



*Centre for
International
Studies
on Economic
Growth*

CEIS Tor Vergata
RESEARCH PAPER SERIES

Working Paper No. 52

April 2004

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Fundamental Values in a Random Coefficient Model"**

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CEIS Tor Vergata - Research Paper Series, Vol. 18, No. 52, April 2004

Industry and Time Specific Deviations from Fundamental Values in a Random Coefficient Model

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March 31, 2004

Abstract

The paper analyzes the relationship between stock prices and fundamentals for a large sample of US stocks in the last ten years using a random coefficient model. Heterogeneity and omitted variable bias are properly taken into account with model coefficients being allowed to vary across time and industries. The random coefficient model allows to track waves of reliance on analysts forecasts and non fundamental stock price components across time.

JEL: C140, C230, G120

Keyword: *Fundamental/Price Relationship, Finite Mixture Models, EM algorithm, Panel Data*

1 Introduction

The paper aims to evaluate the behavior of investors in financial markets, considering the influence of Consensus forecasts on their investment choices. For this purpose we adopt a random coefficient approach and apply it to the constituents of the S&P 500 Composite Index recorded from 1990 to 2002 on quarterly basis. Our economic purpose is to test whether a stable and significant relationship between stock prices and their “fundamental” discounted dividend or discounted cash flow values exists.

Starting from Shiller’s seminal paper (1981) several contributions analyze the price/fundamental relationship using the Net Present Value approach (NPV). From an econometric point of view the question has been tackled by the existing literature alternatively in a cross-section or time series perspective. Our goal is to investigate the relationship with a panel

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data approach that permits to explore both dimensions at the same time. Panel data models are generally based on the assumption that observed responses are independent conditionally upon time. This hypothesis cannot be reasonably accepted for stock prices. This is why we choose to apply a random coefficient approach which allows us to handle such problem. As we better specify in the methodological section, by doing so, we consider observations (stocks) independent once they have been conditioned on both time (year) and space (industry) dimensions. Another advantage of our choice is that it allows to consider the effects of the fundamental component of price taking into account the bias due to time and industry specific omitted environmental variables.

The paper is divided into six sections (including introduction and conclusions). In the second section we present a short survey of the literature on financial market microstructure relevant to the motivation for our inquiry on financial investors reliance on fundamentals. In the third section we illustrate our random coefficient approach. The fourth section presents and comments our descriptive and econometric results. The most relevant results are i) the systematic bias in analysts' forecasts, when comparing effective and expected one year ahead earning growth forecasts and ii) the higher variance of fundamentals with respect to observed prices, with "the excess volatility puzzle" disappearing once we properly take into account interim variations in earnings forecasts. The most interesting econometric results are the significant sensitivities of observed prices to components related and unrelated to fundamentals and the heterogeneity of such sensitivities across time and industries.

2 Expectations and stock prices

Deviations of stock prices from their fundamental values are widely documented in the financial empirical literature. The traditional approach proposed by Shiller (1981), tackles the question by analyzing the variance of prices and of their fundamental (dividend based) net present values with variance bound ratio tests. Results from this approach lead to the formulation of the well known excess volatility puzzle in which, contrary to what expected from the theory, fundamentals vary less than observed prices. Kleidon *et al.*, (1986) formulate the puzzle as follows: "*... dividend changes appear much smoother than those of either prices or earnings. It is conceivable that this phenomenon is the basic puzzle, and that its resolution will explain much of what remains of the apparent evidence against the present value relation.*".¹ Some authors interpret the puzzle by arguing that time varying expectations and time varying risk premia, not incorporated in fun-

¹For other and more recent formulation of the puzzle see West (1988) and Cochrane, (1991)

damentals which simply discount ex post dividends, may help to understand the puzzle (see, among others, Campbell *et al.*, 1997 for a detailed discussion on the topic). Many other contributions confirm that other factors different from dividend based fundamentals contribute to explain volatility in stock prices (see, for example, Campbell and Shiller, 1987, Chow and Liu, 1999, Mankiw *et al.*, 1991, Campbell and Cochrane, 1999, Adriani *et al.*, 2003, Adriani and Becchetti, 2003, Anderson *et al.*, 2003). These findings stimulate a reflection on the theoretical framework of market microstructure which may be compatible with them. Such framework is not that of fully revealing rational expectation theoretical models, which assume that private information is redundant, since the efficient market hypothesis implies the informative equivalence between private and public information. In such case, decision processes take place using information which should be exhaustively propagated on the market by the price vector (Fama, 1970 and Grossman, 1976). As a consequence, when financial markets are characterized by perfect and symmetric information, it is impossible to make profits by trading on the basis of the publicly available information (Milgrom and Stokey, 1985, Malkiel, 1992, Asch *et al.*, 1982). Empirical evidence on the volume of trades and on their profits may be reconciled with financial theory by considering that, in a dynamic and uncertain world, with costly information, the classical hypothesis of fully revealing rational expectations, does not hold. If the price vector is not exhaustively informative and investors have heterogeneous information sets, we typically obtain market equilibria in which more informed agents make their investment in information profitable by trading with less informed ones (see, among others, Grossman and Stiglitz, 1980 and Diamond and Verrecchia, 1981).

If we accept that investors have changing and heterogeneous information and expectations, forecasts on future dynamics of firm specific and economy wide variables become a critical source of knowledge for traders and a critical factor in the formation of their fundamental value. In this framework it becomes essential to investigate which kind of information the investor uses for his trading activity and, if the information is asymmetric, which kind of relationship exists among different classes of information variables. In the next paragraph we argue that I/B/E/S earning forecasts are the main source of information used for estimating the net present value of stocks and we formulate an hypothesis on how this information is incorporated into the fundamental price which, for us, represents the benchmark for investors trading decision.

2.1 Fundamentals and stock prices: the dividend/earning controversy

Many papers in financial literature wonder which kind of publicly available information is valued more by asymmetrically informed investors in their

buying/selling decisions. Pettit (1972), Shiller (1981), Asquith and Mullins 1983, and Bajaj and Vijh (1990), as large part of literature, have stressed that investors interpret dividends as proxies of value-relevant information. Consistently, the empirical literature finds a positive correlation between stock price changes and dividend changes around dividend announcement dates. In particular, Shiller (1981) points out that dividends (instead of cash flow) have to be used to evaluate correctly the fundamental, since dividends are what shareholders effectively cash. By following the alternative path of discounting cash flows (DCF approach), even though non distributed dividends are reinvested with the Discounted Cash Flow (DCF), the risk is to count them twice since their variation is incorporated into earning growth forecasts.

An empirical puzzle, though, is that recent stock market values generate unreasonably low risk premia when the present value of the stocks is calculated by discounting future expected dividends. Using I/B/E/S earnings forecasts and a risk-free rate proxied by long term government bond returns, Claus and Thomas, (2001) estimate the post-1985 equity premium for the US market to be of the order of 3%, much less than the historical difference between secular average yearly returns on stocks and bonds in the US market and less than the 8% observed equity premium since 1926. The authors obtain similar findings for the equity markets of Canada, France, Germany, Japan, and the United Kingdom. On the same line Jagannathan *et al.*, (2001) estimate negative implied risk premia for limited periods in the last two decades. Another argument in favor of the DCF approach is that the value of a stock is the net present value of cash flow in cases of a takeover which involves 100 percent of equity capital. Therefore, it is possible that in periods of stock market boom, with waves of mergers and acquisitions, shareholders rationally anticipate stock values for the bidder and fundamental values move toward DCF values.

In this paper we argue that the issue of dividends versus cash flow has much less relevance when we look at long time horizons and consider the available forecasts on earnings growth as proxies of both dividend and cash flow growth. In such case, the use of initial level of cash flow instead of dividends should cause a bias with second order effects on the fundamental price which is mainly determined by other variables (earning growth forecasts, interest rates, terminal rate of growth, etc.). For the construction of our fundamental we therefore use and compare the two approaches (dividend and cash flow NPV) under the assumption that earnings growth expectations can be proxies of rates of growth of both variables. This restrictive assumption is justified by the fact that earnings are the only source of information systematically available for investors for a large number of companies across time. Results on the sensitivity of market prices to such measure will tell us whether our assumption captures something relevant in the behavior of stock market investors.

2.2 The model

In the formulation of our model, we do not follow the traditional financial literature which analyzes the relationship between price and its fundamental value with variance-bound tests. This is because, as pointed out by Marsh and Merton (1986), that choice does not permit to discriminate between price volatility and dividend smoothing policy. Similar difficulties in discriminating between different hypotheses exist if we use levels of dividend-price ratios. In such case we cannot distinguish between the two sources of variability unless we explicitly assume that log dividend-price ratios are cointegrated stationary (Campbell and Shiller, 1988, 1989).

Our assumption is that variables in levels directly allow to test the hypothesis that the price at time t for the stock i equals its fundamental value avoiding any smoothing problem related to dividend or to non distributed earnings. Therefore, following Anderson *et al.* (2003), our response variable is the price level (rather than the dispersion of stock returns or some ratios such as the dividend-price or earning-price ratios) and the stock price can be derived by looking at both fundamental and non fundamental components. On this basis, the information contained in prices helps investors to select under or over priced stocks. As a first step we can therefore write the relation between price and fundamentals as

$$p_{iItT} = F(f_{iItT}, nf_{iItT}) \quad (1)$$

where, for each stock $i = 1, \dots, n_I$, sector $I = 1, \dots, \mathcal{I}$ and time $t = 1, \dots, n_T$, in year $T = 1, \dots, \mathcal{T}$, f_{iItT} is the fundamental information determining the actual level of the price stock p_{iItT} , and nf_{iItT} evidences idiosyncratic information not directly related to the fundamental value. This information can be also related to a set of “environmental” variables. Following the standard literature on market efficiency and variance-bound tests, the relationship between prices and fundamentals can be explicitly measured by decomposing the stock price into fundamental (persistent) and no fundamental (temporary) components (see among others, Blanchard and Quah, 1989, Lee, 1995). This is true if the relation (1) holds. It is standard in this literature (which examines the problem in a time series perspective) to measure these variables as deviations from fundamental component (among others, Anderson *et al.*, 2003). As better specified in section 3, our approach does not measure directly the non fundamental part of price, but it measures the price/fundamental relationship interpreting residual variability as determined by time and industry varying random components of the coefficient which measures the sensitivity to fundamentals. We can now introduce our model considering that, if the relation (1) holds, then also its linear transformation should be true

$$p_{iItT} = \alpha + \beta f_{iItT} + \varepsilon_{iItT} \quad (2)$$

where f_{iItT} is evaluated by DCF methodology and ε_{iItT} is a residual term assumed to be normally distributed with zero mean and variance σ_ε^2 . Equation (2) describes the behavior of "fundamentalist" investors which take their trading decisions on the basis of the gap between current and fundamental values of stocks or portfolios ². Therefore, if financial markets are efficient and fully informative, $\varepsilon_{iItT} = 0$, while β will be close to 1 and $\alpha = 0$. In fact if the model is correctly specified, and the information contained in the covariate is fully informative, $\hat{p}_{iItT} = \alpha + \beta f_{iItT} = f_{iItT}$, the predicted value of p_{iItT} , will be equal to f_{iItT} . If, on the contrary, β is positive but significantly lower than one, we must argue that financial markets incorporate analysts earnings forecasts but downweight them. This could be interpreted as a rational reaction to the fact that earnings forecasts are systematically overoptimistic (Lim, 2001 and Adriani *et al.*, 2003). To test if equation (2) is invariant across time (years) and sectors, we first run an OLS regression for each combination of I and T ³. Regressions results (not displayed here for sake of space) have shown a strong variability across sectors and time. To give an idea, the 25th and 75th percentiles of the distribution of estimates on the 428 combinations of year/sector observations are: 3.61 - 15.45 for the intercepts, 0.38 - 0.81 for the β s and 24.46 - 123.30 for the error variances. These results are not surprising since financial time series usually exhibit an increasing variability in time, which can be reduced by using a logarithmic transformation of the data. Moreover, Campbell *et al.* (1997) evidence that the log transformation of these series is correct since they show exponential growth over time and can be used also if the linear model allows for unit roots. The strategy of the log-linear specification of (1) is normally adopted in time series as well as in cross-sectional studies and mainly in the literature on dividend-ratio models where the aim is to investigate the relationship among price, its fundamental value and related stock returns. In this respect, it should be also noted that the extreme variability of regression estimates is probably due to population heterogeneity and/or omitted environmental variables affecting the price fundamental relationship. As better described in section 3, to take into account this variability, we employ a model specifying different error variances and regression parameters for each combination of sector and time points (years). Moreover, our model describes a linear relationship among price and fundamental value, but it allows to correctly consider the miss-specification of the link function and of the response distribution through the mixture components (MacLachlan and Peel, 2000, Aitkin, *et al.* 2003). This allows us capture

²Durlauf and Phillips (1988) for example use a similar version of the linear transformation here proposed as an equivalent form of the variance-bound test to estimate the orthogonality between the price gap and the information variables.

³Before running the regression in levels we check that, as expected, the two series are cointegrated stationary in the time dimension for individual stocks with non missing data. Test results are omitted for reasons of space and are available upon request

the non homogeneous behavior of the price-fundamental relationship among sectors and time.

2.3 Specification of the fundamental

To define our fundamental we consider the following two stage "discounted dividend/cash flow" approach.⁴ As in Kaplan and Ruback,(1995) our DCF evaluation of the fundamentals is based on I/B/E/S forecasts and has therefore the advantage of using only current net earnings (or dividends) as accounting variable. We apply a modified version of the two stage DCF model. Inside this model, we use a simple CAPM approach to estimate the expected return from an investment of comparable risk. We can specify the model by considering that, in a simplified world, the expected equity value of firms i at time t in year T can be derived from (Adriani *et al.*(2003), Adriani and Bechetti (2003) and Claus and Thomas, (2001):

$$f_{it} = mv_{it}^e = cf_{it} * \left\{ 1 + \left[\sum_{h=1}^5 \frac{[1 + E_{t,t+4}(g_i)]}{(1+r)^h} + \frac{[1 + E_{t,t+4}(g_i)]^6}{(r_{tv} - g_n)(1+r)^6} \right] \right\} \quad (3)$$

where mv_{it}^e is the expected equity market value of firm i at time t and cf_{it} represents, alternatively, current dividend or current earnings. $E_{t,t+4}$ represents at time t the one year ahead expected earnings rate of growth for firm i , according to the Consensus forecast, while r can be viewed either as the discount rate adopted by equity investors or as the expected return from an investment of comparable risk. To evaluate r we use the standard model of CAPM, i.e., $r_{CAPM} = r_f + \beta E[r_m]$ where r_f , is a short term US treasury bond rate, current beta is calculated on a sample of two year

⁴Accounting and economic literature usually adopt at least three different approaches to calculate the fundamental value of a stock: i) the comparison of balance sheet multiples (EBITDA, EBIT) for firms in the same sector; ii) the residual income method; iii) the discounted cash flow method. The first approach is highly arbitrary as, if market agents have nonhomogeneous information sets or adopt different trading strategies, the benchmark used for comparison may be overvalued or undervalued. The second problem with this method is that industry classifications become always more tricky as far as firms diversify their activities and develop new products or services which cannot be easily classified into traditional taxonomies, (Kaplan and Ruback,1995). The problem with the second approach (residual income method), which is largely used in the literature (see for example Lee *et al.*, 1999; Frankel and Lee, 1998), is that the formula for evaluating the fundamental value of a stock uses a balance sheet measure whose accuracy and capacity of incorporating changes in firm fundamental value is limited. A valuable example of this phenomenon may be given by descriptive evidence provided by Lee *et al.*, (1999) who document the sharp up-trend in the price to book ratio which has risen three times between 1981 and 1996 for the Dow Jones Industrial Average. An interpretation for this result is that accounting methodologies lag behind in adjusting to changes in investors' market value assessments of firms whose share of intangible assets made by human and, more generally, immaterial capital is rising over time. This is the reason why, following Kaplan and Ruback,(1995), we prefer to use the DCF approach.

lagged weekly data and $E[r_m]$ is the risk premium. For the stock market premium we consider that our measure should be between the historical difference in the rates of return of stocks and T-bonds (between 6 and 9 percent) and the minimum implied risk premium for US equity markets in the sample period which is extremely low and around 2 percent.⁵ Both of these two extremes might be incorrect estimates for the risk premium. The former because it assumes that the long term historical premium is equal to the current premium neglecting fundamental institutional and political changes occurred in the last decades. The latter because it assumes that financial markets are correct in evaluating stocks (this is not the case in presence of a bubble or of a nonzero share of noise or liquidity traders trading with arbitrageurs of limited patience). The last part of the formula is the terminal value of the stock. We fix at the sixth year the border between the high growth period to the stable growth period. Sensitivity analysis on this threshold shows nonetheless that this choice is not so crucial for the determination of the value of the stock.⁶ We must in fact consider that the positive impact on value of an additional year of high growth must be traded off with a heavier discount of the terminal value which represents a significant part of the final value. In the terminal value it is assumed that the stock cannot grow more and cannot be riskier than the rest of the economy. The discount rate in the terminal value is calculated as $r_{tv} = r_f + E[r_m]$ considering g_n as the perpetual rate of economic's growth.

3 The econometric model

To estimate (2) we use a random coefficient model. This because in our preliminary analysis, performed by running OLS regressions for each combination of time (year) and sector, we noted a significant variability of parameter estimates across time and sectors. This phenomenon can be due to

⁵To calculate the current implied premium we use the Gordon (1962) formula in which value is equal to: expected dividends next year/(required return on stocks - expected growth rate). In the light of this formula, the extremely low implicit risk premium in the sample period may be interpreted as the availability to pay higher prices even in presence of low dividend payout. The fact that the implied risk premium is lower today than that prevailing in periods (such as the sixties) with similar growth and inflation rates may depend for instance on political factors which may make investors' expectations more optimistic over long time horizons therefore reducing the risk of equity investment. If we just adopt the implied risk premium without weighting it for the historical risk premium, we would implicitly assume that no fundamental value exists or that the fundamental is just what investors believe in that period. In this way changes in the implied risk premium constantly update the fundamental value and the latter becomes just what investors are willing to pay for the stock.

⁶Sensitivity analysis shows that small perturbations of some selected parameters of the fundamental (risk premium, border between high growth and terminal period, risk free rate) do not change substantially results which are presented in the following sections. Evidence is omitted for reasons of space and available upon request.

model misspecifications, like omitted variables (see Aitkin, *et al.* 2003) or, simply, to population heterogeneity. In the first case we can solve this problem by adding a set of unobserved covariates inside the linear predictor. In both cases an alternative is to associate a set of random parameters to some elements of the adopted covariates set. By this manipulation we explicitly adjust the estimate of model parameters for sector and time specific omitted variables which affect the relationship between prices and fundamentals (i.e. the fundamentalists' (chartists') participation rate, the forecasted interest rate term structure or time varying inflation forecasts not perfectly captured in the DCF approach). In statistical terms, indicating with y_{iItT} the price of the stock i in time t for sector I and year T and with x_{iItT} its fundamental counterpart, we insert inside our model these omitted variables by assuming that each sector has its own regression, i.e.

$$y_{iItT} = \alpha_{IT} + \beta_{IT}x_{iItT} + \varepsilon_{iItT} \quad (4)$$

where ε_{iItT} are independent and normal distributed with mean 0 and variance σ_{IT}^2 . Conditionally upon the regression parameters $\boldsymbol{\eta}_{IT} = [\alpha_{IT}, \beta_{IT}, \sigma_{IT}^2]'$, the probability density function of y_{iItT} is

$$f(y_{iItT}|\boldsymbol{\eta}_{IT}) = \frac{1}{\sqrt{2\pi\sigma_{IT}^2}} \exp\left[-\frac{1}{2\sigma_{IT}^2}(y_{iItT} - \alpha_{IT} - \beta_{IT}x_{iItT})^2\right] \quad (5)$$

while that of $\mathbf{y}_{IT} = [y_{1IT}, \dots, y_{n_IT}]'$ is

$$f(\mathbf{y}_{IT}|\boldsymbol{\eta}_{IT}) = \prod_{i=1}^{n_I} \prod_{t=1}^{n_T} f(y_{iItT}|\boldsymbol{\eta}_{IT}) \quad (6)$$

We assume that $\boldsymbol{\eta}_{IT}$ is a random variable with probability function g . The marginal (unconditional) distribution of $\mathbf{y}_{IT}$ is then given by the following integral

$$f(\mathbf{y}_{IT}) = \int f(\mathbf{y}_{IT}|\boldsymbol{\eta}_{IT})g(\boldsymbol{\eta}_{IT})d\boldsymbol{\eta}_{IT} \quad (7)$$

By assuming independence among observations belonging to different combinations of sector by year, we can write the p.d.f. of the whole sample $\mathbf{y} = [\mathbf{y}'_{11}, \dots, \mathbf{y}'_{IT}]'$ as

$$f(\mathbf{y}) = \prod_{I=1}^{\mathcal{I}} \prod_{T=1}^{\mathcal{T}} f(\mathbf{y}_{IT}) \quad (8)$$

The mixture model described allows us to consider explicitly time and sector specific components of prices allowing to capture the sector specific structure by mixtures with heterogeneous variance components. Moreover,

by applying a random coefficient model, we do not need to specify other regressors which can affect the relationship between prices and the underlying fundamental, since we implement our model with random parameters taking into account the effects of these latent variables. The model, by introducing a random distribution of parameters, allows us to estimate an undistorted coefficient of the fundamental conditional to the effects of additional unobserved environmental variables, even though these variables are not specified in the model.

3.1 Parameters estimation

The parameters of our model are estimated by adopting a non parametric maximum likelihood (NPML) approach (Laird, 1978), by avoiding the usual assumption of normality very frequent in the panel data modelling. In particular, we do not assume a particular specification for the p.d.f. g but estimate it together with the other parameters. Following this approach, the mixture (7) is estimated by a finite mixture of the form

$$f(\mathbf{y}_{IT}) = \sum_{k=1}^K \pi_k \prod_{i=1}^{n_I} \prod_{t=1}^{n_T} f(y_{iItT} | \boldsymbol{\eta}_k) \quad (9)$$

The likelihood of the sample is then

$$L(\boldsymbol{\vartheta} | \mathbf{y}) = f(\mathbf{y}) \quad (10)$$

$$= \prod_{I=1}^{\mathcal{I}} \prod_{T=1}^{\mathcal{T}} f(\mathbf{y}_{IT}) \quad (11)$$

$$= \prod_{I=1}^{\mathcal{I}} \prod_{T=1}^{\mathcal{T}} \left(\sum_{k=1}^K \pi_k \prod_{i=1}^{n_I} \prod_{t=1}^{n_T} f(y_{iItT} | \boldsymbol{\eta}_k) \right). \quad (12)$$

The maximum likelihood estimates of the model parameters are computed by using an EM algorithm (Dempster *et al.*, 1977).⁷

The algorithm is based on the maximization of the complete likelihood

$$L_c(\boldsymbol{\vartheta} | \mathbf{y}) = \prod_{I=1}^{\mathcal{I}} \prod_{T=1}^{\mathcal{T}} \prod_{k=1}^K \left(\pi_k \prod_{i=1}^{n_I} \prod_{t=1}^{n_T} f(y_{iItT} | \boldsymbol{\eta}_k) \right)^{z_{ITk}} \quad (13)$$

where z_{ITk} is 1 if the observation $\mathbf{y}_{IT}$ has been sampled from the k^{th} component of the mixture and 0 otherwise. The z 's are not directly observed and so they are treated as missing data, i.e. they are estimated by their expectations

⁷For a detailed survey on the topic see McLachlan and Krishnan, 1997

$$\hat{z}_{ITk} = \frac{\pi_k f(\mathbf{y}_{IT} | \boldsymbol{\eta}_k)}{\sum_{k=1}^K \pi_k f(\mathbf{y}_{IT} | \boldsymbol{\eta}_k)}. \quad (14)$$

The algorithm simply maximizes the complete likelihood (step M) given a set of z 's estimates (step M) and then recalculates by (14) the z 's estimates (step E). It can be shown that at each E or M step the likelihood (12) increases. The algorithm starts from initial estimates of the parameters, iterates the two steps and stops when the likelihood increment between two iterations is less than a given threshold.

In our approach the M step is partitioned into three sub-steps where the complete likelihood is maximized with respect to a sub-set of parameters given the current values of the others. The three sub-steps can be described as follows.

In the first sub-step the log of the complete likelihood

$$\ell_c(\boldsymbol{\vartheta} | \mathbf{y}) = \sum_{I=1}^{\mathcal{I}} \sum_{T=1}^{\mathcal{T}} \sum_{k=1}^K \hat{z}_{ITk} \left(\log(\pi_k) + \sum_{i,t} \log(f(y_{iIT} | \boldsymbol{\eta}_k)) \right) \quad (15)$$

is maximized with respect to the π 's. It is simple to show that rewriting (15) as

$$\ell_c(\boldsymbol{\vartheta} | \mathbf{y}) = \sum_{ITk} \hat{z}_{ITk} \log(\pi_k) + c \quad (16)$$

where c is a constant term independent of the parameters of interest, the log-complete likelihood attains a maximum when

$$\pi_k = \frac{1}{\mathcal{I}\mathcal{T}} \sum_{IT} \hat{z}_{ITk} \quad (17)$$

In the second sub-step we maximize (15) with respect to $\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2$ conditionally upon the other parameters. By rewriting (15) as

$$\ell_c(\boldsymbol{\vartheta} | \mathbf{y}) = \sum_{I,T,k,i,t} -\hat{z}_{ITk} \frac{1}{2} \left(\log(\sigma_k^2) + \frac{1}{\sigma_k^2} (y_{iIT} - \alpha_k - \beta_k x_{iIT})^2 \right) + c, \quad (18)$$

we see that the problem corresponds to K different independent maximizations, each having as solution

$$\sigma_k^2 = \frac{1}{\sum_{I,T,i,t} \hat{z}_{ITk}} \sum_{I,T,i,t} \hat{z}_{ITk} (y_{iIT} - \alpha_k - \beta_k x_{iIT})^2 \quad (19)$$

In the last sub-step, the log-complete likelihood (15) is maximized with respect to the regression coefficients. As in the previous sub-step, we note

that this corresponds to solve K independent maximization problems, one for each couple (α, β) . This can be easily solved by noting that rewriting (15) as

$$\ell_c(\boldsymbol{\vartheta}|\mathbf{y}) = - \sum_{I,T,i,t} \hat{z}_{ITk} \frac{1}{2\sigma_k^2} (y_{iItT} - \alpha_k - \beta_k x_{iItT})^2 + c \quad (20)$$

it follows that

$$\beta_k = \frac{\sum_{I,T,i,t} \hat{z}_{ITk} y_{iItT} (x_{iItT} - \bar{x}_k)}{\sum_{I,T,i,t} \hat{z}_{ITk} (x_{iItT} - \bar{x}_k)^2} \quad (21)$$

$$\alpha_k = \bar{y}_k - \beta_k \bar{x}_k \quad (22)$$

where

$$\bar{x}_k = \frac{1}{\sum_{I,T,i,t} \hat{z}_{ITk}} \sum_{I,T,i,t} \hat{z}_{ITk} x_{iItT}, \quad (23)$$

$$\bar{y}_k = \frac{1}{\sum_{I,T,i,t} \hat{z}_{ITk}} \sum_{I,T,i,t} \hat{z}_{ITk} y_{iItT} \quad (24)$$

Summing up, the econometric model presented here allows to verify statistically the behavior of fundamental prices under the assumption that population heterogeneity affects parameters estimation. Population heterogeneity has been considered by allowing parameters to vary across time (year) and industries. In the model, mixture components have time and industry varying variances. This allows to consider model miss-specification due to unobserved variables or to erroneous assumptions on the link function or response distribution, such as for example the assumption of a linear relationship between prices and fundamentals when the true relationship is log-linear (MacLachlan and Peel, 2000, Aitkin, *et al.* 2003).

4 Results

We have estimated (2) by normal mixture using a semiparametric specification. Our source of data is Datastream for stock prices and I/B/E/S for earning forecasts which are collected for the constituents of S&P500 Composite index from 1:1990 to 12:2002 on quarterly basis. As specified before our model assumes that observations are independent once we condition them to both sectors and time. We employ two mixture models for fundamentals based on dividends per share and earning per share respectively. The two models are

$$\text{EPS : } p_{iIt} = \alpha_{IT}^e + \beta_{IT}^e feps_{iIt} + \varepsilon_{iIt}^e \quad (25)$$

$$\text{DPS : } p_{iIt} = \alpha_{IT}^d + \beta_{IT}^d fdps_{iIt} + \varepsilon_{iIt}^d \quad (26)$$

where $feps$ and $fdps$ are the fundamentals evaluated according to (3) using as starting point earning and dividend per share respectively. Parameter estimates are computed by implementing the EM algorithm described in the previous section, using a code written in Matlab R11. To overcome the local optima problem, a common feature of all iterative algorithms, we started several times our program by using different initializations and choosing the solution giving the maximum value of the likelihood. The starting points, i.e., the initial parameter values, have been selected randomly or by grouping the results of the OLS regressions ran for each combination of sector by year. This procedure has been repeated for different numbers of mixture components ranging from 2 to 25. The correct number of components has been finally chosen by using the BIC criterion (Kerbin, 2000).

It is trivial to observe that in our model the intercept captures components unrelated to the fundamentals. By just looking at descriptive statistics of the dependent and independent variables we find that the volatility of the fundamental is higher than that of the observed price (Table 1). This finding confirms the hypothesis that the “excess volatility puzzle” depends on the erroneous identification of the fundamental with a sum of discounted dividend growth which does not incorporate interim changes of growth forecasts as we do. This view is shared among others by Campbell *et al.* (1997). The relatively higher volatility of the fundamental is confirmed when we compare sector averages of individual stock standard deviations in each industry (figure 1). Looking at parameters in Table (2) we find that estimated expected values of β coefficients are respectively 0.571 and 0.588 for the earning based and for the dividend based fundamental. The significance of these parameters is confirmed by the confidence interval (at 95%) and Likelihood Ratio Tests (LRT). In both models $E(\beta)$ is between 0 and 1, since the LRT rejects for both the two null hypotheses: ($H_0 : E(\beta) = 1$ and $H_0 : E(\beta) = 0$), with p-values equal to 0.000 ⁸

These results suggest that investors are highly sensitive to fundamentals based on analysts forecasts, even though they slightly downweight them. This attitude may be considered not irrational if we take into account, looking at Figure (2), that financial analysts systematically overestimate the expected earning throughout all the considered period. Another interesting finding is that the intercept, i.e. the coefficient measuring the market price component which is unrelated to the fundamental, is significant and

⁸The p -values are obtained by comparing the maximum likelihoods of the full and constrained models. The latter is obtained by estimating the model under the constraint $E(\beta) = \sum_k \beta_k \pi_k = c$, where c can be 0 or 1.

positive. We may therefore argue that prices incorporate a component of time varying euphoria unrelated to forecasts of fundamentals. In Table (3) we find that estimated mean locations for α and β in both specifications are similar. Looking at the locations of the mixture components in Table (3), we do not note significant differences in the α and β distributions. By looking at the variation by industry we observe that confidence on analysts' forecasts is higher in sectors with well known companies, as diversified industries, or in those where stocks are more heavily traded, as in high-tech or financial industries (see Table 4) If we examine the variation in time of the $\hat{\alpha}_{IT}$ and $\hat{\beta}_{IT}$ coefficients (Figures 3 and 4 for fundamental evaluated by earning and dividend respectively) we find that the weight of the intercept (stock value unrelated to fundamentals) gradually but slowly grows, dramatically accelerating in proximity of the peak of the stock bubble, while the sensitivity of the earning and dividend based fundamentals starts to fall before the bubble. Since our fundamental is calculated on the basis of the historical risk premium we may consider it as a proxy of the implied risk premium and therefore argue that, before we reach the proximity of the peak of the stock bubble, we observe a negative correlation between reliance on analysts' forecasts and implied risk premium. In the proximity of the peak the correlation becomes positive since reliance on fundamentals falls, while sensitivity to non fundamental components keeps on rising. After the peak the sensitivity to non fundamental components falls dramatically while the sensitivity to fundamentals keeps on falling.

Remembering that our model allows to capture through time and industry specific coefficients unobserved (time and industry specific) factors affecting the relationship among our variables and prices, a plausible interpretation of our results is that changes in our coefficients proxy time (and industry specific) participation rates of chartists and fundamentalists to the stock market. This would explain the dynamics of the non fundamental component around the peak of the stock bubble with fundamentalists anticipating the fall and leaving the market before chartists.

5 Conclusions

Hope in the future is the fundamental virtue on which prosperity of real and financial markets is based. When we look at stock prices we find that hope in the future concretely and positively affects prices by reducing current implied risk premia. Production of hope in the future may be viewed in the short run as a positive sum game as it creates advantages for both analysts and investors. The risk of the game is that hope is a psychological variable which is highly volatile. The higher the current level of hope, the higher the costs of its sharp fall on financial values and the higher the risk that negative shock induce investors to deem their hopes as unrealistic given the

current levels of prices.

These considerations may help to interpret our results and justify the fact that: i) analysts overestimates are systematic and constant in the last twenty years; ii) the number of analysts forecasts is larger and confidence on analysts' forecasts is higher in sectors with well known companies, as diversified industries, or in which stocks are more heavily traded, as in high-tech or financial industries ; iii) the creation of hope reaches its maximum in particular periods and prepares conditions for its sharp fall. The signs of this are iia) a dramatic rise in the component which is unrelated to expectations on marketwide and firm specific financial variables and iib) a price sensitivity to fundamentals which anticipates price sensitivity to non fundamentals around the stock market peak suggesting that significant changes in fundamentalists/on fundamentalists stock market participation occur before the stock market downfall in the year 2000.

The opportunity of extending our investigation to the dynamics of sensitivity to fundamental and non fundamental components across time is made possible by the adoption of a random effect approach where the vector of regression parameters (intercept, regression coefficient and error variance) is considered as a vector random variable independently and identically distributed across the different combinations of sector by year. For this vector, we avoided the usual assumption of normality (see, among others, Baltagi, 2001) by estimating its conjoint distribution following a non-parametric approach based on mixture modelling. The mean of this vector has been estimated and tested. We also predict the realized values of this vector by computing the posterior means for each regression (i.e., for each combination of sector by year).

However, the model introduces some simplifications of the reality. In fact, we assume that two observations are dependent if they belong to the same combination of sector by year but independent if they belong to different regressions without taking care of a possible serial correlation among observations related to the same firm in different years. Nevertheless, we could not test the presence of this correlation because the panel is unbalanced and with some missing data (supposed to be "at random") even if we suspect its existence. In spite of those limits, we think that the model has been able to capture the essential information present in the data.

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Descriptive Statistics				
	mean	Std. Dev.	Skewness	kurtosis
p	27.563	18.163	1.387	8.017
$feps$	29.932	24.541	2.328	12.570
$fdps$	29.156	24.023	2.364	12.959
Obs	10826			

Table 1: Stock Prices and Fundamentals

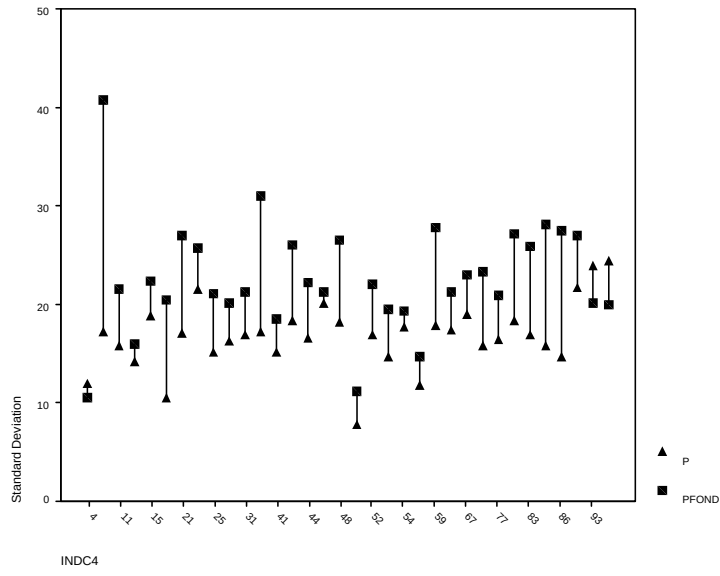


Figure 1: Volatility of observed and fundamental prices. Sector averages of individual stock standard deviations. See Table (4) for the Industries legend

	Estimated Parameters									
	$\hat{E}(\alpha)$	95% conf. int.		$\hat{E}(\beta)$	95% conf. int.		$\hat{V}(\alpha)$	$\hat{V}(\beta)$	$\hat{E}(\sigma^2)$	n_k
EPS	11.698	11.450	11.950	0.571	0.564	0.576	97.421	0.058	116.690	15
DPS	11.647	11.400	11.900	0.588	0.582	0.594	95.718	0.062	117.290	24
$H_0 : E(\beta^e) = 1 \quad p - value = 0.000$										
$H_0 : E(\beta^d) = 1 \quad p - value = 0.000$										

Table 2: Estimated Parameters for equations (25) and (26) where:

- the two estimated models are:
 - EPS: $p_{iItT} = \alpha_{IT}^e + \beta_{IT}^e feps_{iItT} + \varepsilon_{iItT}^e$
 - DPS: $p_{iItT} = \alpha_{IT}^d + \beta_{IT}^d fdps_{iItT} + \varepsilon_{iItT}^d$
- $E(\alpha)$ and $V(\alpha)$ are the mean and variance of the random intercept;
- $E(\beta)$ and $V(\beta)$ are the mean and variance of the random regression coefficient;
- n_k identifies the number of mixture components which have been selected by using the BIC criteria.

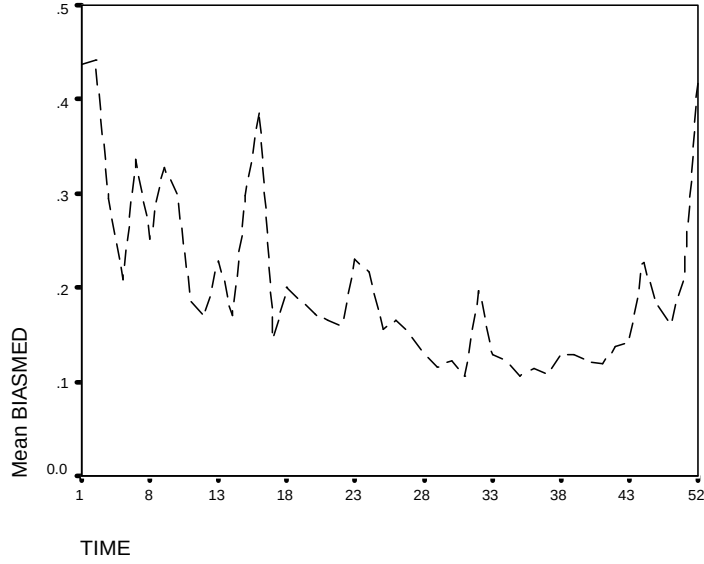


Figure 2: dynamics of the average earning forecast bias, where:

- $bias_{jt} = \frac{E_t[eps_{jt+1}] - eps_{jt}}{p_{jt}}$

k	EPS				DPS			
	α_k	β_k	π_k	σ_k^2	α_k	β_k	π_k	σ_k^2
1	15.139	0.124	0.025	60.5	50.358	0.122	0.016	1379.0
2	50.008	0.130	0.016	1382.8	15.147	0.123	0.024	60.6
3	29.056	0.239	0.059	207.7	25.177	0.301	0.091	179.7
4	16.068	0.305	0.066	149.2	16.608	0.303	0.067	150.6
5	20.017	0.383	0.056	120.1	28.875	0.385	0.094	324.6
6	28.382	0.388	0.086	328.5	5.766	0.444	0.066	12.1
7	5.553	0.444	0.067	11.7	7.792	0.458	0.083	72.3
8	7.659	0.451	0.079	70.9	9.115	0.585	0.085	45.1
9	9.575	0.559	0.074	44.3	4.679	0.614	0.138	28.1
10	4.762	0.600	0.141	29.0	9.425	0.782	0.111	106.0
11	8.834	0.779	0.102	108.1	4.103	0.865	0.068	46.9
12	4.2797	0.828	0.072	48.0	2.086	0.870	0.069	9.1
13	2.0843	0.840	0.071	9.4	3.526	1.028	0.052	39.0
14	3.407	1.003	0.052	39.1	7.484	1.078	0.035	140.7
15	7.711	1.038	0.032	144.5				

Table 3: Locations and Prior Probabilities of Mixtures' components

		EPS	DPS
<i>Sectors</i>	<i>INDC</i>	β^e	β^d
Mining	4	0.4382	0.4492
Oil & Gas	7	0.2484	0.2607
Chemicals	11	0.6325	0.6667
Construction & Building Materials	13	0.6148	0.6331
Forestry & Paper	15	0.5684	0.5724
Steel & Other Metals	18	0.4816	0.4955
Aerospace & Defence	21	0.4884	0.5087
Diversified Industrials	24	0.7148	0.7441
Electronic & Electrical	25	0.3422	0.3514
Engineering & Machinery	26	0.5743	0.5902
Automobiles & parts	31	0.528	0.541
Household Goods & Textiles	34	0.6514	0.6661
Beverages	41	0.481	0.5084
Food Producers & Processors	43	0.698	0.7167
Health	44	0.6224	0.63
Personal Care & Household Products	47	0.5153	0.5377
Pharmaceutical & Biotechnology	48	0.4278	0.4495
Tobacco	49	0.5284	0.5424
Retailers, General	52	0.61	0.6259
Leisure and Hotels	53	0.6769	0.7036
Media and Entertainment	54	0.5821	0.5833
Support Services	58	0.6365	0.67
Transport	59	0.6134	0.6254
Food & Drug Retailers	63	0.6584	0.6753
Telecom Services	67	0.6089	0.6337
Electricity	72	0.5205	0.5552
Utilities, Other	77	0.6513	0.6614
Banks	81	0.6259	0.6302
Insurance	83	0.5984	0.613
Life Assurance	84	0.666	0.6846
Real Estate	86	0.6128	0.6315
Speciality & Other Finance	87	0.6612	0.6935
Information Technology Hardware	93	0.6801	0.6951
Software Computer Services	97	0.4327	0.431

Table 4: Industry specific sensitivity to fundamentals

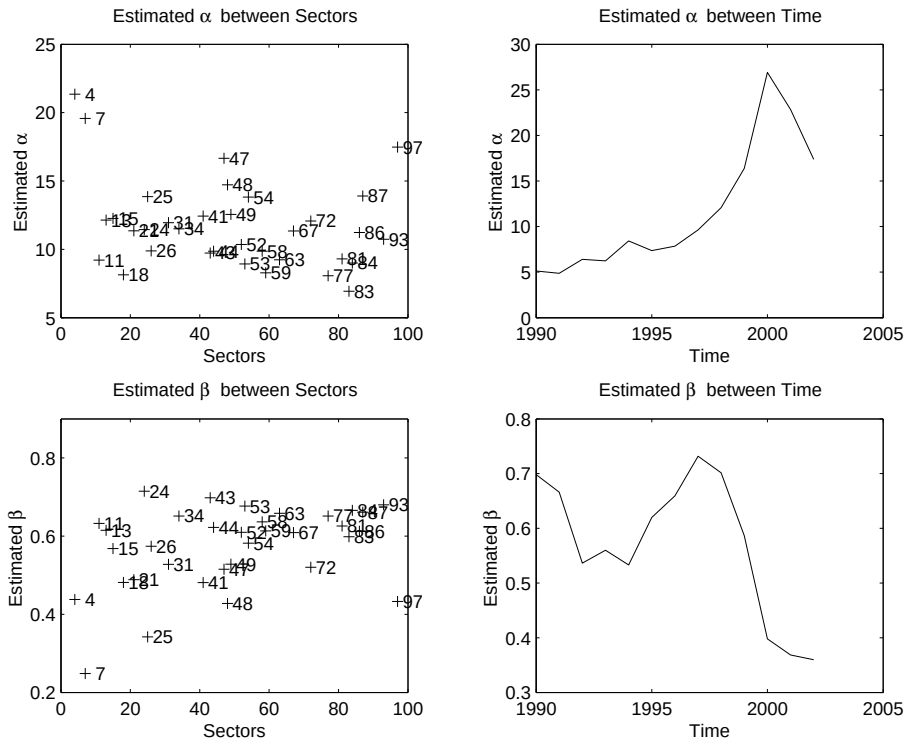


Figure 3: Time and industry varying sensitivities to fundamental and non fundamental components (fundamental evaluated by discounting earnings).

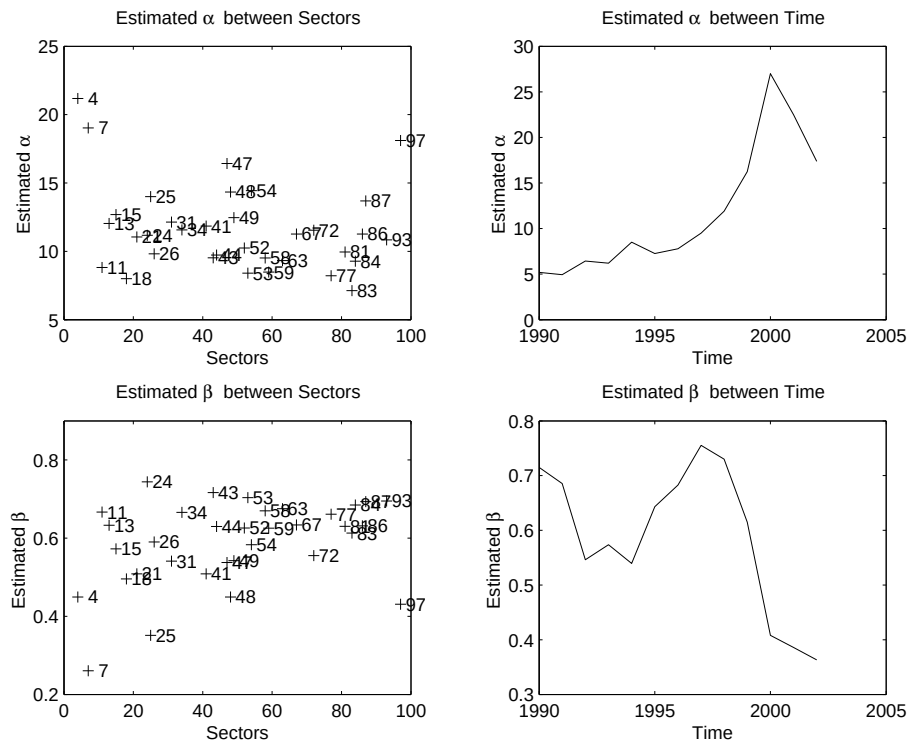


Figure 4: Time and industry varying sensitivities to fundamental and non fundamental components (fundamental evaluated by discounting dividends)