way, forecast averaging yields so different results. In particular, it is interesting to understand which variables have largely contributed in reducing the nowcasting and forecasting errors, in what period and most importantly why, see Bok et al. (2018). The big picture can be obtained from the word clouds displayed in figure 9, which are constructed by attributing a larger font size and different colors to the series depending on the average weight w_i across the test sample, used to construct the nowcasts (the results for forecasting are not qualitatively dissimilar and are omitted). The most used variables are shown in red. Panels (a)-(c) refer to the best performing DEV averaging method and are populated mostly, if not exclusively, by a subset of monthly indicators belonging to the industrial production set, which remains stable as the information set becomes richer $(m = 1, 2, 3)$. Panels (d)-(f) refer to the best performing LW averaging scheme. Interestingly, in the first and second month of the quarter the information that is more relevant for nowcasting GDP is carried monthly indicators belonging to CS and ICS, like business expectations. When information enlarges $(m = 3)$, so that industrial production for the first month of the reference quarter is available (see figure 7), the indicators of the IP group become prominent. A likely cause of the differences between the two averaging schemes lies in the fact that the weights of the DEV consider the historical performance of the N nowcasting models, whereas the LW averaging scheme estimates the weights on the basis of the out-of-sample predictive performance.

Additional insight is obtained by figure 10, which displays the heatmaps of the cross-sectional aggregation weights for nowcasting and forecasting GDP over the test period, using a small information set $(m = 1)$; similar considerations applying to the other values of m. The distribution of the nowcasting weights is rather sparse, with an important role played by the industry and construction surveys, in particular during period up to 2012 including the great recession and the sovereign debt crisis; consumer confidence, turnover and retail sales, and industrial production tend to receive more relevance towards the end of the sample. As far as the forecasting weights are concerned, the distribution of the weights is less sparse. ICS and CS play a lesser role, especially for the last part of the sample and industrial production is more informative. The DEV averaging weights, not reported for brevity, are much more heterogeneous and stable, concentrating on the IP indicators.

8 Conclusions

The paper has proposed a model averaging methodology for nowcasting and forecasting monthly GDP with high-dimensional time series, which, given the availability of a large number N of monthly indicators, pools the estimates of monthly GDP arising from the N bivariate models of GDP and each indicator in turn. The bivariate model is a mixed-frequency dynamic factor model, that incorporates the temporal aggregation constraints.

We have considered alternative model averaging strategies and conducted a recursive forecasting experiment for their assessment, which enables the following conclusions to be drawn.

Figure 8: 3-year rolling RMSEs of best performing DEV averaging method (solid line), LW averaging method (solid line with circles) and AR benchmark (dash and dotted line). Panels (a), (b) and (c) refer to the nowcasting performance using the information sets available at month $m = 1, 2, 3$, of the reference quarter,
reconctively. Panels (d), (a) and (f) to the one quarter respectively. Panels (d), (e) and (f) to the one-quarter-ahead predictions using the information sets available at month $m = 1, 2, 3$, respectively.

Figure 9: Each figure shows a cloud of words related to the variable names used in the forecasting exercise (January 2008 - December 2018). Panels (a), (b) and (c) refer to the nowcasting performance using the information sets available at month $m = 1, 2, 3$, of the reference quarter, respectively. Panels (d), (e) and (f) to the one-quarter-ahead predictions using the information sets available at month $m = 1, 2, 3$, respectively.

Figure 10: Best performing LW averaging method: heatmaps of the cross-sectional aggregation weights in the test sample for producing the nowcast (a) and the forecast (b), on the basis of the information set available in month $m = 1$ of quarter τ .

- The optimal model averaging strategy depends on the objective of the analysis. While for historical estimation of monthly GDP the weights can be validly obtained by the in sample predictive performance, as measured by the conditional deviance of the quarterly GDP estimation error, for nowcasting and forecasting GDP in real time the best pooling method estimates the minimum mean square error weights using a Ledoit and Wolf estimator of the out-of-sample prediction error covariance matrix.
- The optimal combinations of the individual nowcasts and forecasts need to adapt to the information available at the time the prediction is made: for instance, if the target is nowcasting the GDP of the current quarter and the prediction is made at the end of the first month of the quarter, the consumer and business survey indicators provide preliminary and timely indication of the most recent evolution, and thus receive more weight, compared to when the nowcast is made at the end of the quarter, for which the information on hard indicators has already accrued.
- The optimal weights vary according to the phase of the business cycle. During the great recession and the sovereign debt crisis the soft indicators played a more relevant role, providing a more timely signal of adverse economic conditions.
- While soft indicators can make up for the missing information on hard indicators for nowcasting, their role is less relevant for forecasting.
- The adoption of the unweighted average is not supported by the empirical evidence: while this is often considered as a standard in the context of forecast combination, in our case it proves suboptimal for nowcasting and forecasting GDP in real time, as the information available has a different role.

The advantages of our approach are twofold: it makes it feasible handling a very large number of time series, as the same model is estimated as many times as there are indicators in an efficient and timely way. Secondly, it is possible to assess the role of the indicators by evaluating the weight by which they concur to the estimation of monthly GDP. This evaluation has lead to several interesting discoveries.

We plan to extend this approach to produce monthly estimates of the individual GDP components, by output and expenditure type, and to obtain an indirect GDP estimate by aggregating the GDP components according to the the annual overlap method, that is used in the production of the national accounts, and that ensures the consistency in cross-sectional aggregation with the published total GDP at market prices. The direct and indirect approaches can be performed in conjunction and compared. The obvious advantage of the indirect approach is the ability to decompose the sources of growth (growth accounting).

Another important extension deals with density forecasting with many potential indicators, following Aastveit et al. (2014) and Proietti et al. (2017). In particular, the nowcasting and forecasting densities arising from the bivariate models could be aggregated according to approach introduced by Gneiting and Ranjan (2013).

A Statistical treatment

A.1 State space representation of the bivariate complete data model

The state space representation of the monthly model (4) is constructed as follows. We start from the state space model for the $ARMA(1,1)$ common component χ_{it} :

$$
\chi_{it} = \mathbf{e}' \mathbf{g}_t, \quad \mathbf{g}_t = \mathbf{T}_{\chi} \mathbf{g}_{t-1} + \vartheta \eta_t,
$$
\n(8)

where

$$
\mathbf{e} = \left[\begin{array}{c} 1 \\ 0 \end{array} \right], \quad \mathbf{T}_{\chi} = \left[\begin{array}{c} \phi & 1 \\ 0 & 0 \end{array} \right], \quad \boldsymbol{\vartheta} = \left[\begin{array}{c} 1 \\ \boldsymbol{\vartheta} \end{array} \right].
$$

Let us denote $\mathbf{y}_t = [x_{it} \ y_t]'$, $\mathbf{m} = [m_i \ m]'$ and $\boldsymbol{\xi}_t = [\xi_{it} \ \xi_{yt}]'$. Then (4) is written

$$
\begin{array}{rcl}\mathbf{y}_t & = & \mathbf{y}_{t-1} + \mathbf{m} + \vartheta \mathbf{e}' \mathbf{g}_t + \boldsymbol{\xi}_t \\
 & = & \mathbf{y}_{t-1} + \mathbf{m} + \vartheta \mathbf{e}' \mathbf{T}_\chi \mathbf{g}_{t-1} + \vartheta \eta_t + \boldsymbol{\xi}_t,\n\end{array}
$$

where the second line follows by direct substitution from 8.

Defining

$$
\alpha_t = \begin{bmatrix} \mathbf{y}_t \\ \mathbf{g}_t \\ \xi_t \end{bmatrix}, \quad \beta = \begin{bmatrix} \mathbf{y}_0 \\ \mathbf{m} \end{bmatrix}, \quad \epsilon_t = \begin{bmatrix} \eta_t \\ \xi_t \end{bmatrix},
$$

and the system matrices $\mathbf{Z} = [\mathbf{I} \ \mathbf{0} \ \mathbf{0}],$

$$
\mathbf{T} = \left[\begin{array}{ccc} \mathbf{I} & \vartheta e' \mathbf{T}_\chi & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_\chi & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array} \right], \quad \mathbf{W} = \left[\begin{array}{ccc} \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array} \right], \quad \mathbf{H} = \left[\begin{array}{ccc} \vartheta & \mathbf{I} \\ \vartheta & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{array} \right],
$$

we obtain the state space form of the complete model (4),

$$
\mathbf{y}_t = \mathbf{Z}\boldsymbol{\alpha}_t, \quad \boldsymbol{\alpha}_t = \mathbf{T}\boldsymbol{\alpha}_{t-1} + \mathbf{W}\boldsymbol{\beta} + \mathbf{H}\boldsymbol{\epsilon}_t,\tag{9}
$$

where the second equation holds for $t = 2, \ldots, n$. The initial state vector is

$$
\alpha_1 = \mathbf{W}_1 \boldsymbol{\beta} + \mathbf{H} \tilde{\boldsymbol{\epsilon}}_1, \tag{10}
$$

where

$$
\mathbf{W}_1 = \left[\begin{array}{cc} \mathbf{I} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array} \right], \quad \tilde{\boldsymbol{\epsilon}} = \left[\begin{array}{c} \mathbf{e}'\mathbf{g}_1 \\ \boldsymbol{\xi}_1 \end{array} \right],
$$

so that $\tilde{\epsilon} \sim N\left(0, \text{diag}\left(\sigma_{\eta}^2\left[1+(\phi+\vartheta)^2/(1-\phi^2)\right], \sigma_i^2, \sigma_y^2\right)\right)$.

The elements of β are taken as diffuse, in accordance to de Jong (1991), i.e. it is assumed that $\beta \sim N(0, V)$, where V^{-1} converges to a zero matrix.

A.2 Temporal aggregation

The SSF of the monthly model is now modified so as to take into account temporal aggregation. The second element of the vector $\mathbf{y}_t = [x_{it}, y_t]'$, corresponding to the GDP component, is not observed at time t, but at times $t = 3\tau, \tau = 1, 2, \ldots, [n/3]$, we observe the quarterly total

$$
Y_{\tau} = y_{3\tau} + y_{3\tau - 1} + y_{3\tau - 2},
$$

where τ denotes the quarters and $\lceil \cdot \rceil$ denotes integer division.

To incorporate this observational constraint, following Harvey (1989) we define the cumulator variable, y_t^c :

$$
y_t^c = \rho_t y_{t-1}^c + y_t, \quad \rho_t = \begin{cases} 0, & \text{if } t = 3(\tau - 1) + 1, \\ 1, & \text{otherwise.} \end{cases}
$$

At times $t = 3\tau$ this coincides with the (observed) aggregated series, so that only a systematic sample of y_t^c is available for the month corresponding to the end of the quarter (March, June, September and December).

Partitioning $\mathbf{Z} = [\mathbf{z}_1, \mathbf{z}_2]'$, substituting $y_t = \mathbf{z}'_2 \boldsymbol{\alpha}_t$ in the expression for y_t^c , and using (9) gives $y_t^c = \rho_t y_{t-1}^c + \mathbf{z}_2' \mathbf{T} \alpha_{t-1} + \mathbf{z}_2' \mathbf{W} \beta + \mathbf{z}_2' \mathbf{H} \epsilon_t$. The cumulator y_t^c is then used to create new augmented state and observation vectors, α_t^* and y_t^{\dagger} t_i , respectively:

$$
\boldsymbol{\alpha}^*_t = \left[\begin{array}{c} \boldsymbol{\alpha}_t \\ y^c_t \end{array} \right], \quad \mathbf{y}^{\dagger}_t = \left[\begin{array}{c} x_{it} \\ y^c_t \end{array} \right].
$$

The relevant SSF has measurement and transition equations given respectively by:

$$
\mathbf{y}_t^{\dagger} = \mathbf{Z}^* \alpha_t^*, \quad \alpha_t^* = \mathbf{T}_t^* \alpha_{t-1}^* + \mathbf{W}_t^* \boldsymbol{\beta} + \mathbf{H}^* \boldsymbol{\epsilon}_t,\tag{11}
$$

where the latter holds for $t = 2, \ldots, n$. The system matrices are

$$
\mathbf{Z}^* = \begin{bmatrix} \mathbf{z}'_1 & 0 \\ \mathbf{0}' & 1 \end{bmatrix}, \quad \mathbf{T}_t^* = \begin{bmatrix} \mathbf{T} & \mathbf{0} \\ \mathbf{z}'_2 \mathbf{T} & \rho_t \end{bmatrix},
$$

$$
\mathbf{W}_t^* = \begin{bmatrix} \mathbf{W} \\ \mathbf{z}'_2 \mathbf{W} \end{bmatrix}, \quad \mathbf{H}^* = \begin{bmatrix} \mathbf{H} \\ \mathbf{z}'_2 \mathbf{H} \end{bmatrix}.
$$
(12)

For $t = 1$ the state vector is

 $\boldsymbol{\alpha}^*_1 = \mathbf{W}^*_1\boldsymbol{\beta} + \mathbf{H}^*$ $\tilde{\epsilon}_1,$ (13)

with

$$
\mathbf{W}_1^*=\left[\begin{array}{c} \mathbf{W}_1 \\ \mathbf{z}_2'\mathbf{W}_1 \end{array}\right]
$$

.

A.3 Estimation

The state space model $(11)–(12)$ is linear and, assuming that the disturbances have a Gaussian distribution, the unknown parameters can be estimated by maximum likelihood using the prediction error decomposition performed by the Kalman filter. Given the parameter values, the Kalman filter and smoother (KFS) will provide the minimum mean square error estimates of the states α_t^* . See Harvey (1989), Durbin and Koopman (2012), and the next section for details.

The KFS thus also provides the best linear estimate of the sequence $\{y_t^c, t = 1, \ldots, n\}$, given the available observed time series. The estimates of y_t^c can be then "decumulated" using $y_t = y_t^c - \rho_t y_{t-1}^c$, so as to be converted into estimates of y_t , i.e. the monthly indicator of the GDP component.

Note that, in order to take temporal aggregation into account, the 2×1 vectors with observations \mathbf{y}^{\dagger}_t $t, t = 1, \ldots, n$, have been defined in section A.2, such that the series in second position is missing systematically in the first and second month of each quarter.

The sequential processing estimation method is based on converting the original multivariate state space model into a univariate one. The measurement equation for the i-th element of the vector \mathbf{y}_t^{\dagger} $_{t}^{+}$ is:

$$
y_{t,i}^{\dagger} = \mathbf{z}_i^{*'} \alpha_{t,i}^*, \quad t = 1, \dots, n, \quad i = 1, 2,
$$
\n(14)

where $\mathbf{z}_{i}^{*'}$ ^{*'} denotes the *i*-th row of **Z**^{*}. Notice also that $y_{t,1}^{\dagger} = x_{it}$. The transition equation is

$$
\begin{array}{rcl}\n\alpha_{t,1}^* &=& \mathbf{T}_t^* \alpha_{t-1,N}^* + \mathbf{W}_t^* \boldsymbol{\beta} + \mathbf{H}^* \boldsymbol{\epsilon}_{t,1}, \quad i = 1, \\
\alpha_{t,i}^* &=& \alpha_{t,i-1}^*, \qquad i = 2,\n\end{array} \tag{15}
$$

with $\epsilon_{t,1} \sim N(0, \Sigma_{\epsilon}), \Sigma_{\epsilon} = \text{diag}(\sigma_{\eta}^2, \sigma_{i}^2, \sigma_{y}^2) =$. The state space form is completed by the specification of the initial state vector, that, according to (13), is written as $\alpha_{1,1}^* = \mathbf{W}_1^* \boldsymbol{\beta} + \mathbf{H}_1^* \tilde{\boldsymbol{\epsilon}}_{1,1}$, where $\text{Var}(\tilde{\boldsymbol{\epsilon}}_{1,1})$ is equal to $\Sigma_{\tilde{\epsilon}} = \text{diag}(\sigma_{\eta}^2 \left[1 + (\phi + \vartheta)^2 / (1 - \phi^2)\right], \sigma_i^2, \sigma_y^2)$, which was given at the end of Section A.2. Using the state space model (14) and (15), the Kalman filter with sequential processing can be applied to obtain the predictions of the state vector and its estimation error covariance matrix. Estimation of the unknown parameters is carried out by the maximum likelihood via the prediction error decomposition. These algorithms, as well as the algorithms for obtaining the real–time and smoothed estimates of the state vector $\alpha_{t,i}^*$, and its covariance matrix $\mathbf{P}_{t,i}^*$, are illustrated in this appendix.

A.4 Augmented Kalman filter with sequential processing

The augmented Kalman filter computes recursively the predictions of the state vector and its covariance matrix. Estimation of the unknown parameters is carried out by the maximum likelihood via prediction error decomposition.

The augmented Kalman filter, accounting for the presence of missing values, is given by the following definitions and recursive formulae. The initial conditions are set equal to

$$
\mathbf{a}_{1,1}^* = \mathbf{0}, \ \mathbf{A}_{1,1}^* = \mathbf{W}_1^*, \ \mathbf{P}_{1,1}^* = \mathbf{H}_1^* \mathbf{\Sigma}_{\tilde{\epsilon}} \mathbf{H}_1^{*'}, \ q_{1,1} = 0, \ \mathbf{s}_{1,1} = \mathbf{0}, \ \mathbf{S}_{1,1} = \mathbf{0}, \ n_o = 0, \ d_{1,1} = 0.
$$

The initial state vector can be thus written as $\alpha_{1,1}^* = \mathbf{a}_{1,1}^* + \mathbf{A}_{1,1}^* \boldsymbol{\beta} + \mathbf{H}_1^* \tilde{\boldsymbol{\epsilon}}_{1,1}$. Then, for $t = 1, \ldots, n$,

 $i = 1, \ldots, N - 1$, if $y_{t,i}^{\dagger}$ is observed:

$$
\nu_{t,i} = y_{t,i}^{\dagger} - \mathbf{z}_{i}^{*'} \mathbf{a}_{t,i}^{*}, \qquad \mathbf{V}_{t,i}' = -\mathbf{z}_{i}^{*'} \mathbf{A}_{t,i}^{*}, \nf_{t,i} = \mathbf{z}_{i}^{*'} \mathbf{P}_{t,i}^{*} \mathbf{z}_{i}^{*}, \qquad \mathbf{K}_{t,i+1} = \mathbf{P}_{t,i}^{*} \mathbf{z}_{i}^{*}/f_{t,i}, \n\mathbf{a}_{t,i+1}^{*} = \mathbf{a}_{t,i}^{*} + \mathbf{K}_{t,i+1} \nu_{t,i}, \qquad \mathbf{A}_{t,i+1}^{*} = \mathbf{A}_{t,i}^{*} + \mathbf{K}_{t,i+1} \mathbf{V}_{t,i}'^{*}, \n\mathbf{P}_{t,i+1}^{*} = \mathbf{P}_{t,i}^{*} - \mathbf{K}_{t,i+1} \mathbf{K}_{t,i+1}^{t} f_{t,i}, \ng_{t,i+1} = g_{t,i} + \nu_{t,i}^{2}/f_{t,i}, \qquad d_{t,i+1} = d_{t,i} + \ln f_{t,i}, \n\mathbf{s}_{t,i+1} = \mathbf{s}_{t,i} + \mathbf{V}_{t,i} \nu_{t,i}/f_{t,i}, \qquad \mathbf{S}_{t,i+1} = \mathbf{S}_{t,i} + \mathbf{V}_{t,i} \mathbf{V}_{t,i}/f_{t,i}, \nn_{o} = n_{o} + 1.
$$
\n(16)

 $\mathbf{V}_{t,i}$ is a vector with 4 elements; $\mathbf{A}_{t,i}^*$ is a $(\mathcal{M}+1)\times(4)$ matrix, and n_o counts the number of effective observations. Else, if $y_{t,i}^{\dagger}$ is missing:

$$
\mathbf{a}_{t,i+1}^{*} = \mathbf{a}_{t,i}^{*}, \qquad \mathbf{A}_{t,i+1}^{*} = \mathbf{A}_{t,i}^{*}, \n\mathbf{P}_{t,i+1}^{*} = \mathbf{P}_{t,i}^{*}, \nq_{t,i+1} = q_{t,i}, \qquad d_{t,i+1} = d_{t,i}, \n\mathbf{s}_{t,i+1} = \mathbf{s}_{t,i}, \qquad \mathbf{S}_{t,i+1} = \mathbf{S}_{t,i}.
$$
\n(17)

Then, for $i=2$, if $y_{t,N}^{\dagger}$ is observed:

$$
\nu_{t,N} = y_{t,N}^{\dagger} - \mathbf{z}_{N}^{*} \mathbf{a}_{t,N}^{*},
$$
\n
$$
f_{t,N} = \mathbf{z}_{N}^{*} \mathbf{P}_{t,N}^{*} \mathbf{z}_{N}^{*},
$$
\n
$$
f_{t,N} = \mathbf{z}_{N}^{*} \mathbf{P}_{t,N}^{*} \mathbf{z}_{N}^{*},
$$
\n
$$
\mathbf{K}_{t+1,1} = \mathbf{T}_{t+1}^{*} \mathbf{P}_{t,N}^{*} \mathbf{z}_{N}^{*}/f_{t,N},
$$
\n
$$
\mathbf{a}_{t+1,1}^{*} = \mathbf{T}_{t+1}^{*} \mathbf{a}_{t,N}^{*} + \mathbf{K}_{t+1,1} \nu_{t,N},
$$
\n
$$
\mathbf{A}_{t+1,1}^{*} = \mathbf{W}_{t+1}^{*} + \mathbf{T}_{t+1}^{*} \mathbf{A}_{t,N}^{*} + \mathbf{K}_{t+1,1} \mathbf{V}_{t,N}',
$$
\n
$$
\mathbf{P}_{t+1,1}^{*} = \mathbf{T}_{t+1}^{*} \mathbf{P}_{t,N}^{*} \mathbf{T}_{t+1}^{*} + \mathbf{H}^{*} \mathbf{\Sigma}_{\epsilon} \mathbf{H}^{*} - \mathbf{K}_{t+1,1} \mathbf{K}_{t+1,1} f_{t,N},
$$
\n
$$
g_{t+1,1} = g_{t,N} + \nu_{t,N}^{2} / f_{t,N},
$$
\n
$$
\mathbf{s}_{t+1,1} = \mathbf{s}_{t,N} + \mathbf{V}_{t,N} \nu_{t,N} / f_{t,N},
$$
\n
$$
\mathbf{S}_{t+1,1} = \mathbf{S}_{t,N} + \mathbf{V}_{t,N} \mathbf{V}_{t,N} / f_{t,N},
$$
\n
$$
n_{o} = n_{o} + 1.
$$
\n(18)

Else, if $y_{t,N}^{\dagger}$ is missing:

$$
\mathbf{a}_{t+1,1}^{*} = \mathbf{T}_{t+1}^{*} \mathbf{a}_{t,N}^{*}, \quad \mathbf{A}_{t+1,1}^{*} = \mathbf{W}_{t+1}^{*} + \mathbf{T}_{t+1}^{*} \mathbf{A}_{t,N}^{*}, \n\mathbf{P}_{t+1,1}^{*} = \mathbf{T}_{t+1}^{*} \mathbf{P}_{t,N}^{*} \mathbf{T}_{t+1}^{*'} + \mathbf{H}^{*} \mathbf{\Sigma}_{\epsilon} \mathbf{H}^{*'}, \nq_{t+1,1} = q_{t,N}, \quad d_{t+1,1} = d_{t,N}, \n\mathbf{s}_{t+1,1} = \mathbf{s}_{t,N}, \quad \mathbf{S}_{t+1,1} = \mathbf{S}_{t,N}.
$$
\n(19)

The diffuse estimate of the vector β , and its covariance matrix, are, respectively,

$$
\hat{\boldsymbol{\beta}} = \mathbf{S}_{n+1,1}^{-1} \mathbf{s}_{n+1,1}, \quad \text{Var}(\hat{\boldsymbol{\beta}}) = \mathbf{S}_{n+1,1}^{-1}.
$$
\n(20)

The diffuse likelihood, based on de Jong (1991) and denoted \mathcal{L}_{∞} , takes the expression:

$$
\mathcal{L}_{\infty} = -0.5 \left[d_{n+1,1} + (n_o - K) \ln(2\pi) + \ln |\mathbf{S}_{n+1,1}| + q_{n+1,1} - \mathbf{s}'_{n+1,1} \mathbf{S}_{n+1,1}^{-1} \mathbf{s}_{n+1,1} \right].
$$
 (21)

The latter is maximised with respect to the unknown parameters.

Diagnostics and goodness of fit for the GDP component are based on the conditional innovations,

that are given by $\tilde{\nu}_{t,i} = \nu_{t,i} + \mathbf{V}'_{t,i} \mathbf{S}_{t,i}^{-1} \mathbf{s}_{t,i}$, for $i = 2$, with variance $\tilde{f}_{t,i} = f_{t,i} + \mathbf{V}'_{t,i} \mathbf{S}_{t,i}^{-1} \mathbf{V}_{t,i}$. The innovations have the following interpretation: $\tilde{\nu}_{t,i} = y_{t,i}^{\dagger} - \text{E}(y_{t,i}^{\dagger}|\mathbf{Y}_{t}^{\dagger})$ $_{t-1}^{\dagger}, y_{t,j}^{\dagger}, j < i$, where \mathbf{Y}_{t}^{\dagger} denotes the information set $\{y_1^{\dagger}$ $\frac{1}{1}, \ldots, \mathbf{y}_{t}^{\dagger}$ [†]_t}. The standardised innovations, $\tilde{\nu}_{t,i}/\sqrt{\tilde{f}_{t,i}}$ can be used to check for residual autocorrelation and departure from the normality assumption.

A.5 Real–time and smoothed estimates

This appendix explains how the real–time and smoothed estimates of the state vector $\alpha_{t,i}^*$ and its covariance matrix $\mathbf{P}_{t,i}^*$ are obtained, based on the quantities derived from the Kalman filter recursions presented in Appendix A.4.

The filtered, or real–time, estimates of the state vector and its estimation error matrix are given by:

$$
\tilde{\boldsymbol{\alpha}}_{t,i}^* = \mathrm{E}(\boldsymbol{\alpha}_t^* | \mathbf{Y}_{t-1}^\dagger, y_{t,j}^\dagger, j \leq i), \quad \tilde{\mathbf{P}}_{t,i}^* = \mathrm{Var}(\boldsymbol{\alpha}_t^* | \mathbf{Y}_{t-1}^\dagger, y_{t,j}^\dagger, j \leq i).
$$

If $y_{t,i}^{\dagger}$ is observed and $i < N$, these quantities are computed as follows:

$$
\tilde{\boldsymbol{\alpha}}_{t,i}^* = \mathbf{a}_{t,i}^* + \mathbf{A}_{t,i}^*\mathbf{S}_{t,i+1}^{-1}\mathbf{s}_{t,i+1} + \mathbf{P}_{t,i}^*\mathbf{z}_i^*\tilde{\nu}_{t,i}/f_{t,i}, \quad \tilde{\mathbf{P}}_{t,i}^* = \mathbf{P}_{t,i}^* + \mathbf{A}_{t,i}^*\mathbf{S}_{t,i+1}^{-1}\mathbf{A}_{t,i}^{*'} - \mathbf{P}_{t,i}^*\mathbf{z}_i^*\mathbf{z}_i^{*'}\mathbf{P}_{t,i}^*/f_{t,i}.
$$

For $i = N$,

$$
\tilde{\boldsymbol{\alpha}}_{t,N}^* = \mathbf{a}_{t,N}^* + \mathbf{A}_{t,N}^*\mathbf{S}_{t+1,1}^{-1}\mathbf{s}_{t+1,1} + \mathbf{P}_{t,N}^*\mathbf{z}_N^*\tilde{\nu}_{t,N}/f_{t,N}, \quad \tilde{\mathbf{P}}_{t,N}^* = \mathbf{P}_{t,N}^* + \mathbf{A}_{t,N}^*\mathbf{S}_{t+1,1}^{-1}\mathbf{A}_{t,N}^{*'} - \mathbf{P}_{t,N}^*\mathbf{z}_N^*\mathbf{z}_N^*\mathbf{P}_{t,N}^*/f_{t,N}.
$$

For $i = 1$, $\tilde{\alpha}_{t,i}^*$ coincides with the one-step-ahead prediction $\tilde{\alpha}_{t,i+1}^* = E(\alpha_t^* | Y_t^{\dagger})$ $_{t-1}^{\dagger}, y_{t,j}^{\dagger}, j \leq i$) as the transition equation is $\alpha^*_{t,i} = \alpha^*_{t,i-1}, i = 2$. For $i = 2$,

$$
\begin{array}{lll} \tilde{\boldsymbol{\alpha}}^{*}_{t+1,1} & = & \boldsymbol{\rm a}^{*}_{t+1,1} + \boldsymbol{\rm A}^{*}_{t+1,1} \boldsymbol{\rm S}^{-1}_{t+1,1} \boldsymbol{\rm s}_{t+1,1} + \boldsymbol{\rm P}^{*}_{t+1,1} \boldsymbol{\rm z}^{*}_{1} \tilde{\nu}_{t,N} / f_{t,N}, \\ \tilde{\boldsymbol{\rm P}}^{*}_{t+1,1} & = & \boldsymbol{\rm P}^{*}_{t+1,1} + \boldsymbol{\rm A}^{*}_{t+1,1} \boldsymbol{\rm S}^{-1}_{t+1,1} \boldsymbol{\rm A}^{*'}_{t+1,1} - \boldsymbol{\rm P}^{*}_{t+1,1} \boldsymbol{\rm z}^{*}_{1} \boldsymbol{\rm z}^{*'}_{1} \boldsymbol{\rm P}^{*}_{t+1,1} / f_{t,N}, \end{array}
$$

are respectively the one–step–ahead prediction $\tilde{\boldsymbol{\alpha}}_{t+1,1}^* = \text{E}(\boldsymbol{\alpha}_{t+1}^*|\textbf{Y}^{\dagger}_{t}$ t_{t-1}) and the predictive variance $\tilde{\mathbf{P}}_{t+1,1}^* = \text{Var}(\boldsymbol{\alpha}_{t+1}^*|\mathbf{Y}^{\dagger}_t)$ (t_{t-1}^{\dagger}) . The corresponding expressions for $y_{t,i}^{\dagger}$ missing are straightforward.

The smoothed estimates are obtained from the augmented smoothing algorithm proposed by de Jong (1988), appropriately adapted here to handle missing values and sequential processing of the observations. Defining $\mathbf{r}_{n,2} = \mathbf{0}, \mathbf{R}_{n,2} = \mathbf{0}, \mathbf{N}_{n,2} = \mathbf{0}$, for $t = n, \ldots, 1$, and $i = 2, 1$, if $y_{t,i}^{\dagger}$ is available:

$$
\begin{array}{lll} \mathbf{L}_{t,i} & = & \mathbf{I}-\mathbf{K}_{t,i}{\mathbf{z}_{i}^{*}}'\\ \mathbf{r}_{t,i-1} & = & \mathbf{z}_{i}^{*}\nu_{t,i}/f_{t,i}+\mathbf{L}_{t,i}\mathbf{r}_{t,i}, \ \ \mathbf{R}_{t,i-1}=\mathbf{z}_{i}^{*}\mathbf{V}_{t,i}'/f_{t,i}+\mathbf{L}_{t,i}\mathbf{R}_{t,i},\\ \mathbf{N}_{t,i-1} & = & \mathbf{z}_{i}^{*}\mathbf{z}_{i}^{*}/f_{t,i}+\mathbf{L}_{t,i}\mathbf{N}_{t,i}\mathbf{L}_{t,i}'. \end{array}
$$

Else, if $y_{t,i}^{\dagger}$ is missing,

$$
\begin{aligned} \mathbf{r}_{t,i-1} &= \mathbf{r}_{t,i}, \ \ \mathbf{R}_{t,i-1} = \mathbf{R}_{t,i}, \quad \mathbf{N}_{t,i-1} = \mathbf{N}_{t,i}, \\ \mathbf{r}_{t-1,N} &= \mathbf{T}_{t+1}^{*'} \mathbf{r}_{t,i}, \ \ \mathbf{R}_{t,i-1} = \mathbf{T}_{t+1}^{*'} \mathbf{R}_{t,i}, \quad \mathbf{N}_{t,i-1} = \mathbf{T}_{t+1}^{*'} \mathbf{N}_{t,i} \mathbf{T}_{t+1}^{*} .\end{aligned}
$$

The smoothed estimates are obtained as follows:

$$
\begin{array}{rcl}\tilde{\boldsymbol{\alpha}}^{*}_{t|n} & = & \boldsymbol{\rm a}^{*}_{t,1}+\boldsymbol{\rm A}^{*}_{t,1}\hat{\boldsymbol{\beta}}+\boldsymbol{\rm P}^{*}_{t,1}(\boldsymbol{\rm r}_{t-1,N}+\boldsymbol{\rm R}_{t-1,N}\hat{\boldsymbol{\beta}}),\\ \boldsymbol{\rm P}^{*}_{t|n} & = & \boldsymbol{\rm P}^{*}_{t,1}+\boldsymbol{\rm A}^{*}_{t,1}\boldsymbol{\rm S}^{-1}_{n+1}\boldsymbol{\rm A}^{*'}_{t,1}-\boldsymbol{\rm P}^{*}_{t,1}\boldsymbol{\rm N}_{t-1,N}\boldsymbol{\rm P}^{*}_{t,1},\end{array}
$$

where $\hat{\boldsymbol{\beta}}$ is as defined in (20).

B Optimal estimator of the shrinkage intensity parameter

Ledoit and Wolf (2004a) have derived the optimal estimator of the shrinkage intensity parameter $\lambda \in [0,1].$

Let $q_{ij,\tau} = (\nu_{i\tau} - \bar{\nu}_i)(\nu_{j\tau} - \bar{\nu}_j) - \hat{\sigma}_{ij}$ and define

$$
\hat{\pi}_{ij} = \frac{1}{T} \sum_{\tau} q_{ij,\tau}^2, \quad \hat{t}_{ii,ij} = \frac{1}{T} \sum_{\tau} q_{ii,\tau} q_{ij} \quad i,j = 1,\dots,N,
$$

$$
\hat{\pi} = \sum_{i}^{N} \sum_{j}^{N} \hat{\pi}_{ij}, \quad \hat{\rho} = \sum_{i=1}^{N} \hat{\pi}_{ii} + \sum_{i=1}^{N} \sum_{j=1,j \neq i}^{N} \frac{\bar{r}}{2} \left(\sqrt{\frac{\hat{\sigma}_{jj}}{\hat{\sigma}_{ii}}} \hat{t}_{ii,ij} + \sqrt{\frac{\hat{\sigma}_{ii}}{\hat{\sigma}_{jj}}} \hat{t}_{jj,ij} \right),
$$

$$
N \quad N
$$

and

$$
\hat{\gamma} = \sum_{i=1}^{N} \sum_{j=1}^{N} (\hat{\sigma}_{ij} - \tilde{\omega}_{ij})^2,
$$

where $\hat{\pi}$ estimates the sum of the variances of the elements of the sample covariance matrix, $\hat{\rho}$ estimates the sum of the asymptotic covariances of the elements of the shrinkage target with those of the sample covariance matrix, and $\hat{\gamma}$ estimates the deviation of the shrinkage target from the sample covariance matrix. Then, a consistent estimator of λ is

$$
\lambda^* = \max \left\{ 0, \min \left\{ \frac{1}{T} \frac{\hat{\pi} - \hat{\rho}}{\hat{\gamma}}, 1 \right\} \right\}.
$$

C List of monthly indicators

Hereby we provide the complete list of the monthly indicators used for the estimation of the euro area monthly GDP. The third column provides the reference area and the last column the publication delay in months with respect to their reference month.

Continued from previous page

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