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Suboptimality of Probability Matching – A Formal Proof, a Graphical Analysis and an Impulse Balance Interpretation

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Suboptimality of probability matching — A formal proof, a graphical analysis and an impulse balance interpretation

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Abstract

We prove suboptimality of probability matching in prediction tasks with an arbitrary (finite) number of outcomes and repetitions. For the popular case of binary prediction tasks, we also provide a graphical representation of the result. Finally, we relate probability matching to impulse balance equilibrium theory and show when probability matching is consistent with its predictions.

Keywords: probability matching, individual decision making, impulse balance equilibrium

JEL Classification: D81, D83

1 Introduction

Probability matching is "the tendency to match choice proportions to outcome proportions in binary prediction tasks" (Koehler and James, 2014, page 104). This phenomenon was observed in early experiments based on variations of a common design in which subjects had to repeatedly predict the outcome of a random event (see, e.g., Grant et al., 1951, Hake and Hyman, 1953, and Siegel and Goldstein, 1959).[1] After the initial findings, evidence of probability matching has been reported by experimental psychologists and economists. The debate on probability matching is still lively (for selective reviews of the literature, see Vulkan, 2000 and Koehler and James, 2014) and also

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[1]Classical tasks include guessing the color of a randomly selected ball or of randomly flashing light bulbs.

considers factors like incentivized choices, feedback and training, which are effective in limiting this phenomenon (see, e.g., Shanks et al., 2002).

This evidence is relevant since probability matching is inconsistent with rational decision making.[2] In fact, when correct guesses are monetarily rewarded, subjects who prefer more money to less[3] and maximize expected payoff being aware of outcome probabilities[4] should always choose the outcome with the greatest likelihood. This rather straightforward conclusion is usually supported by numerical examples in which the expected payoff from probability matching is directly compared to the maximal payoff[5] or by a detailed verbal reasoning which leads to identifying the optimal choice.

However, to the best of our knowledge, a formal proof of the suboptimality of probability matching in prediction tasks with an arbitrary number of outcomes and repetitions is not available in the literature. With this note we aim at filling this gap. To this end, we provided a complete characterization of suboptimality of probability matching in both, ex-ante and sequential prediction tasks. As an additional original contribution, we propose a graphical analysis of this result for the popular case of binary prediction tasks. Finally, we show that, in some instances of the prediction task, probability matching is the guessing strategy which minimize the maximal expected foregone payoff and we relate this result to learning direction and impulse balance theory.

2 The Ex-Ante Prediction Task

The prediction task is formulated as follows: a lottery over a finite number $n \geqslant 2$ of outcomes with probability distribution $p = (p_1, ..., p_n)$ is to be repeated $\tau \geqslant 1$ times. Each lottery play constitute a trial. Being aware of p, the decision maker chooses a possibly different outcome $i \in \{1, ..., n\}$ for each trial. If the chosen outcome is realized the decision maker obtains a positive prize δ, otherwise she gets nothing. Trials begin only after the decision maker has made her choices. In this prediction task, an array of outcome choices constitute a strategy, denoted by $\gamma = (\gamma_1, \ldots \gamma_t, \ldots \gamma_\tau)$ with $\gamma_t \in \{1, \ldots, n\}$ for $t = 1, \ldots, \tau$. Given a strategy γ, the probability of the outcome chosen in the t-th trial is p_{γ_t}, hence the expected payoff of γ is $\sum_{t=1}^{\tau} p_{\gamma_t}\delta$.

Given a strategy γ, let γ_t^i be defined as $\gamma_t^i = 1$ if $\gamma_t = i$ and $\gamma_t^i = 0$ if $\gamma_t \neq i$. The expected payoff

[2] See, for instance, the discussion by Hirshleifer and Riley (1992, pages 33–36).

[3] More generally, if the subjective evaluation of a correct guess is larger than the evaluation of an incorrect one.

[4] Awarness of probabilities can be the result of a learning process through repeated trials or be based on direct information provided by the experimenters, as in Koehler and James (2010).

[5] See, for instance, Koehler and James (2014, page 104)

of γ can be written as

$$\sum_{i=1}^{n}\sum_{t=1}^{\tau}\gamma_t^i p_i\delta.$$

that is, as the sum over all outcomes $i \in \{1, ..., n\}$ of the absolute frequency of trials in which the decision maker chooses outcome i (i.e., $\sum_{t=1}^{\tau}\gamma_t^i$) times the probability of outcome i in the lottery (i.e., p_i). The relative frequency of outcome i in strategy γ is $\hat{g}_i(\gamma) = \left(\sum_{i=1}^{\tau}\gamma_t^i\right)/\tau$. Hence the expected payoff of γ can be rewritten as

$$\tau\sum_{i=1}^{n}\hat{g}_i(\gamma)p_i\delta,$$

so that it is apparent that if two strategies, γ and γ', are such that $\hat{g}_i(\gamma) = \hat{g}_i(\gamma')$ for every $i \in \{1, ..., n\}$, that is if they result in the same distribution of relative frequencies of outcome choices across trials, then they have the same expected payoff.[6] Therefore, the prediction task can be analyzed directly via payoff-equivalent strategies.

To this end we reformulate it in terms of arrays of relative frequencies of outcome choices across trials $(g_1, ..., g_n)$. With some abuse of notation and terminology, these will be denoted by g and referred to as a *guessing strategies*. The set of feasible guessing strategies is $\mathcal{G} = \{(g_1, ..., g_n) \in \mathbb{Q}^n : g_i \geqslant 0 \text{ for } i = 1, ..., n,\ \sum_{i=1}^{n} g_i = 1\}$.[7] The decision maker chooses a feasible guessing strategy g so as to maximize $\tau\sum_{i=1}^{n} g_i p_i\delta$. Since $\tau > 0$, this is equivalent to maximize the expected payoff $e(g) = \sum_{i=1}^{n} g_i p_i\delta$. Hence the decision maker solves

$$\max_{g\in\mathcal{G}}\ e(g), \tag{1}$$

whose set of solution(s), given δ, is $g(p)$.

In this formulation of the prediction task, *probability matching* is denoted by $g^p = (g_1^p, ..., g_n^p)$ and it is the feasible guessing strategy such that $g_i^p = p_i$ for every $i \in \{1, ..., n\}$. Since g_i^p is by definition a rational number, for this strategy to be well defined we assume that the probability of each outcome is a rational number, i.e. $p \in \Delta_{\mathbb{Q}}^{n-1} = \{\hat{p} \in \mathbb{Q}^n : \hat{p}_i \geqslant 0 \text{ for } i = 1, ..., n,\ \sum_{i=1}^{n}\hat{p}_i = 1\}$, and that τ is a common multiple of the denominators of probabilities expressed as irreducible fractions.[8]

[6]We are implicitly assuming that the order of guesses does not matter for the decision maker, even though behaviorally this may not be the case.

[7]$\mathbb{Q}^n$ is the set of n-tuples of rational numbers.

[8]From the latter assumption it follows that $\tau \geqslant n$.

Let $\mathcal{M}(p)$ be the set of maximally likely outcomes in the lottery, defined as $\mathcal{M}(p) = \{j \in \{1, ..., n\} \ : \ p_j = \max\{p_1, ..., p_n\}\}$ for any given $p \in \Delta_{\mathbb{Q}}^{n-1}$. Observe that $\mathcal{M}(p)$ is non empty for every p and $1 \leqslant \#\mathcal{M}(p) \leqslant n$, i.e. there is at least one (since n is finite) and at most n (when $p_i = 1/n$ for every $i \in \{1, ..., n\}$) maximally likely outcomes. The following proposition provides a complete characterization of suboptimality of probability matching:

Proposition 1. For any positive prize δ and any outcome probability distribution $p \in \Delta_{\mathbb{Q}}^{n-1}$, if $g^* \in g(p)$ then $e(g^*) \geqslant e(g^p)$ and equality holds if and only if

$$j \notin \mathcal{M}(p) \ \text{ implies } \ p_j = 0 \tag{NS}$$

Proof. The weak suboptimality of the probability matching strategy, i.e. the fact that $e(g^*) \geqslant e(g^p)$ whenever $g^* \in g(p)$, follows directly from $g^p \in \mathcal{G}$, i.e. from the fact that g^p is a feasible guessing strategy and therefore cannot yield an expected payoff greater than g^*. To prove that (NS) is a necessary and sufficient condition for $e(g^*) = e(g^p)$ to hold whenever $g^* \in g(p)$, first of all observe that $g(p) = \{g \in \mathcal{G} \ : \ i \notin \mathcal{M}(p) \ \text{ implies } \ g_i = 0\}$, since no strategy can be optimal if it includes guesses on outcomes whose probability is not maximal. Furthermore, observe that (NS) implies that, for all $i \in \mathcal{M}(p)$, it must be true that $p_i = \bar{p}$, with $\bar{p} = 1/\#\mathcal{M}(p)$.

To prove sufficiency, assume (NS) holds and pick $g^* \in g(p)$. In this case,

$$e(g^*) = \sum_{i=1}^{n} g_i^* p_i \delta = \left(\sum_{i \in \mathcal{M}(p)} g_i^* p_i \delta + \sum_{i \notin \mathcal{M}(p)} g_i^* p_i \delta \right) = \sum_{i \in \mathcal{M}(p)} g_i^* \bar{p} \delta = \bar{p}\delta,$$

since $g_i^* = 0$ for every $i \notin \mathcal{M}(p)$ and $\sum_{i \in \mathcal{M}(p)} g_i^* = 1$. Similarly,

$$e(g^p) = \sum_{i=1}^{n} g_i^p p_i \delta = \left(\sum_{i \in \mathcal{M}(p)} g_i^p p_i \delta + \sum_{i \notin \mathcal{M}(p)} g_i^p p_i \delta \right) = \sum_{i \in \mathcal{M}(p)} p_i^2 \delta = \bar{p}\delta,$$

since $p_i = 0$, hence $g_i^p = 0$, for every $i \notin \mathcal{M}(p)$ and $\sum_{i \in \mathcal{M}(p)} p_i^2 = \#\mathcal{M}(p)\bar{p}^2 = \bar{p}$.

To prove necessity, suppose that $e(g^*) = e(g^p)$ whenever $g^* \in g(p)$. Since $g^p \in \mathcal{G}$, it follows that $g^p \in g(p)$. Therefore $j \notin \mathcal{M}(p)$ implies $g_j^p = 0$, hence $p_j = 0$. ■

Proposition 1 provides a complete characterization of the suboptimality of probability matching in a prediction task with an arbitrary (finite) number of outcomes and trials.

When $n = 2$, this characterization can be illustrated by solving problem (1) graphically. To this end, in Figure 1 and 2 we measure g_1 on the horizontal axis and g_2 on the vertical one. The downward sloping solid line represents, therefore, the set $\mathcal{G}$ of feasible guessing strategies. The point (g_1^p, g_2^p) lies below the 45° degree line in Figure 1 since $g_1^p = p_1 > p_2 = g_2^p$, and above it in Figure 2 since $p_1 < p_2$. The dashed lines represent iso-expected payoff curves, i.e. combinations of guessing strategies such that $e(g)$ is constant (their slopes differ across figures because of the difference in probability ratios). Since $e(g)$ increases as we move north-east on the diagram, the expected payoff is maximized when $g_1^* = 1$ and $g_2^* = 0$ in Figure 1 and when $g_1^* = 0$ and $g_2^* = 1$ in Figure 2. In both figures it is apparent that probability matching is suboptimal since the iso-expected curve through (g_1^p, g_2^p) is lower than that through (g_1^*, g_2^*).

If $p_1 = p_2$, then the slope of the indifference curves is equal to the slope of the line representing the feasible set. In this case (not displayed in the figures), condition (NS) holds and indeed the probability matching strategy is optimal, since every feasible guessing strategy results in the same maximal expected payoff.

Figure 1: Outcome 1 more likely

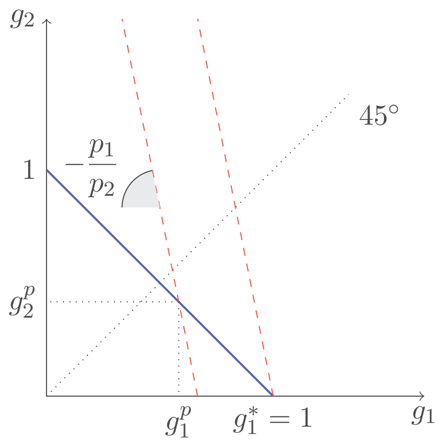
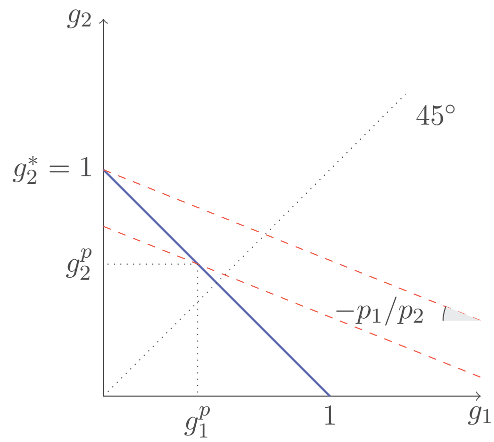


Figure 2: Outcome 2 more likely



3 Randomized and Sequential Guessing

In the prediction task analyzed above the decision maker chooses a single outcome for each trial, hence randomization within trials it not allowed. Furthermore, since the decision maker chooses all outcomes before trials begin, (s)he cannot condition the choice to outcome realizations in previous trials. In this section, we expand our analysis and consider two reformulations of the prediction task which encompass these cases. By adapting the definition of guessing strategy and probability matching strategy, we show that Proposition 1 extends to these reformulations as well.

In the first reformulation of the prediction task, the decision maker still chooses outcomes before any trial. However, she can randomize her choice in every trial. To illustrate one (of the many) protocol to implement this task, suppose the experimenter fills an urn with $b \geqslant 2$ balls of $n \geqslant 2$ different colors. The decision maker, who is aware of the composition of the experimenter's urn, is assigned an empty urn as well as with b balls of each of the n colors. In each trial, the decision maker has to fill the own urn with colored balls. Trials consists in randomly selecting one ball from each of the two urns and, if the drawn balls have the same color, the decision maker wins the prize, otherwise (s)he gets nothing.

In this task, a strategy is $\gamma = (\gamma_1, ..., \gamma_t..., \gamma_\tau)$ with $\gamma_t = (\gamma_t^1, \dots, \gamma_t^n) \in \Delta^{n-1}$ being the probability distribution over outcomes that the decision maker chooses in trial t.[9] The expected payoff of γ is

$$\sum_{t=1}^{\tau} \sum_{i=1}^{n} \gamma_t^i p_i \delta$$

The average probability of choosing outcome i across all trials is $(\sum_{t=1}^{\tau} \gamma_t^i)/\tau$. Strategies resulting in the same array of average probabilities have the same expected payoff. Therefore also this reformulation can be analyzed via payoff-equivalent strategies, hence via a guessing strategy $g = (g_1, ...g_n)$, with $g_i = \sum_{t=1}^{\tau} \gamma_t^i$ for every $i = 1, ..., n$. The set $\mathcal{G}$ of feasible guessing strategies, the expected payoff $e(g)$ of a guessing strategy and the probability matching strategy g^p are defined as in the previous section. In this reformulation of the prediction task, probability matching implies that the decision maker matches average probability of outcome to actual probability.

The second formulation of the prediction task that we investigate is sequential guessing. In this case, the decision maker can condition her guesses on the realizations of the lottery in previous trial(s), since the information on outcome realizations is available at the beginning of every trial. In this reformulation, a strategy consists in a probability distribution over outcomes for each possible history of realizations across trials. Formally, $r_t \in \{1, ..., n\}$ is the outcome realized in trial $t \geq 1$, so that $(r_1, r_2 \dots, r_t)$ is the history of realizations up to trial t. The set of possible histories up to trial t is H^t and its generic element is denoted by h. The set of all histories in the prediction task is $H = \cup_{t=0}^{\tau-1} H^t$.[10] A strategy $\gamma : H \to \Delta^{n-1}$ is a mapping from the set of histories into the set probability distributions over outcomes. Given $h \in H^{t-1}$, $\gamma(h) = (\gamma^1(h), \dots, \gamma^n(h)) \in \Delta^{n-1}$ is the probability distribution over outcomes that the decision maker chooses in trial t.[11] The expected

[9] Δ^{n-1} is the ordinary $n-$dimensional simplex. If we impose that, in every trial, $\gamma_t^i = 1$ for some outcome i and $\gamma_t^j = 0$ for all outcomes $j \neq i$, then this reformulation is equivalent to the one in the previous section.

[10] The (empty) history at the beginning of the prediction task is denoted by h^0 and we let $H^0 = \{h^0\}$.

[11] This reformulation encompasses both, unconditional strategies ($\gamma(h) = \gamma(h')$ for $h, h' \in H^{t-1}$) and deterministic

payoff of γ is

$$\sum_{t=1}^{\tau} \sum_{h \in H^{t-1}} \sum_{i=1}^{n} \phi(h)\gamma^i(h)p_i\delta$$

in which $\phi(h)$ denotes the probability of history $h \in H^{t-1}$ (as observed at the beginning of trial t).

Once again, strategies resulting in the same distribution of average outcome probabilities $\left(\sum_{t=1}^{\tau} \sum_{h \in H^{t-1}} \gamma^i(h)\right)/\tau$ across all trials are payoff equivalent. Therefore, to analyze the problem of the decision maker we can refer to guessing strategies that consist of average probability over outcomes: $g = (g_1, \ldots, g_n)$ with $g_i = \sum_{t=1}^{\tau} \sum_{h \in H^{t-1}} \gamma^i(h)$. We define the set $\mathcal{G}$ of feasible guessing strategies, the expected payoff $e(g)$ of a guessing strategy and the probability matching guessing strategy g^p as in the previous section. In this reformulation of the prediction task probability matching implies that the decision maker matches average probability of outcome to actual probability.

It is straightforward to check that the proof of Proposition 1 extends to these reformulations of the prediction task, hence its conclusions holds for them as well.

4 Probability matching as impulse balance guessing strategy

In the previous sections we have shown that probability matching is a (weakly) sub-optimal guessing strategy if the decision maker aims at maximizing the expected payoff of her choices. In this section, we show that, in some instances of the prediction task, it is an optimal strategy when a different criterion is used to evaluate her choices.[12] In particular, we show that in binary prediction tasks and also in prediction tasks with an arbitrary (finite) number of outcomes, provided that they are equally likely, probability matching is the guessing strategy which minimizes the maximal foregone expected payoff.

To justify this criterion, we relate it to the impulse balance equilibrium theory. This is a behavioral theory derived from learning direction theory and used to interpret experimental data in repeated choice tasks.[13] It is based on the ex-post rational evaluation of counter-factual, i.e. foregone, payoffs from choices not made in the previous trial: feedback information causes upward (due to foregone gains) and downward (due to foregone losses) impulses and equilibrium choices balance them.

outcome choices ($\gamma^i(h) = 1$ and $\gamma^j(h) = 0$ for some $i \in \{1, ..., n\}$, $j \neq i$ and $h \in H^{t-1}$).

[12] We are grateful to Werner Güth for his suggestion to relate probability matching to impulse balance equilibrium theory and for the discussions on its implementation.

[13] Both theories have been proposed by Selten and several (co-)authors. See, e.g., Selten and Stoecker (1986), Selten and Butcha (1999), Selten et al. (2005), Ockenfels and Selten (2014) and the summary list in Selten (2004).

This property of the equilibrium choices is embedded in the minimization of the maximal expected foregone payoffs since the solution to this problem, in some instances of the prediction task we consider, implies equal, hence balanced, foregone expected payoffs. Furthermore, since probability matching is the guessing strategy which corresponds to this solution, in these cases it is possible to interpret probability matching as an impulse balance guessing strategy.

Given an outcome probability distribution with $p_i > 0$ for every $i \in \{1, ..., n\}$ and a strategy $\gamma \in \Delta^{n-1}$, the foregone expected payoff associated to outcome i is $p_i(1-\gamma_i)$, since by ex-post rationality, the optimal strategy in case outcome i occurred should have assigned probability 1 to this outcome. Therefore, in this case the foregone payoff is $1-\gamma_i$ weighted by the probability p_i.[14]

When the decision maker aims at the lowest maximal foregone expected payoff, (s)he chooses γ so as to solve

$$\min_{\gamma \in \Delta^{n-1}} \max_i \{p_i(1-\gamma_i)\} \tag{2}$$

The main result of this section is summarized in the following:

Proposition 2. Probability matching is a solution to problem (2) when either $n = 2$ or $n > 2$ and $p_i = p_j$ for every $i, j \in \{1, ..., n\}$.

Proof. Assume provisionally that the constraints $0 \leqslant \gamma_i \leqslant 1$ are not binding at the solution of problem (2), so that it is rewritten as

$$\min_{\gamma_i} \max_i \{p_i(1-\gamma_i)\} \quad \text{s.t.} \quad \sum_{i=1}^{n} \gamma_i = 1 \tag{3}$$

Let $F_i = p_i(1-\gamma_i)$, so that $\gamma_i = 1-(F_i/p_i)$ and the constraint is rewritten as $\sum_i F_i/p_i = (n-1)$.[15] Let $\mathcal{H}$ denote the harmonic mean of the outcome probabilities: $\mathcal{H} = n\left(\sum_i p_i^{-1}\right)^{-1}$. Observe that $\sum_i (\mathcal{H}/np_i) = 1$. Therefore $(\mathcal{H}/np_1, ..., \mathcal{H}/np_n)$ is a tuple of non-negative weights which adds up to one. Using these weights, problem (3) is rewritten in terms of F_i as follows

$$\min_{F_i} \max_i \{F_i\} \quad \text{s.t.} \quad \sum_{i=1}^{n} \left(\frac{\mathcal{H}}{np_i}\right) F_i = \mathcal{H}\left(\frac{n-1}{n}\right) \tag{4}$$

By a standard result on weighted means,[16] $\max_i \{F_i\} \geqslant \sum_i (\mathcal{H}/np_i) F_i$ with equality when $F_1 = F_2 = ... = F_n$. Therefore the solution to problem (4) is $F_i^* = \mathcal{H}(n-1)/n$ for every $i \in \{1, .., n\}$.

[14] We disregard the prize δ, since it does not affect problem (2).

[15] $\sum_i$ stands for $\sum_{i=1}^{n}$.

[16] See e.g. Steele (2004, chapter 8)

Since $F_i^* = p_i(1-\gamma_i^*)$, the solution to problem (3) is

$$\gamma_i^* = 1 - \left(\frac{\mathcal{H}}{p_i}\right)\left(\frac{n-1}{n}\right) \quad \forall i \in \{1, ..., n\} \tag{5}$$

and this is also the (unique) solution to problem (2) provided that $0 \leqslant \gamma_i^* \leqslant 1$ for every $i \in \{1, ..., n\}$. To check whether this is true or not, we consider three cases. Firstly, when $n = 2$ equation (5) implies $\gamma_i^* = p_i \in (0, 1)$. Secondly, when $n > 2$ and $p_i = 1/n$ for every i it implies $\gamma_i^* = 1/n = p_i$ for every i. Therefore, probability matching is the unique guessing strategy which minimizes the maximum expected foregone payoff both, in the binary prediction task for any outcome probability distribution or in the prediction task with an arbitrary number of outcomes if they are equally likely. It remains to show that when $n > 2$ and outcomes are not all equally likely, probability matching cannot be a solution to problem (3). If this is not the case, equation (5) implies

$$p_i = 1 - \left(\frac{\mathcal{H}}{p_i}\right)\left(\frac{n-1}{n}\right) \quad \forall i \in \{1, ..., n\} \tag{6}$$

hence $p_i(1-p_i) = p_k(1-p_k)$ for all $i, k \in \{1, ..., n\}$ and therefore

$$p_i\left(p_k + \sum_{j \neq i,k} p_j\right) = p_k\left(p_i + \sum_{j \neq i,k} p_j\right) \quad \forall i, k \tag{7}$$

However, (7) holds only if all outcomes are equally likely, which is a contradiction. To conclude the proof, we have to consider the case in which at least one of the constraints $0 \leq \gamma_i^* \leq 1$ for an outcome i is binding. In this case the result is straightforward since $\gamma_i^* \in \{0, 1\}$ and therefore $\gamma_i^* \neq p_i \in (0, 1)$. ■

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