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Consistent estimation of the “true” fixed-effects stochastic frontier model [☆]

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Abstract

The classic stochastic frontier panel data models provide no mechanism to disentangle individual time invariant unobserved heterogeneity from inefficiency. Greene (2005a,b) proposed a fixed-effects model specification that distinguishes these two latent components and allows a time varying inefficiency distribution. However, the maximum likelihood estimator proposed by Greene leads to biased inefficiency estimates due to the incidental parameters problem. In this paper, we propose two alternative estimation procedures that, by relying on a first difference data transformation, achieve consistency for $n \rightarrow \infty$ with fixed T . Evidence from Monte Carlo simulations shows good finite sample performances of both approaches even in presence of small samples.

Keywords: Stochastic frontiers, Fixed-effects, Panel data, Marginal simulated likelihood, Pairwise differencing

JEL: C13, C16, C23

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1. Introduction

The analysis of efficiency is an important issue in many empirical studies and the stochastic frontier (SF) models, introduced by Aigner et al. (1977) and by Meeusen & van den Broeck (1977), represent a popular tool to measure this unobservable economic indicator.¹ Since then, several improvements on SF modeling have led to a large empirical literature, most of which is based on panel data.² Compared to cross-sectional data, longitudinal data has an important advantage as it allows one to follow the same unit over time, thereby allowing to control and model unobserved heterogeneity.

Depending on how this form of heterogeneity is treated, SF panel data literature can be divided into two different groups of models.³ The first does not provide any mechanism to disentangle unobserved time invariant heterogeneity from inefficiency. Therefore, these specifications interpret unit-specific unobserved heterogeneity as inefficiency. On the other hand, the second group offers a different interpretation, distinguishing between these two latent components. This distinction allows to take into account time invariant omitted explanatory variables, such as unobservable environmental factors, unrelated with the production process but affecting the output.

This study reconsiders the estimation of the “true” fixed-effects (TFE) panel data SF model proposed by Greene (2005a,b). This model belongs to the aforementioned second group since time invariant unobserved heterogeneity is jointly specified with a time varying inefficiency. Greene proposed a maximum likelihood dummy variables estimator (MLDVE), in which unit-specific intercepts are estimated jointly with the other model parameters. This approach leads to inconsistent variance parameter estimates due to the incidental parameters problem (Neyman & Scott, 1948; Lancaster, 2002). Since these parameters represent the key ingredients in the post-estimation of inefficiencies, a solution to this problem plays a crucial role in the efficiency analysis.

In this paper, we propose two consistent estimators of the TFE model that, by relying on a first-difference data transformation, avoid the incidental parameters problem. Moreover, both estimators allow the inefficiency to be heteroscedastic. The first approach is based on the marginalization of the inefficiency term via simulation. This strategy is attractive because it allows the estimation of both Normal-Half Normal and Normal-Exponential models. However, the use of Maximum Simulated Likelihood (MSL) principle imposes some restrictions on the heteroscedastic specification: the scale parameter of the inefficiency distribution must only be expressed as a function of time invariant explanatory variables. In the second inferential procedure, we propose a different estimation solution by exploiting the analytical closeness property of the marginal likelihood in a two-period panel for a Normal-Exponential SF model. This estimator is defined as the maximizer of a second-order U-statistic built from all the pairwise quasi likelihood contributions.

The paper proceeds as follows. Section 2 reviews the TFE model and describes the derivation of the marginal likelihood in the first-difference setup. Section 3 presents our estimators. In section 4, we present the finite sample properties of both estimators via Monte Carlo simulations as well as a comparison with the

¹Production frontiers represent the maximum amount of output that can be obtained from a given level of inputs, while a cost frontier characterizes the minimum expenditure required to produce a bundle of outputs given the prices of the inputs used in its production.

²See Kumbhakar & Lovell (2000) or Greene (2008) for recent surveys.

³See Greene (2005a,b) for a detailed review on the treatment of unobserved heterogeneity in SF models.

Greene's MLDVE. In section 5, we illustrate our estimators through an application on hospitals technical efficiency, while section 6 offers some conclusions.

2. The model

Consider the following specification for a fixed-effects stochastic production frontier model

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + \varepsilon_{it}, \quad (1)$$

$$\varepsilon_{it} = v_{it} - u_{it}, \quad (2)$$

$$v_{it} \sim IID \mathcal{N}(0, \tau), \quad (3)$$

$$u_{it} \sim IID \mathcal{F}_u(\sigma), \quad i = 1, \dots, n, \quad t = 1, \dots, T, \quad (4)$$

where, for each unit i and period t , y_{it} represents the level of output, $\mathbf{x}_{it}$ is a vector of inputs, $\boldsymbol{\beta}$ is a vector of technology parameters and α_i is the unit fixed-effect.⁴ The composite error term ε_{it} is the difference between the idiosyncratic error v_{it} , and the one-sided disturbance u_{it} which represents inefficiency.⁵ As customary in SF literature, we assume that v_{it} and u_{it} are independently distributed of each other, and of the regressors. The symbol $\mathcal{F}_u$ denotes a member of the “one-parameter” family of distributions with a support defined over $\mathbb{R}^+$ (e.g., Half-Normal or Exponential) and scale parameter equal to σ .

As already mentioned, Greene (2005a,b) proposed a MLDVE for this model in which the unit specific intercepts are estimated along with the structural parameters, and showed that the maximization of the likelihood function is computationally feasible also in presence of a large number of nuisance parameters ($> 1,000$). However, as Greene's simulations suggest, in short panel this approach may produce inconsistent variance estimates due to the incidental parameters problem. The latter has a critical impact on the SF analysis since an accurate estimation of inefficiency scores relies on the precision of these estimates.

One of the common solutions to this problem is to eliminate the nuisance parameters through a data transformation. This strategy is followed by Wang & Ho (2010) (W&H) for a SF model similar to (1–4), except that the inefficiency is assumed to change deterministically over time, i.e. $u_{it} = h_{it}u_i$ where $h_{it} = \exp(z'_{it}\boldsymbol{\delta})$ and $u_i \sim \mathcal{N}^+(\mu, \sigma_u^2)$. As W&H stressed, this assumption is critical for deriving the marginal likelihood since the distribution of u_i is clearly not affected by the first-difference transformation. Our assumption about u_{it} does not impose any constraint on the inefficiency variability at the cost of a more challenging marginal likelihood derivation.

The first-difference transformation implies that model (1–4) may be rewritten as

$$\Delta y_i = \Delta X'_i \boldsymbol{\beta} + \Delta \varepsilon_i, \quad (5)$$

$$\Delta \varepsilon_i = \Delta v_i - \Delta u_i, \quad (6)$$

$$\Delta v_i \sim IID \mathcal{N}_{T-1}(\mathbf{0}, \Sigma_v), \quad (7)$$

$$\Delta u_i \sim IID \mathcal{F}_{\Delta u}(\sigma), \quad i = 1, \dots, n, \quad (8)$$

⁴For notational simplicity, we assume balanced panels but all the results can be easily generalised to the unbalanced case.

⁵Notice that when the sign of the last term in (2) is positive, the model describes a stochastic cost frontier.

where $\Delta \mathbf{y}_i = (\Delta y_{i2}, \dots, \Delta y_{iT})$ with $\Delta y_{it} = y_{it} - y_{it-1}$, $\forall t = 2, \dots, T$.⁶ The normality assumption for v_{it} implies that $\Delta \mathbf{v}_i$ has a $T - 1$ -variate Normal distribution with covariance matrix $\Sigma_v = \tau^2 \Lambda_{T-1}$, where Λ_{T-1} is the symmetric tridiagonal $T - 1 \times T - 1$ matrix

$$\Lambda_{T-1} = \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 \\ -1 & 2 & -1 & \cdots & 0 \\ 0 & -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & -1 \\ 0 & 0 & \cdots & -1 & 2 \end{pmatrix}. \quad (9)$$

On the contrary, the multivariate distribution of $\Delta \mathbf{u}_i$ is generally unknown. Nevertheless, given the independence assumption between $\Delta \mathbf{v}_i$ and $\Delta \mathbf{u}_i$, the marginal likelihood contribution L_i^* can be defined in general terms as

$$\begin{aligned} L_i^*(\boldsymbol{\theta}) &= \int f(\Delta \mathbf{v}_i, \Delta \mathbf{u}_i | \boldsymbol{\theta}) d\Delta \mathbf{u}_i = \int f(\Delta \mathbf{v}_i | \boldsymbol{\theta}) f(\Delta \mathbf{u}_i | \sigma) d\Delta \mathbf{u}_i = \\ &= \int f(\Delta \mathbf{y}_i | \boldsymbol{\beta}, \tau, \Delta X_i, \Delta \mathbf{u}_i) f(\Delta \mathbf{u}_i | \sigma) d\Delta \mathbf{u}_i, \end{aligned} \quad (10)$$

where $\boldsymbol{\theta} = (\boldsymbol{\beta}', \sigma, \tau)'$. As pointed out by W&H, marginalization of $\Delta \mathbf{u}_i$ hides two challenges: the multivariate density function $f(\Delta \mathbf{u}_i | \sigma)$ is unknown; the integral (10) does not lead to a closed-form expression. In the next section we present two estimation strategies that overcome these problems.

3. Estimation

3.1. Marginal maximum simulated likelihood estimation

The marginalization (10) can be performed by simulation, in a similar way to the elimination of nuisance parameters in nonlinear random-effects models. We propose to estimate model (5–8) treating the marginal likelihood function as an expectation with respect to the random vector $\Delta \mathbf{u}_i$. Lemma 1 explicitly states the distributional assumptions required to apply the MSL estimation technique.

Lemma 1 *Assume that (i) the distribution of the inefficiency $\mathcal{F}_u$ belongs to a one-parameter family of distributions with a support defined over $\mathbb{R}^+$ and scale parameter σ ; (ii) $\mathcal{F}_u$ exhibits a scaling property, i.e. $\mathbf{u}_i = \sigma \tilde{\mathbf{u}}_i$ where the rescaled inefficiency $\tilde{\mathbf{u}}_i \sim \mathcal{F}_u(1)$. Then the multivariate distribution of the rescaled first-differenced inefficiency $F_{\Delta \tilde{\mathbf{u}}}$ is parameter free, such that $\Delta \mathbf{u}_i = \sigma \Delta \tilde{\mathbf{u}}_i$.*

As far as the first assumption is concerned, Ritter & Simar (1997) claim that cross-sectional Normal-Gamma SF models are associated in small samples with severe identification problems due to the flat surface of the profile likelihood. Similarly to the cross-sectional model, in the “true” specification the sample information is only informative on the difference (or the sum) between v_{it} and u_{it} , thus we believe that a one-parameter inefficiency distribution is in general a preferable choice. Furthermore, the most common

⁶A similar notation is also used for the other transformed vectors and matrices.

one-parameter inefficiency distributions (Half Normal and Exponential) are characterised by the scaling property and therefore also the second assumption does not appear restrictive.

The marginal likelihood contribution L_i^* can be expressed in term of its simulated counterpart as

$$L_i^*(\theta) = \int f(\Delta y_i | \theta, \Delta X_i, \Delta u_i) f(\Delta u_i | \sigma) d\Delta u_i = \quad (11)$$

$$= \mathbb{E}_{\Delta \tilde{u}} \left[\phi_{T-1} \left(\Delta y_i - \Delta X_i' \beta + \sigma \Delta \tilde{u}_i; \mathbf{0}, \Sigma_v \right) \right] = \quad (12)$$

$$\approx \frac{1}{D} \sum_{d=1}^D \left[\phi_{T-1} \left(\Delta y_i - \Delta X_i' \beta + \sigma \Delta \tilde{u}_{id}; \mathbf{0}, \Sigma_v \right) \right], \quad (13)$$

where $\phi_g(\cdot; \mathbf{m}, V)$ is the g -variate Gaussian density with mean vector $\mathbf{m}$ and covariance matrix V , and D is the number of draws from the multivariate distribution of the first-differenced rescaled inefficiency $\mathcal{F}_{\Delta \tilde{u}}$. Therefore, in order to apply this estimation method, we need to simulate from the distribution of $\Delta \tilde{u}_i$. This is straightforward if one is able to simulate from the univariate distribution of u_{it} . Indeed, the vector

$$\Delta \tilde{u}_{id} = (\tilde{u}_{i2d} - \tilde{u}_{i1d}, \dots, \tilde{u}_{iTd} - \tilde{u}_{i(T-1)d})'$$

is, by construction, a valid draw from $F_{\Delta \tilde{u}}$.

The resulting Marginal Maximum Simulated Likelihood Estimator (MMSLE) is asymptotically equivalent to the classical Marginal Maximum Likelihood Estimator (MMLE), but it requires the number of draws D as well as the sample size n to go to infinity with T fixed. It then follows that the accuracy of this approach relies on whether D is sufficiently large to guarantee that the average (13) is a good approximation of the expectation (12).⁷ It is worth noting that MMSLE and MLDVE can be considered complementary as the computational burden of the former discourages its use in favor of the latter in cases where the impact of the incidental parameters problem becomes negligible (panel data with $T > 15$).

3.1.1. Heteroscedastic case

The estimation procedure described in previous section can be easily generalised to the case of heteroscedastic inefficiency. This extension is important since both frontier parameters and inefficiency estimates may be adversely affected by neglected heteroscedasticity. Moreover, the inclusion in the model of explanatory variables correlated with inefficiency but not with unobserved heterogeneity could help to enhance parameters identification. However, in order to allow $\mathbf{u}$ to be heteroscedastic we need to impose an additional assumption.

Assumption iii) *The scale parameter of the inefficiency distribution $\mathcal{F}_u$ is time invariant for each unit, i.e. $\sigma_i = g(\mathbf{z}_i' \delta)$ where $g(\cdot)$ is a positive monotonic function, $\mathbf{z}_i$ a set of time invariant explanatory variables and δ is a vector of parameters to be estimated.*

In this case, the time invariant nature of the variables that affect inefficiency ensures that the simulated draws $\Delta \tilde{u}_{id}$ are identically distributed for each unit of the panel. The usefulness of this heteroscedasticity specification relies also on assumption iii), whose validity is strictly related to the empirical application.

⁷We conducted a simulation study to assess the minimum number of draws needed to get a suitable approximation of the expectation (12). The required number of draws quickly grows with T , making the MMSLE very time-consuming when $T > 15$ ($t = 5, D = 100,000$; $t = 10, D = 250,000$; $t = 15, D = 550,000$).

3.2. Pairwise Difference Estimator

As mentioned before, our second inferential strategy to estimate model (5–8) is based on the closed-form expression of the integral (10) when the inefficiency is exponentially distributed and $T = 2$.⁸ The difference $\Delta U = U_2 - U_1$ between two i.i.d. exponential random variables with scale parameter σ is known to have a Laplace distribution with p.d.f.

$$f(\Delta u|\sigma) = \frac{1}{2\sigma} \exp\left(\frac{|\Delta u|}{\sigma}\right). \quad (14)$$

Using this result, the marginal likelihood function for a two-period panel is

$$\begin{aligned} L^*(\theta) &= \prod_{i=1}^n f(\Delta y_{it}|\theta, \Delta \mathbf{x}_{it}) = \\ &= \prod_{i=1}^n \left\{ \int_{\mathbb{R}} f(\Delta y_{it}|\boldsymbol{\beta}, \tau, \Delta \mathbf{x}_{it}, \Delta u_{it}) f(\Delta u_{it}|\sigma) d\Delta u_{it} \right\} = \\ &= \prod_{i=1}^n \left\{ \int_{\mathbb{R}} \frac{1}{\sqrt{4\pi\tau^2}} \exp\left(-\frac{(\Delta \mu_{it} + \Delta u_{it})^2}{4\tau^2}\right) \frac{1}{2\sigma} \exp\left(\frac{|\Delta u_{it}|}{\sigma}\right) \Delta u_{it} \right\} = \\ &= \prod_{i=1}^n \left\{ \int_{\mathbb{R}^-} \frac{1}{\sqrt{4\pi\tau^2}} \exp\left(-\frac{(\Delta \mu_{it} + \Delta u_{it})^2}{4\tau^2}\right) \frac{1}{2\sigma} \exp\left(\frac{-\Delta u_{it}}{\sigma}\right) \Delta u_{it} \right. \\ &\quad \left. + \int_{\mathbb{R}^+} \frac{1}{\sqrt{4\pi\tau^2}} \exp\left(-\frac{(\Delta \mu_{it} + \Delta u_{it})^2}{4\tau^2}\right) \frac{1}{2\sigma} \exp\left(\frac{\Delta u_{it}}{\sigma}\right) \Delta u_{it} \right\} = \\ &= \prod_{i=1}^n \left\{ \frac{1}{2\sigma} \left[\exp\left(\frac{\tau^2}{\sigma^2} - \frac{\Delta \mu_{it}}{\sigma}\right) \Phi\left(\frac{\Delta \mu_{it}}{\sqrt{2}\tau} - \frac{\sqrt{2}\tau}{\sigma}\right) + \right. \right. \\ &\quad \left. \left. + \exp\left(\frac{\tau^2}{\sigma^2} + \frac{\Delta \mu_{it}}{\sigma}\right) \Phi\left(-\frac{\Delta \mu_{it}}{\sqrt{2}\tau} - \frac{\sqrt{2}\tau}{\sigma}\right) \right] \right\}, \end{aligned} \quad (15)$$

where $\Delta \mu_{it} = \Delta y_{it} - \Delta \mathbf{x}_{it}'\boldsymbol{\beta}$ and $\Phi(\cdot)$ is the cumulative density function of the Standard Normal distribution.

The marginal likelihood function (15) implies the existence of $H = \binom{T}{2}$ consistent “subsample” estimators. In fact, if we limit the inference to a subsample extracted considering any two waves of the panel, we can still consistently estimate the vector of parameters θ . Even if each of these H estimators shares the asymptotic properties of a MMLE over the whole sample, it exploits only a portion of the available information implying high inefficiency in finite samples. Similarly to Abrevaya (1999), we can produce a more efficient estimator by combining the H marginal log-likelihoods in just one objective function. The resulting estimator can be viewed as a quasi MMLE for the whole sample in which the correlation between the subsamples is ignored. As pointed out by Poirier & Ruud (1988), the application of a quasi MLE in a nonlinear panel data model in which the dependency structure among observations is mis-specified, often yields a consistent and asymptotically normal estimator.

Before discussing in detail the estimator, let us present this inferential process in a simplified case where only subsamples extracted from consecutive pairs of waves are considered. Similarly to the Partial

⁸We derive for the general case ($T \geq 2$) the p.d.f. of the random vector $\Delta \mathbf{u}_i$. However, the subsequent marginalization cannot be performed in closed form. These details are available from the authors upon request.

Maximum Likelihood approach used in Wang et al. (2009), by combining the marginal likelihood function defined in (15) we can define a “split-sample” estimator $\check{\theta}$ as

$$\check{\theta} = \operatorname{argmax}_{\theta \in \Theta} \frac{1}{nT^*} \sum_{i=1}^n \sum_{t=2}^T \log f(\Delta y_{it} | \theta, \Delta \mathbf{x}_{it}), \quad (16)$$

where $T^* = \text{int}(T/2)$. The distinguishing feature of this estimator from the MMLE is that we are *not* assuming a full sample likelihood factorization in term of the product of the subsamples likelihood contributions. Therefore this estimator is clearly not a Maximum Likelihood Estimator (MLE) but, more properly, an M-estimator.

As pointed out by Honoré & Powell (1994), the split-sample estimator is inefficient compared to the estimator $\tilde{\theta}$ defined as the maximizer of the following objective function

$$U_n(\theta) = n^{-1} \binom{T}{2}^{-1} \sum_{i=1}^n \sum_{t=2}^T \sum_{s < t} \log f(\Delta_t^s y_i | \theta, \Delta_t^s \mathbf{x}_i), \quad (17)$$

where $\Delta_t^s y_i = y_{it} - y_{is}$ and $\Delta_t^s \mathbf{x}_i = \mathbf{x}_{it} - \mathbf{x}_{is}$. Following previous literature (Honoré & Powell, 1994; Abrevaya, 1999, among the others), we refer to the maximizer of (17) as Pairwise Difference Estimator (PDE). This estimator maximizes an objective function which involves all pairs of waves, exploiting more information with respect to the split-sample estimator. Moreover, consistently with Abrevaya (1999), we find that the criterion function (17) is much smoother than the MMLE’s objective function.

As already discussed, the relevant property of this inferential procedure is that the asymptotics are also determined by $n \rightarrow \infty$ with fixed T . Even if we do not formally derive the large sample properties of the PDE, these proofs are a direct consequence of the consistency and the asymptotic normality of the corresponding subsample estimators.⁹ As the PDE belongs to the class of U-estimators, its asymptotic variance is equal to $A_0^{-1} B_0 A_0^{-1}$, where

$$A_0 = - \sum_{t=2}^T \sum_{s < t} \mathbb{E} (\nabla_{\theta\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \theta_0)), \quad (18)$$

and

$$B_0 = \mathbb{E} \left\{ \left[\sum_{t=2}^T \sum_{s < t} \nabla_{\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \theta_0) \right] \left[\sum_{t=2}^T \sum_{s < t} \nabla_{\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \theta_0) \right]' \right\}, \quad (19)$$

with ∇_{θ} and $\nabla_{\theta\theta}$ denote the vector of first derivatives and the Hessian matrix of the objective function respectively and θ_0 is the true parameter value. Estimation of the asymptotic variance of the PDE is straightforward, because for each pairwise difference we have that

$$-\mathbb{E}[\nabla_{\theta\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \theta_0)] = \mathbb{E}[\nabla_{\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \theta_0) \nabla_{\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \theta_0)'].$$

⁹We derive the large sample properties of the PDE estimator assuming the validity of the subsamples MMLEs’ asymptotic properties. A formal proof of these large sample properties, available upon request, follows the arguments used by Martins-Filho & Yao (2010) for the Normal-Half Normal cross-sectional case.

Therefore, the estimator of the asymptotic variance of $\tilde{\theta}$ is given by

$$\widehat{Avar}(\tilde{\theta}) = n^{-1} \hat{A}_0^{-1} \hat{B}_0 \hat{A}_0^{-1}, \quad (20)$$

where

$$\hat{A}_0 = \frac{1}{n} \sum_{i=1}^n \sum_{t=2}^T \sum_{s < t} \nabla_{\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \tilde{\theta}) \nabla_{\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \tilde{\theta}) \quad (21)$$

and

$$\hat{B}_0 = \hat{A}_0 + \frac{1}{n} \sum_{i=1}^n \sum_{t=2}^T \sum_{s < t} \sum_{(k,h) \neq (t,s)} \nabla_{\theta} \log f(\Delta_t^s y_i | \Delta_t^s \mathbf{x}_i, \tilde{\theta}) \nabla_{\theta} \log f(\Delta_k^h y_i | \Delta_k^h \mathbf{x}_i, \tilde{\theta}). \quad (22)$$

Notice that the consistency property of $\tilde{\theta}$ can be obtained under either $n \rightarrow \infty$ with fixed T , or $T \rightarrow \infty$ with fixed n . The convergence arguments used to informally justify the asymptotic properties of $\tilde{\theta}$ cannot be used for the $T \rightarrow \infty$ case, because the subsample MMLEs are no longer consistent. However, the proof can directly use the large-sample theory for minimizers of U-processes developed by Honoré & Powell (1994).

3.2.1. Heteroscedastic case

As in Section (3.1.1), the marginal likelihood (15) can be generalised to the case of heteroscedastic inefficiency. The difference $\Delta U = U_2 - U_1$ between two independent but not identically distributed exponential random variables $U_i \sim \mathcal{E}(\sigma_i)$, $i = 1, 2$, has the following p.d.f.

$$f(\Delta u | \sigma_1, \sigma_2) = \begin{cases} \frac{1}{\sigma_1 + \sigma_2} \exp\left(\frac{\Delta u}{\sigma_1}\right), & \text{if } \Delta u < 0, \\ \frac{1}{\sigma_1 + \sigma_2} \exp\left(-\frac{\Delta u}{\sigma_2}\right), & \text{otherwise.} \end{cases} \quad (23)$$

Thus, if we define the scale parameter of the inefficiency distribution in terms of a linear predictor as

$$\sigma_{it} = \exp(\mathbf{z}'_{it} \boldsymbol{\gamma}),$$

the resulting marginal likelihood, derived using the same approach of the homoscedastic case, can be expressed as

$$\begin{aligned} L^*(\boldsymbol{\theta}) &= \prod_{i=1}^n \left\{ \frac{\exp[-(\mathbf{z}_{i1} + \mathbf{z}_{i2})' \boldsymbol{\gamma}]}{\exp(-\mathbf{z}'_{i1} \boldsymbol{\gamma}) + \exp(-\mathbf{z}'_{i2} \boldsymbol{\gamma})} \times \right. \\ &\quad \times \left[\exp\left[\tau^2 \exp(-2\mathbf{z}'_{i2} \boldsymbol{\gamma}) - \exp(-\mathbf{z}'_{i2} \boldsymbol{\gamma}) \Delta \mu_{i2}\right] \Phi\left(\frac{\Delta \mu_{i2}}{\sqrt{2}\tau} - \sqrt{2} \exp(-\mathbf{z}'_{i2} \boldsymbol{\gamma}) \tau\right) + \right. \\ &\quad \left. \left. + \exp\left[\tau^2 \exp(-2\mathbf{z}'_{i1} \boldsymbol{\gamma}) + \exp(-\mathbf{z}'_{i1} \boldsymbol{\gamma}) \Delta \mu_{i2}\right] \Phi\left(-\frac{\Delta \mu_{i2}}{\sqrt{2}\tau} - \sqrt{2} \exp(-\mathbf{z}'_{i1} \boldsymbol{\gamma}) \tau\right) \right] \right\}. \quad (24) \end{aligned}$$

This marginal likelihood function can be used to define an objective function like (17). Therefore, the heteroscedastic case is completely analogous to the homoscedastic one.

3.3. Fixed-effects and inefficiency scores

A fundamental feature of SF models is the estimation of technical (cost) inefficiency. The standard approach is to post-estimate inefficiency exploiting the conditional distribution of u_{it} given ε_{it} . Following Jondrow et al. (1982) (JLMS) a point estimate of u_{it} can be obtained using the mean of this conditional distribution, $\mathbb{E}(u_{it}|\varepsilon_{it})$, evaluated at the regression residual $\hat{\varepsilon}_{it}$, where $\hat{\varepsilon}_{it} = y_{it} - \hat{\alpha}_i - \mathbf{x}'_{it}\hat{\boldsymbol{\beta}}$.

Since we consider a transformed model in which the α_i are ruled out from the parameter space, the estimation of the fixed-effects has to be performed in a second stage. An efficient estimator for $\boldsymbol{\alpha}$ can be obtained by maximizing the log-likelihood function of the untransformed model where the other parameters are substituted by a consistent estimates.¹⁰ A simpler alternative is given by

$$\hat{\alpha}_i = \frac{1}{T} \sum_{t=1}^T (y_{it} - \mathbf{x}'_{it}\hat{\boldsymbol{\beta}}) + \mathbb{E}(u_{it}|\hat{\sigma}) \quad i = 1, \dots, n, \quad (25)$$

where $\hat{\boldsymbol{\beta}}$ and $\hat{\sigma}$ are consistent estimates. This estimator is equivalent to the mean-adjusted estimator of α_i in the fixed-effects linear model and, therefore, it should be consistent as $T \rightarrow \infty$.

4. A Monte Carlo study

The Monte Carlo (MC) analysis in this section integrates previous simulation studies by Greene (2005a,b) and W&H. We investigate finite sample properties of the two inferential procedures described in section 3, comparing their results with those of the Greene's MLDVE. To this aim we consider as Data Generating Process (DGP) the following Normal-Exponential stochastic production frontier model

$$y_{it} = \alpha_i + \beta_1 x_{1it} + \beta_2 x_{2it} + v_{it} - u_{it}, \quad (26)$$

$$v_{it} \sim \mathcal{N}(0, \tau), \quad (27)$$

$$u_{it} \sim \mathcal{E}(\sigma_{it}), \quad (28)$$

$$\sigma_{it} = \exp(\gamma_0 + z_i \gamma_1), \quad i = 1, \dots, n, \quad t = 1, \dots, T, \quad (29)$$

where the fixed-effect parameters $\alpha_1, \dots, \alpha_n$ are drawn from a uniform distribution in $[0, 1]$, the two regressors x_{1it} and x_{2it} are distributed according to a Normal distribution centered in the corresponding fixed-effect α_i with variances equal to 1 and 4, respectively. As remarked by W&H, this DGP ensures correlation between regressors and fixed-effects, a typical scenario in which the fixed-effects specification is called upon. The inefficiency determinant z_i is a time invariant random draw from a Normal distribution with zero mean and variance equal to 0.25. Even though the PDE estimator allows time varying regressors in equation (29), the time invariance of z_i is imposed in order to compare the two proposed estimators under the same DGP.

The analysis is conducted under different dimensions. First, we consider the effect of different sample sizes ($n=100, 250$ and 500) and panel lengths ($T=5$ for the MSMLE and $T=5, 10, 15$ and 30 for the PDE). While the chosen cross-sectional dimensions allow to investigate the consistency property of the estimators

¹⁰Although our case is equivalent to the one reported in the paragraph (2.2.2) of W&H paper, an explicit formula for α_i cannot be obtained from the first-order condition of the log-likelihood due to the presence of the individual effect also in the arguments of the inverse Mills ratio.

when T is fixed, the motivation for choosing high values for the longitudinal dimension is twofold: they allow to mimic long panel datasets (common in macro applications of the SF model) and to consider a benchmark in which the impact of the incidental parameter bias on the JLMS scores is negligible.

Second, while we keep fixed in each experiment the values of the frontier parameters $\beta_1 = 0.3$ and $\beta_2 = 0.7$, the value of $\tau = 0.25$ and the value of $\gamma_1 = 1$, the design values for γ_0 are chosen to cover a wide set of signal to noise (STN) ratios ranging from 0.5 to 6. This setup gives a total of 18 experiments for the MMSLE and 72 experiments for the PDE. With respect to previous studies, we opt for a wider STN range in order to increase the validity of the MC analysis.¹¹

The evaluation of the PDE, MMSLE and MLDVE is based on 1000, 250 and 1000 converged replications for each experiment, respectively.¹² Tables from 2 to 5 summarize simulation results reporting the mean, the average standard error, the Mean Squared Error (MSE) and the Mean Absolute Percentage Error (MAPE) of the parameter estimates, as well as the linear and the Spearman rank correlation coefficient between the “true” inefficiencies and the estimated scores. Finally, the convergence rate of the optimizer is also reported.

4.1. Simulation results for the PDE

We start our discussion from the PDE simulation results as the broader set of experiments we have conducted offers a more comprehensive picture of the estimator properties. Table 2 reports the simulation results for all the experiments in which $T = 5$.¹³ When $\sigma/\tau = 2$, the PDE performs quite well even in presence of small samples ($n = 100$). All parameters are accurately estimated, as shown by an average absolute percentage error that never exceeds 20 percentage points. The correlation coefficient between the true inefficiency vector $\mathbf{u}$ and its estimate $\hat{\mathbf{u}}$ is 0.87 (the corresponding Spearman rank correlation coefficient is 0.67), which is quite large given the small sample size. The increase in the length of the panel (table 3) does not produce a substantial increase in the correlation coefficient suggesting that, even in short panels, the bias of the JLMS estimated scores induced by the fixed-effects additional estimation step appears to be negligible.

As expected, the estimator’s performances improve when T gets larger. The MSEs of γ_0 and γ_1 fall from 0.026 and 0.061 to 0.01 and 0.026 when the panel length increases from 5 to 10. The better performances stem from both a smaller variance and a smaller bias of the parameter estimates. Most important, the consistency property for $n \rightarrow \infty$ with fixed T is suggested by the reduction of all parameter’s MSE in these cases. For example, the MSEs of γ_0 and γ_1 decrease from 0.026 and 0.061 to 0.008 and 0.019 when n increases from 100 to 250.

When the signal is lower than the noise ($\sigma/\tau = 0.5$), the inference on the inefficiency determinants becomes more critical. The γ_1 estimate shows a large downward bias associated with a relatively large average standard error, especially when both the time and cross-sectional dimensions are small. This problem does

¹¹Even if previous simulation studies focus on STN between 1 and 4, extreme values can be often observed in empirical applications as, for instance, in the hospitals efficiency estimation. The same is true for the capital investment model estimated by W&H, in which the authors report a STN ratio $\sigma/\tau \approx 77$.

¹²We discard all the samples with a positive skewness. All estimators were programmed with Stata 11 by means of an extensive use of Mata. The programs are available from the authors upon request.

¹³For space reasons we do not include in the reported tables the results for the experiments in which the STN ratio is equal to 1.5 and 6 and in which $T > 10$, but these cases are summarised in figures 1 - 3.

not appear to be severe for larger n : the reduction in the bias associated with the case $n = 500$ and $T = 5$ looks very similar to the corresponding one with $n = 100$ but $T = 30$, ensuring an accurate inference in most SF empirical applications.

In the opposite extreme ($\sigma/\tau = 6$), it is the inference on τ and γ_0 that becomes markedly problematic. In particular in the $n = 100$ and $T = 5$ configuration, these two parameters are poorly estimated showing a MAPE of about 30 and 66 percentage points respectively, the highest bias found among all the PDE experiments. On the contrary, all other parameters including the inefficiency effect γ_1 are accurately estimated. These results clearly show that in presence of large STN ratios and small samples, reliable conclusions can be drawn from the effect of inefficiency determinants but not from the estimated average level of inefficiency.

These simulation evidences are summarised in figure 1: if the signal is lower than the noise the bias is concentrated on γ_1 and on the related inefficiency ranking, while in the opposite extreme case the average level of inefficiency is mismeasured, due to the relevant bias on γ_0 .

Finally, it is worth noting that the inferential performance of the JLMS estimator heavily depends on the STN ratio (Wang & Schmidt, 2009). In particular, when τ is relatively larger than σ , the distribution of the estimated inefficiency $\hat{u}$ is more shrunk toward the mean of the true distribution of u . Hence, the resulting bias implies a lower correlation between $\hat{u}$ and the true u that does not depend on the accuracy of variance estimates, but on the JLMS estimator intrinsic features. This is also suggested by the comparison between table 2 and table 3: an increase in the length of the panel does not produce a significant increase in the correlation between $\hat{u}$ and u .¹⁴

4.2. Simulation results for the MMSLE

The MMSL estimator performs satisfactorily when $1 \leq \sigma/\tau \leq 4$, which are the cases more frequently encountered in empirical applications. The τ estimates show a slight upward bias, with a MAPE ranging between 6 and 14 per cent and quite small average standard errors. However, variance parameter estimates seem to be strongly biased when the STN ratio attains extreme values ($\sigma/\tau < 1$ or $\sigma/\tau > 4$). Figure 2 offers a concise representation of the inferential performances of $\hat{\tau}$, $\hat{\gamma}_0$ and $\hat{\gamma}_1$ for different STN ratios. Consistently with PDE results, γ_0 appears to suffer higher ratios, while the opposite is true for γ_1 . In particular, when $n = 100$ and $\sigma/\tau < 1$, the MAPE of $\hat{\gamma}_1$ is very high (97 per cent). The graph also offers an informal proof of the estimator consistency as n gets larger.

The MMSLE shows high convergence rates (almost 100 per cent in all experiments) even in samples characterised by a skewness of the compounded error term very close to zero in which the other estimators do not converge. Our choice of reporting the MAPE implies that few outliers may strongly affect the simulation results. Actually, when $\sigma/\tau = 0.5$ or $\sigma/\tau = 6$, we observe extremely biased estimates in just two per cent of the simulated samples. By considering, for instance, the median absolute percentage error, the bias of the estimator in the aforementioned cases is more than halved.

We conclude this section comparing the two proposed estimation procedures. Surprisingly, the inferential performances of the PDE are very similar to those of the MMSLE for the intermediate STN ratio scenarios ($1 \leq \sigma/\tau \leq 4$) with efficiency losses that appear to be negligible. Moreover, the PDE seems to

¹⁴Similar conclusions can be drawn in the $T > 10$ cases. The corresponding tables are available upon request.

provide robustness against the STN ratio induced bias. Finally, we do not notice significant differences in the accuracy of the JLMS inefficiency scores.

4.3. A comparison with MLDVE

This section compares PDE and MMSLE with the Greene’s MLDVE. Two intertwined differences emerge: the first is represented by the effect of the incidental parameters problem; the second is related to the convergence rate of the criterion function optimization.¹⁵

Consistently with Greene (2005b), we find that the incidental parameters bias does not affect the frontier’s estimates. In this sense, the SF model behaves as a linear panel data model, in which the bias affects only the variance parameters.

When $\sigma/\tau \leq 1$ (tables 4 vs 2 and 1), the finite sample performances of the three estimation procedures look similar. However, in short panels it still remains a significant bias in the estimate of τ and γ_1 due to the incidental parameters problem: the MAPE of $\hat{\tau}$ is about 17 per cent with respect to a maximum of 7.5 per cent of the PDE and 4.5 of the MMSLE; similarly, the bias of the MLDVE for γ_1 is one third higher than that of the PDE.

As the STN ratio increases, the incidental parameters bias appears in the MLDVE with all its force, pushing $\hat{\tau}$ toward zero. An intuitive explanation for this behaviour is that the nuisance parameters bias affects the estimation in the same direction of STN ratio induced bias.¹⁶

Even in presence of a relatively long panel ($T = 10$, tables 5 vs 3), the PDE performs equivalently to the MLDVE for intermediate STN ratio scenarios (it is slightly preferable when $n \geq 250$), while it clearly outperforms the MLDVE in the extreme scenarios. As expected, in long panels ($T \geq 15$) the MLDVE shows very good performances, but nonetheless the two estimators still perform equivalently.

As far as the optimization of the criterion function is concerned, figure 4 shows the existence of a huge gap between the two proposed procedures and MLDVE. In particular, the optimization of the MLDVE’s criterion function shows critical problems in short panels and for extreme STN scenarios: the average rate of convergence in short panels ($T = 5$) is about 30 per cent, with a minimum of only 1.5 per cent in the case of the highest considered STN ratio and $n = 500$. Convergence performances largely improve as the panel length increases but, in the $\sigma/\tau = 0.5$ cases, still in two-third of the cases the convergence is not achieved. These results may explain the rare use of the “true” fixed-effects specification in empirical applications.

5. Empirical application: hospital’s technical efficiency

In this section, we apply the PDE estimator in an empirical study of Italian hospitals activity. Our analysis integrates the work of Daidone & D’Amico (2009) (D&D), to which we refer the reader for further details on data sources and variable definitions. In particular, we investigate how labor structure, ownership and level of specialization affect hospital’s technical efficiency. In the remainder of this section, we provide a brief discussion of the data, outline the model and, finally, present and discuss the efficiency analysis results.

¹⁵In order to produce a fair comparison, we use the same starting values and the same stopping rules for all estimators.

¹⁶In small samples in which the true STN ratio is high, even the standard MLE for cross-sectional models usually reports a significant downward bias in the estimate of τ .

5.1. Data

The data set considered in D&D consists of a yearly unbalanced panel of Italian hospitals located in the Lazio's region from 2000 to 2005. The panel contains 107 hospitals observed over a 6 years period, for a total of 622 observations. Hence, this data mirrors the sample size of one of the Monte Carlo scenarios presented in section 4 ($n = 100$ and $T = 5$).

The output variable is the number of acute patients discharges adjusted for its case-mix complexity through the Diagnosis Related Groups (DRG) weights. In order to represent the multi-output nature of the hospitals industry, we classify adjusted discharges using the information contained in the DRG categories. The classification system of the hospital activity in the study period is made up of 492 DRGs. In order to keep the estimation feasible, discharges have been aggregated into the following five output variables: Complex Surgery (Y_1), Emergency Room Treatments (Y_2), Cancers and HIV (Y_3), General Surgery (Y_4) and General Medicine (Y_5). We use as inputs the number of beds (X_4) as a measure of capital, while the number of physicians (X_1), nurses (X_2) and other personnel (X_3 , comprising teaching and ancillary staff) are used as a measure of the workforce.

One of the policy questions of this empirical application is to assess the role of specialization as determinant of technical efficiency. Following D&D, we have chosen the Gini ratio as an indicator of the level of specialization, which is equal to zero in the case of generalist hospitals with perfect equidistribution of health care services (e.g., polispecialistic medical center) and equal to one in the case of specialist hospitals characterised by a single specialty.

The second research question is about the link between ownership structure and inefficiency, thus we distinguish the hospitals in public, private but not-for-profit and private for profit.¹⁷ This classification reflects not only the ownership status, but also the founding sources. Finally, in order to analyze the impact of the labor structure on the inefficiency, both the nurses/beds and the physicians/beds ratios are included. Descriptive statistics and a brief summary of variable definitions are reported in table 6.

5.2. Model and estimation

A Stochastic Distance Function is the easiest solution to represent the multi-output production technology in a single SF equation.¹⁸ A general stochastic distance panel data model can be defined as

$$(y_{5it})^{-1} = D\left(\mathbf{x}_{it}, \frac{y_{it}}{y_{5it}}; \beta\right) \exp(u_{it} - v_{it}). \quad (30)$$

The dependent variable is the reciprocal of the chosen normalizing output y_{5it} (General Medicine), while the covariates are the inputs and the remaining normalised outputs, with an appropriately functional form selected for $D(\cdot)$. If we specify the functional form of the distance as a flexible Translog function, then the

¹⁷Public hospitals include hospitals directly managed by Local Health Authorities, independent public hospitals (Law Decree 502/92) and assimilated to public structures (Law 833/78).

¹⁸For a detailed discussion of distance functions and their properties see Kumbhakar & Lovell (2000).

general model defined in equation (30) can be written as

$$\begin{aligned}
 -y_{5it} = & \alpha_i + \sum_{m=1}^5 \delta_m y_{mit}^* + \sum_{k=1}^4 \beta_k x_{kit} + \frac{1}{2} \sum_{m=1}^5 \sum_{p=1}^5 \delta_{mp} y_{mit}^* y_{pit}^* + \frac{1}{2} \sum_{k=1}^4 \sum_{j=1}^4 \beta_{kj} x_{kit} x_{jit} + \\
 & + \sum_{k=1}^4 \sum_{m=1}^5 \beta_k \delta_m x_{kit} y_{mit}^* + \sum_{t=2001}^{2005} d_t - u_{it} + v_{it},
 \end{aligned} \tag{31}$$

where y_{mit}^* are the normalised outputs, d_t are year dummies, δ_{mp} and β_{kj} are technological parameters with the index i denoting hospital, t representing time, k and j labeling the input variables and m and p indicating outputs.¹⁹

The inefficiency u_{it} is assumed to be exponentially distributed with a scale parameter σ_{it} defined as

$$\sigma_{it} = \exp \left(\sum_{r=1}^R \gamma_r z_{rit} \right), \tag{32}$$

where the design matrix of the inefficiency determinants Z contains the Gini ratio (*Gini*), the nurses and the physicians per bed ratios (*Nurses/Bed* and *Physicians/Bed*), dummies for the quartiles of the number of beds (with the 1st quartile as the base category), ownership status dummies (public hospitals represent the base category), year dummies (with 2000 as the base year) and geographical dummies for each Local Health Authority in the region.

Our analysis differs from the one of D&D in two aspects. First, we estimate a “true” fixed-effects stochastic frontier model, while the authors present cross-sectional estimates from pooled data.²⁰ Secondly, since the investigation of ownership, time and geographical effects answers to some of the motivating questions of the analysis, we consider them as inefficiency determinants in equation (32). On the contrary, D&D used a two-step approach, regressing the estimated inefficiency scores on a set of explanatory variables, which may lead to be severely biased estimates (Wang & Schmidt, 2002).

It is worth to emphasize that Greene’s “true” fixed and random effects estimators, W&H estimators (both the within and the first-difference variants) and standard cross-sectional Normal-Exponential frontier estimator fail to converge. Hence, this empirical illustration provides an extreme STN ratio scenario in which the classical likelihood based estimators have convergence problems. Thus, in order to compare the PDE with an alternative estimator, we present a “pooled” version of the former in which the objective function becomes

$$U_n(\boldsymbol{\theta}) = \binom{N}{2}^{-1} \sum_{i=2}^N \sum_{j<i} \log f(\Delta_i^j y | \boldsymbol{\theta}, \Delta_i^j \mathbf{x}), \tag{33}$$

¹⁹Notice that all output and input variables are expressed in logarithms and that symmetry constraints have been imposed on the interaction terms, i.e $\delta_{mp} = \delta_{pm}$ and $\beta_{kj} = \beta_{jk}$.

²⁰D&D justify the data pooling using the empirical evidence that most of input and output variability is between rather than within the hospitals. However, the importance of including the fixed-effects in the model is to control for unobservable environmental factors, which otherwise will be captured by the inefficiency component. In SF analysis, panel data methods are therefore appropriate from a neglected heterogeneity perspective.

where, treating the sample as a cross-section, $\Delta_i^j y = y_i - y_j$, $\Delta_i^j \mathbf{x} = \mathbf{x}_i - \mathbf{x}_j$ and N is the total number of observations. In this way, we estimate a cross-sectional model using all the combinations of pairwise observations, exploiting the smoothness of the proposed U-statistic criterion function.²¹

5.3. Results and Discussion

The existing literature on Italian's hospitals efficiency found a declining trend of the average technical efficiency in the decade from 1996 to 2005, in particular for private not-for-profit structures. Recently, D&D found that public and private not-for-profit hospitals are significantly more efficient than private for-profit structures. A previous study of Barbeta et al. (2007), whose analysis is limited to the second part of the nineties, found that not-for-profit hospitals are more efficient than their public counterparts.

Table 7 summarizes technological parameter estimates using average elasticities of the distance function with respect to outputs and inputs. For the pooled case, the results significantly point to increasing returns to scale, since the null hypothesis of constant returns to scale is rejected ($H_0 : \epsilon_{Y,X} = 1$). Nurses and beds are positive while the other staff elasticity is not significantly different from zero. As expected, General Medicine presents the highest elasticity value among all the outputs, a result that could be economically interpreted as an indirect indication of the importance of this department in terms of hospital activity. This empirical evidence, both in terms of elasticities and return to scale, is in line with the estimates of the D&D study.

Also the elasticities from PDE estimates suggest a similar “technological” picture: return to scale are increasing and the number of beds still remains the most important input in the production process. However, the elasticity of the ancillary staff becomes relevant and negative. This result is not surprising, since this input is less involved in the production process. Hence, this evidence suggests that larger hospitals are more productive, but that the oversizing of ancillary staff may be considered as a ballast for the production process.

Regarding the determinants of the inefficiency, table 8 shows how controlling for the unobserved heterogeneity may largely affect the magnitude and the significance of the estimates. An interesting result from the “true” fixed-effects specification is that the level of specialization and the labor force structure assume a significant role in determining the hospital's inefficiency. PDE estimates shed some light on the importance of a balanced labor structure. In this case, the latter is altered by both a particularly high physicians/beds ratio and a small number of nurses per bed.

Looking at the effect of the ownership structure, results clearly show the key role played by the unobserved heterogeneity. In fact, while the pooled estimates suggest that private not-for-profit hospitals are less efficient than public ones, this effect vanishes in the “true” fixed-effects specification. This evidence may be justified by the fact that private hospitals are subject to caps on production and to discount on DRG tariffs for cost-containment reasons, leading for some private not-for-profit hospitals to a suboptimal level of production and to a higher level of inefficiency. This policy restriction has been constant over the study period and, therefore, its effect is captured by the individual effects.

Finally, it is worth noting that the estimated average STN ratio is relatively high. As discussed in section 4, the average inefficiency level could be poorly estimated in this case due to a severe bias in the intercept

²¹Following Honoré & Powell (1994), we also correspondingly adapt the asymptotic variance-covariance matrix estimator.

γ_0 . Thus, while we are confident about the reliability of the inefficiency effect estimates, simulation results from the upper left panel of table 2 recommend against commenting this kind of average evidence.

6. Conclusions

The influential paper of Greene (2005b) emphasizes the importance of disentangling inefficiency from unobservable individual heterogeneity in a SF panel setting. In this paper, we reconsider the estimation of the “true” fixed-effects stochastic frontier panel data model proposing two different estimators which achieve consistency as $n \rightarrow \infty$ with fixed T .

Monte Carlo evidence suggests that the incidental parameters problem does not only affect the estimation of the variance parameters, but also generates severe convergence problems in the likelihood optimization. We found a huge gap between the two proposed procedures and the MLDVE, with the latter showing critical problems especially in short panels and for extreme STN ratio scenarios.

Simulation results suggest that both the MMSLE and the PDE provide meaningful inference in settings where both n and T are small, as is the case in many empirical applications. Surprisingly, the inferential performances of the PDE are very similar to those of the MMSLE, with negligible efficiency losses. Moreover, the PDE appears to be robust to the bias induced by high STN ratios. In presence of long panels ($T > 15$), where the incidental parameters problem vanishes, the efficiency losses with respect to the MLDVE also appears to be negligible.

We apply the PDE to a sample of Italian hospitals in order to investigate the effect of ownership status, level of specialization and labor force structure on the inefficiency. This application provides a real world example in which the PDE’s objective function shows its favorable properties with respect to classical likelihood-based estimators, which in this case failed to converge. We find a fairly robust evidence that the level of specialization and the labor force structure assume a significant role in determining the hospital’s inefficiency while, after controlling for individual unobservable effects, the impact of the ownership status loses its statistical significance.

References

- Abrevaya, J. (1999). Leapfrog estimation of a fixed-effects model with unknown transformation of the dependent variable. *Journal of Econometrics*, 93, 203–228.
- Aigner, D., Lovell, C., & Schmidt, P. (1977). Formulation and estimation of stochastic frontier production function models. *Journal of Econometrics*, 6, 21–37.
- Aradillas-Lopez, A., Honoré, B. E., & Powell, J. L. (2007). Pairwise difference estimation with nonparametric control variables. *International Economic Review*, 48, 1119–1158.
- Barbetta, G. P., Turati, G., & Zago, A. M. (2007). Behavioral differences between public and private not-for-profit hospitals in the Italian national health service. *Health Economics*, 16, 75–96.
- Daidone, S., & D’Amico, F. (2009). Technical efficiency, specialization and ownership form: Evidences from a pooling of Italian hospitals. *Journal of Productivity Analysis*, 32, 203–216.
- Greene, W. (2005a). Fixed and random effects in stochastic frontier models. *Journal of Productivity Analysis*, 23, 7–32.
- Greene, W. (2005b). Reconsidering heterogeneity in panel data estimators of the stochastic frontier model. *Journal of Econometrics*, 126, 269–303.
- Greene, W. (2008). The econometric approach to efficiency analysis. In H. O. Fried, C. A. K. Lovell, & S. S. Schmidt (Eds.), *The Measurement of Productive Efficiency and Productivity Change* chapter 2. (pp. 92–250). Oxford University Press. New York.
- Honoré, B. E., & Powell, J. L. (1994). Pairwise difference estimators of censored and truncated regression models. *Journal of Econometrics*, 64, 241–278.

- Jondrow, J., Lovell, C., Materov, I., & Schmidt, P. (1982). On the estimation of technical efficiency in the stochastic production function model. *Journal of Econometrics*, 19, 233–238.
- Kumbhakar, S., & Lovell, C. (2000). *Stochastic frontier analysis*. Cambridge University Press.
- Lancaster, T. (2002). The incidental parameters problem since 1948. *Journal of Econometrics*, 95, 391–414.
- Martins-Filho, C., & Yao, F. (2010). *A note on some properties of a skew-normal density*. Working Papers 10-10 Department of Economics, West Virginia University.
- Meeusen, W., & van den Broeck, J. (1977). Efficiency estimation from cobb-douglas production function with composed errors. *International Economic Review*, 18, 435–444.
- Neyman, J., & Scott, E. (1948). Consistent estimates based on partially consistent observations. *Econometrica*, 16, 1–32.
- Poirier, D. J., & Ruud, P. A. (1988). Probit with dependent observations. *Review of Economic Studies*, 55, 593–614.
- Ritter, C., & Simar, L. (1997). Pitfalls of normal-gamma stochastic frontier models. *Journal of Productivity Analysis*, 8, 167–182.
- Wang, H., & Ho, C. (2010). Estimating fixed-effect panel stochastic frontier models by model transformation. *Journal of Econometrics*, 157, 286–296.
- Wang, H., Iglesias, E., & Wooldridge, J. (2009). Partial maximum likelihood estimation of a spatial probit model. *Mimeo*, .
- Wang, H., & Schmidt, P. (2002). One-step and two-step estimation of the effects of exogenous variables on technical efficiency levels. *Journal of Productivity Analysis*, 18, 129–144.
- Wang, W., & Schmidt, P. (2009). On the distribution of estimated technical efficiency in stochastic frontier models. *Journal of Econometrics*, 148, 36–45.

Tables and Graphs

Table 1: Simulation results of MMSL estimator. Simulation design: $T = 5$; $\tau = 0.25$; based on 250 converged Monte Carlo replications. Average standard errors are in parenthesis. $MSE(\hat{u}_{it}) = (NT)^{-1} \sum_{i=1}^n \sum_{t=1}^T (E(u_{it}|\epsilon_{it}) - u_{it})^2$ where $E(u_{it}|\epsilon_{it})$ is the JLMS estimator evaluated at $\epsilon_{it} = \hat{\epsilon}_{it}$, ρ represents the linear correlation between $E(u_{it}|\epsilon_{it})$ and u_{it} (the corresponding Spearman rank correlation coefficient is in parenthesis). The “-” denotes averages over Monte Carlo replications.

N=100	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.301 (0.014)	1.8e-04	3.584	0.302 (0.016)	2.2e-04	3.952	0.302 (0.020)	3.7e-04	5.042	0.298 (0.027)	8.9e-04	8.048
$\hat{\beta}_2$	0.700 (0.013)	1.7e-04	1.398	0.699 (0.015)	2.4e-04	1.717	0.699 (0.019)	3.8e-04	2.218	0.698 (0.027)	7.4e-04	3.214
$\tilde{\gamma}_0$	-3.165 (0.808)	12.913	40.949	-1.834 (0.258)	0.181	13.982	-1.018 (0.109)	0.013	8.863	-0.339 (0.070)	0.009	22.661
$\tilde{\gamma}_1$	1.596 (0.778)	9.804	96.778	1.109 (0.319)	0.201	30.244	0.985 (0.155)	0.033	14.176	0.901 (0.091)	0.032	14.923
$\bar{\sigma}_v$	0.252 (0.016)	2.1e-04	4.485	0.254 (0.019)	4.2e-04	6.650	0.253 (0.026)	6.5e-04	8.187	0.257 (0.039)	0.002	13.810
$MSE(\hat{u}_{it})$	0.013 (0.005)			0.029 (0.009)			0.065 (0.011)			0.182 (0.044)		
ρ	0.600 (0.417)			0.763 (0.538)			0.871 (0.675)			0.912 (0.754)		
Conv. Rate	0.990			1.000			1.000			1.000		
N=250	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.299 (0.008)	6.3e-05	2.151	0.300 (0.010)	7.6e-05	2.298	0.300 (0.012)	1.2e-04	2.999	0.302 (0.016)	2.5e-04	4.238
$\hat{\beta}_2$	0.701 (0.009)	7.2e-05	0.962	0.701 (0.010)	9.1e-05	1.091	0.702 (0.012)	1.3e-04	1.248	0.702 (0.017)	2.6e-04	1.771
$\tilde{\gamma}_0$	-2.470 (0.371)	0.219	13.931	-1.698 (0.135)	0.022	6.869	-1.000 (0.070)	0.005	5.591	-0.348 (0.048)	0.004	14.836
$\tilde{\gamma}_1$	1.043 (0.401)	0.220	35.853	0.997 (0.189)	0.037	14.415	1.008 (0.098)	0.015	9.635	0.980 (0.057)	0.022	12.126
$\bar{\sigma}_v$	0.248 (0.011)	1.5e-04	3.786	0.247 (0.013)	1.9e-04	4.161	0.248 (0.016)	2.7e-04	5.394	0.263 (0.024)	9.7e-04	10.100
$MSE(\hat{u}_{it})$	0.011 (0.002)			0.026 (0.002)			0.064 (0.007)			0.180 (0.030)		
ρ	0.614 (0.427)			0.779 (0.546)			0.873 (0.670)			0.910 (0.748)		
Conv. Rate	1.000			1.000			1.000			0.990		
N=500	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.006)	3.9e-05	1.686	0.300 (0.007)	4.9e-05	1.929	0.300 (0.009)	7.4e-05	2.352	0.299 (0.012)	1.4e-04	3.067
$\hat{\beta}_2$	0.700 (0.006)	3.3e-05	0.633	0.699 (0.007)	4.5e-05	0.747	0.699 (0.009)	7.6e-05	0.968	0.699 (0.012)	1.5e-04	1.406
$\tilde{\gamma}_0$	-2.498 (0.283)	0.142	10.469	-1.725 (0.095)	0.013	4.938	-1.005 (0.048)	0.002	3.727	-0.340 (0.038)	0.002	9.695
$\tilde{\gamma}_1$	1.087 (0.305)	0.169	29.336	1.023 (0.132)	0.026	12.225	1.004 (0.076)	0.008	7.633	0.975 (0.043)	0.014	9.060
$\bar{\sigma}_v$	0.249 (0.008)	6.0e-05	2.528	0.250 (0.009)	7.4e-05	2.785	0.251 (0.011)	9.1e-05	2.965	0.261 (0.018)	4.2e-04	6.255
$MSE(\hat{u}_{it})$	0.010 (0.001)			0.026 (0.002)			0.064 (0.005)			0.179 (0.022)		
ρ	0.631 (0.440)			0.783 (0.553)			0.876 (0.675)			0.912 (0.753)		
Conv. Rate	1.000			1.000			1.000			1.000		

Table 2: Simulation results of PD estimator. Simulation design: $T = 5$; $\tau = 0.25$; based on 1,000 converged Monte Carlo replications. Average standard errors are in parenthesis. $MSE(\hat{u}_{it}) = (NT)^{-1} \sum_{i=1}^n \sum_{t=1}^T (E(u_{it}|\epsilon_{it}) - u_{it})^2$ where $E(u_{it}|\epsilon_{it})$ is the JLMS estimator evaluated at $\epsilon_{it} = \hat{\epsilon}_{it}$, ρ represents the linear correlation between $E(u_{it}|\epsilon_{it})$ and u_{it} (the corresponding Spearman rank correlation coefficient is in parenthesis). The “—” denotes averages over Monte Carlo replications.

N=100	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.010)	1.9×10^{-4}	3.645	0.301 (0.012)	2.5×10^{-4}	4.214	0.301 (0.015)	4.4×10^{-4}	5.569	0.301 (0.022)	9.4×10^{-4}	8.113
$\hat{\beta}_2$	0.700 (0.010)	1.9×10^{-4}	1.562	0.700 (0.012)	2.6×10^{-4}	1.809	0.700 (0.015)	4.4×10^{-4}	2.387	0.700 (0.022)	9.4×10^{-4}	3.466
$\tilde{\gamma}_0$	-2.255 (1.042)	0.134	12.526	-1.747 (0.241)	0.057	11.096	-1.050 (0.111)	0.026	11.847	-0.358 (0.068)	0.010	23.107
$\tilde{\gamma}_1$	0.773 (0.838)	0.203	35.653	1.027 (0.318)	0.097	24.974	1.059 (0.181)	0.058	18.533	1.035 (0.126)	0.030	13.532
$\hat{\sigma}_v$	0.241 (0.024)	4.4×10^{-4}	6.383	0.249 (0.021)	5.5×10^{-4}	7.504	0.255 (0.029)	0.001	11.956	0.258 (0.051)	0.004	20.852
$MSE(\hat{u}_{it})$	0.012 (0.003)			0.028 (0.005)			0.068 (0.013)			0.182 (0.045)		
ρ	0.587 (0.414)			0.768 (0.542)			0.869 (0.668)			0.909 (0.752)		
Conv. Rate	0.711			0.923			0.997			1.000		
N=250	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.006)	7.1×10^{-5}	2.252	0.301 (0.007)	9.8×10^{-5}	2.610	0.301 (0.010)	1.6×10^{-4}	3.383	0.301 (0.014)	3.4×10^{-4}	4.910
$\hat{\beta}_2$	0.701 (0.006)	6.8×10^{-5}	0.930	0.701 (0.007)	9.3×10^{-5}	1.087	0.701 (0.010)	1.6×10^{-4}	1.463	0.701 (0.014)	3.4×10^{-4}	2.156
$\tilde{\gamma}_0$	-2.293 (0.377)	0.078	9.587	-1.724 (0.138)	0.026	7.349	-1.014 (0.064)	0.008	6.703	-0.339 (0.041)	0.004	13.998
$\tilde{\gamma}_1$	0.864 (0.387)	0.101	25.367	1.026 (0.182)	0.045	16.500	1.024 (0.103)	0.020	10.897	1.019 (0.074)	0.011	8.201
$\hat{\sigma}_v$	0.244 (0.013)	1.8×10^{-4}	4.074	0.250 (0.013)	2.4×10^{-4}	4.964	0.251 (0.018)	5.3×10^{-4}	7.329	0.252 (0.029)	0.002	12.520
$MSE(\hat{u}_{it})$	0.011 (0.002)			0.026 (0.003)			0.064 (0.008)			0.176 (0.027)		
ρ	0.617 (0.436)			0.781 (0.549)			0.875 (0.672)			0.912 (0.753)		
Conv. Rate	0.742			0.972			1.000			1.000		
N=500	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.005)	3.6×10^{-5}	1.584	0.300 (0.005)	4.9×10^{-5}	1.870	0.300 (0.007)	8.4×10^{-5}	2.423	0.300 (0.010)	1.7×10^{-4}	3.470
$\hat{\beta}_2$	0.700 (0.005)	3.7×10^{-5}	0.685	0.700 (0.005)	4.7×10^{-5}	0.774	0.700 (0.007)	8.0×10^{-5}	1.022	0.700 (0.010)	1.7×10^{-4}	1.471
$\tilde{\gamma}_0$	-2.312 (0.242)	0.048	7.503	-1.711 (0.092)	0.013	5.261	-1.009 (0.044)	0.004	4.589	-0.336 (0.028)	0.002	9.332
$\tilde{\gamma}_1$	0.894 (0.255)	0.062	20.082	1.007 (0.122)	0.024	12.177	1.009 (0.071)	0.010	7.671	1.006 (0.051)	0.005	5.700
$\hat{\sigma}_v$	0.245 (0.009)	9.8×10^{-5}	3.017	0.248 (0.009)	1.2×10^{-4}	3.535	0.250 (0.013)	2.5×10^{-4}	5.015	0.251 (0.021)	7.0×10^{-4}	8.287
$MSE(\hat{u}_{it})$	0.010 (0.001)			0.026 (0.002)			0.063 (0.005)			0.174 (0.018)		
ρ	0.631 (0.444)			0.784 (0.553)			0.876 (0.675)			0.912 (0.755)		
Conv. Rate	0.789			0.992			1.000			1.000		

Table 3: Simulation results of PD estimator. Simulation design: $T = 10$; $\tau = 0.25$; based on 1,000 converged Monte Carlo replications. Average standard errors are in parenthesis. $MSE(\hat{u}_{it}) = (NT)^{-1} \sum_{i=1}^n \sum_{t=1}^T (E(u_{it}|\epsilon_{it}) - u_{it})^2$ where $E(u_{it}|\epsilon_{it})$ is the JLMS estimator evaluated at $\epsilon_{it} = \hat{\epsilon}_{it}$, ρ represents the linear correlation between $E(u_{it}|\epsilon_{it})$ and u_{it} (the corresponding Spearman rank correlation coefficient is in parenthesis). The “--” denotes averages over Monte Carlo replications.

N=100	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\bar{\beta}_1$	0.300 (0.005)	7.9×10^{-5}	2.389	0.301 (0.005)	1.0×10^{-4}	2.712	0.301 (0.007)	1.8×10^{-4}	3.557	0.301 (0.010)	3.8×10^{-4}	5.216
$\bar{\beta}_2$	0.700 (0.005)	8.4×10^{-5}	1.047	0.700 (0.005)	1.1×10^{-4}	1.218	0.701 (0.007)	1.9×10^{-4}	1.554	0.701 (0.010)	3.9×10^{-4}	2.223
$\tilde{\gamma}_0$	-2.317 (0.297)	0.084	10.037	-1.742 (0.103)	0.036	8.495	-1.020 (0.047)	0.009	7.471	-0.341 (0.029)	0.004	14.715
$\tilde{\gamma}_1$	0.862 (0.305)	0.128	28.841	1.042 (0.136)	0.062	19.376	1.025 (0.075)	0.024	12.203	1.016 (0.053)	0.013	8.983
$\bar{\sigma}_v$	0.245 (0.010)	1.7×10^{-4}	4.091	0.250 (0.009)	2.6×10^{-4}	5.153	0.251 (0.013)	6.2×10^{-4}	7.918	0.253 (0.021)	0.002	12.813
$MSE(\hat{u}_{it})$	0.010 (0.002)			0.024 (0.003)			0.050 (0.007)			0.113 (0.022)		
ρ	0.633 (0.443)			0.806 (0.570)			0.905 (0.714)			0.945 (0.812)		
Conv. Rate	0.747			0.977			1.000			1.000		
N=250	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\bar{\beta}_1$	0.300 (0.003)	3.1×10^{-5}	1.490	0.300 (0.003)	3.9×10^{-5}	1.676	0.300 (0.004)	6.6×10^{-5}	2.155	0.300 (0.006)	1.4×10^{-4}	3.053
$\bar{\beta}_2$	0.700 (0.003)	3.1×10^{-5}	0.640	0.700 (0.003)	4.1×10^{-5}	0.731	0.700 (0.004)	6.9×10^{-5}	0.935	0.700 (0.006)	1.4×10^{-4}	1.342
$\tilde{\gamma}_0$	-2.346 (0.158)	0.045	7.277	-1.720 (0.060)	0.014	5.308	-1.008 (0.028)	0.003	4.523	-0.335 (0.018)	0.001	9.122
$\tilde{\gamma}_1$	0.937 (0.169)	0.060	19.710	1.022 (0.079)	0.025	12.076	1.010 (0.046)	0.009	7.157	1.007 (0.033)	0.005	5.288
$\bar{\sigma}_v$	0.246 (0.005)	7.5×10^{-5}	2.630	0.250 (0.006)	1.1×10^{-4}	3.367	0.250 (0.008)	2.4×10^{-4}	4.942	0.251 (0.014)	6.3×10^{-4}	8.052
$MSE(\hat{u}_{it})$	0.010 (0.001)			0.023 (0.002)			0.048 (0.004)			0.108 (0.013)		
ρ	0.650 (0.452)			0.808 (0.570)			0.906 (0.713)			0.945 (0.812)		
Conv. Rate	0.791			0.995			1.000			1.000		
N=500	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\bar{\beta}_1$	0.300 (0.002)	1.6×10^{-5}	1.087	0.300 (0.002)	2.1×10^{-5}	1.216	0.300 (0.003)	3.4×10^{-5}	1.560	0.300 (0.005)	7.0×10^{-5}	2.245
$\bar{\beta}_2$	0.700 (0.002)	1.6×10^{-5}	0.463	0.700 (0.002)	2.1×10^{-5}	0.527	0.700 (0.003)	3.5×10^{-5}	0.676	0.700 (0.005)	7.3×10^{-5}	0.983
$\tilde{\gamma}_0$	-2.367 (0.111)	0.027	5.595	-1.706 (0.041)	0.007	3.804	-1.003 (0.020)	0.002	3.215	-0.331 (0.013)	7.0×10^{-4}	6.378
$\tilde{\gamma}_1$	0.964 (0.118)	0.032	14.607	1.010 (0.054)	0.012	8.700	1.007 (0.032)	0.004	5.107	1.005 (0.023)	0.002	3.819
$\bar{\sigma}_v$	0.248 (0.004)	3.6×10^{-5}	1.845	0.250 (0.004)	6.0×10^{-5}	2.446	0.250 (0.006)	1.2×10^{-4}	3.547	0.251 (0.010)	3.1×10^{-4}	5.683
$MSE(\hat{u}_{it})$	0.009 (6.2×10^{-4})			0.023 (0.001)			0.048 (0.003)			0.110 (0.009)		
ρ	0.661 (0.458)			0.813 (0.574)			0.907 (0.715)			0.946 (0.812)		
Conv. Rate	0.869			1.000			1.000			1.000		

Table 4: Simulation results of MLDV estimator. Simulation design: $T = 5$; $\tau = 0.25$; based on 1,000 converged Monte Carlo replications. Average standard errors are in parenthesis. $MSE(\hat{u}_{it}) = (NT)^{-1} \sum_{i=1}^n \sum_{t=1}^T (E(u_{it}|\epsilon_{it}) - u_{it})^2$ where $E(u_{it}|\epsilon_{it})$ is the JLMS estimator evaluated at $\epsilon_{it} = \hat{\epsilon}_{it}$, ρ represents the linear correlation between $E(u_{it}|\epsilon_{it})$ and u_{it} (the corresponding Spearman rank correlation coefficient is in parenthesis). The “—” denotes averages over Monte Carlo replications.

N=100	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.014)	2.0×10^{-4}	3.759	0.300 (0.016)	2.7×10^{-4}	4.334	0.303 (0.024)	6.0×10^{-4}	6.594	0.302 (0.033)	0.001	8.751
$\hat{\beta}_2$	0.700 (0.014)	1.9×10^{-4}	1.586	0.700 (0.017)	2.7×10^{-4}	1.876	0.702 (0.024)	6.0×10^{-4}	2.766	0.701 (0.033)	0.001	3.818
$\hat{\gamma}_0$	-2.197 (0.361)	0.172	14.488	-1.610 (0.248)	0.070	12.065	-0.827 (0.143)	0.050	21.168	-0.349 (0.044)	0.002	11.410
$\hat{\gamma}_1$	0.686 (0.382)	0.245	41.003	0.823 (0.301)	0.122	28.799	0.608 (0.225)	0.204	42.009	0.733 (0.096)	0.081	26.717
$\hat{\sigma}_v$	0.206 (0.026)	0.003	17.518	0.189 (0.053)	0.006	24.366	0.038 (0.081)	0.051	84.846	5.1×10^{-9} (8.9×10^{-10})	0.062	100.000
$MSE(\hat{u}_{it})$	0.015 (0.009)			0.033 (0.017)			0.088 (0.019)			0.135 (0.025)		
ρ	0.565 (0.392)			0.759 (0.527)			0.849 (0.639)			0.933 (0.779)		
Conv. Rate	0.342			0.570			0.350			0.239		
N=250	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.008)	7.2×10^{-5}	2.249	0.300 (0.010)	9.6×10^{-5}	2.576	0.301 (0.016)	2.7×10^{-4}	4.324	0.301 (0.020)	3.9×10^{-4}	5.170
$\hat{\beta}_2$	0.700 (0.009)	7.3×10^{-5}	0.976	0.700 (0.010)	1.0×10^{-4}	1.124	0.701 (0.015)	2.2×10^{-4}	1.736	0.701 (0.020)	3.9×10^{-4}	2.278
$\hat{\gamma}_0$	-2.205 (0.282)	0.117	12.082	-1.560 (0.162)	0.046	10.254	-0.774 (0.061)	0.055	22.942	-0.348 (0.028)	0.001	8.160
$\hat{\gamma}_1$	0.720 (0.295)	0.166	34.348	0.782 (0.192)	0.085	24.619	0.526 (0.105)	0.236	47.512	0.727 (0.061)	0.078	27.333
$\hat{\sigma}_v$	0.210 (0.014)	0.002	16.002	0.191 (0.034)	0.005	23.624	0.008 (0.039)	0.060	96.631	2.1×10^{-9} (3.2×10^{-10})	0.062	100.000
$MSE(\hat{u}_{it})$	0.013 (0.004)			0.030 (0.011)			0.093 (0.011)			0.132 (0.016)		
ρ	0.591 (0.408)			0.772 (0.530)			0.842 (0.629)			0.935 (0.779)		
Conv. Rate	0.290			0.683			0.236			0.143		
N=500	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.006)	3.9×10^{-5}	1.681	0.300 (0.007)	5.1×10^{-5}	1.889	0.302 (0.011)	1.3×10^{-4}	2.976	0.300 (0.014)	2.0×10^{-4}	3.752
$\hat{\beta}_2$	0.700 (0.006)	3.9×10^{-5}	0.707	0.700 (0.007)	4.9×10^{-5}	0.779	0.701 (0.011)	1.3×10^{-4}	1.282	0.700 (0.014)	1.9×10^{-4}	1.571
$\hat{\gamma}_0$	-2.186 (0.214)	0.092	10.892	-1.556 (0.106)	0.032	9.186	-0.764 (0.025)	0.056	23.634	-0.347 (0.021)	7.2×10^{-4}	6.525
$\hat{\gamma}_1$	0.703 (0.226)	0.139	32.231	0.771 (0.127)	0.069	23.453	0.507 (0.052)	0.246	49.304	0.727 (0.043)	0.076	27.312
$\hat{\sigma}_v$	0.210 (0.011)	0.002	16.185	0.193 (0.014)	0.003	22.903	9.5×10^{-4} (0.013)	0.062	99.620	1.0×10^{-9} (1.7×10^{-10})	0.062	100.000
$MSE(\hat{u}_{it})$	0.012 (0.002)			0.028 (0.004)			0.094 (0.005)			0.132 (0.013)		
ρ	0.601 (0.414)			0.778 (0.534)			0.843 (0.629)			0.936 (0.781)		
Conv. Rate	0.198			0.743			0.118			0.062		

Table 5: Simulation results of MLDV estimator. Simulation design: $T = 10$; $\tau = 0.25$; based on 1,000 converged Monte Carlo replications. Average standard errors are in parenthesis. $MSE(\hat{u}_{it}) = (NT)^{-1} \sum_{i=1}^n \sum_{t=1}^T (E(u_{it}|\epsilon_{it}) - u_{it})^2$ where $E(u_{it}|\epsilon_{it})$ is the JLMS estimator evaluated at $\epsilon_{it} = \hat{\epsilon}_{it}$, ρ represents the linear correlation between $E(u_{it}|\epsilon_{it})$ and u_{it} (the corresponding Spearman rank correlation coefficient is in parenthesis). The “--” denotes averages over Monte Carlo replications.

N=100	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.301 (0.009)	8.5×10^{-5}	2.487	0.301 (0.010)	1.1×10^{-4}	2.737	0.301 (0.013)	1.6×10^{-4}	3.348	0.302 (0.019)	3.7×10^{-4}	5.037
$\hat{\beta}_2$	0.700 (0.009)	8.2×10^{-5}	1.026	0.700 (0.010)	1.1×10^{-4}	1.184	0.700 (0.013)	1.6×10^{-4}	1.461	0.701 (0.018)	3.3×10^{-4}	2.078
$\hat{\gamma}_0$	-2.318 (0.254)	0.071	9.270	-1.685 (0.139)	0.019	6.541	-0.981 (0.073)	0.006	6.040	-0.270 (0.080)	0.010	23.363
$\hat{\gamma}_1$	0.845 (0.306)	0.117	27.973	0.941 (0.201)	0.044	17.055	0.938 (0.119)	0.018	10.921	0.858 (0.139)	0.039	15.689
$\hat{\sigma}_v$	0.231 (0.011)	4.8×10^{-4}	7.644	0.229 (0.013)	6.1×10^{-4}	8.630	0.216 (0.017)	0.001	13.770	0.134 (0.088)	0.021	46.359
$MSE(\hat{u}_{it})$	0.011 (0.002)			0.023 (0.002)			0.043 (0.004)			0.081 (0.020)		
ρ	0.635 (0.441)			0.812 (0.573)			0.918 (0.729)			0.961 (0.842)		
Conv. Rate	0.395			0.795			0.992			0.651		
N=250	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.006)	3.4×10^{-5}	1.522	0.300 (0.006)	3.9×10^{-5}	1.664	0.300 (0.007)	5.6×10^{-5}	1.980	0.301 (0.011)	1.3×10^{-4}	2.984
$\hat{\beta}_2$	0.700 (0.006)	3.3×10^{-5}	0.654	0.700 (0.006)	3.9×10^{-5}	0.706	0.700 (0.008)	5.9×10^{-5}	0.876	0.700 (0.011)	1.3×10^{-4}	1.287
$\hat{\gamma}_0$	-2.343 (0.178)	0.035	6.447	-1.680 (0.079)	0.007	3.820	-0.977 (0.043)	0.002	4.003	-0.276 (0.060)	0.006	17.975
$\hat{\gamma}_1$	0.882 (0.216)	0.061	20.050	0.940 (0.116)	0.017	10.425	0.932 (0.070)	0.010	8.137	0.876 (0.103)	0.026	12.741
$\hat{\sigma}_v$	0.232 (0.007)	3.5×10^{-4}	7.026	0.229 (0.008)	5.2×10^{-4}	8.576	0.215 (0.011)	0.001	13.885	0.150 (0.069)	0.015	40.143
$MSE(\hat{u}_{it})$	0.010 (9.8×10^{-4})			0.022 (0.001)			0.042 (0.002)			0.075 (0.015)		
ρ	0.648 (0.449)			0.815 (0.573)			0.918 (0.727)			0.963 (0.845)		
Conv. Rate	0.358			0.824			0.997			0.590		
N=500	$\sigma/\tau = 0.5$			$\sigma/\tau = 1$			$\sigma/\tau = 2$			$\sigma/\tau = 4$		
	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE	Mean	MSE	MAPE
$\hat{\beta}_1$	0.300 (0.004)	1.6×10^{-5}	1.063	0.300 (0.004)	2.0×10^{-5}	1.199	0.300 (0.006)	3.1×10^{-5}	1.487	0.300 (0.008)	6.0×10^{-5}	2.050
$\hat{\beta}_2$	0.700 (0.004)	1.7×10^{-5}	0.467	0.700 (0.004)	1.9×10^{-5}	0.498	0.700 (0.005)	2.9×10^{-5}	0.610	0.700 (0.008)	5.9×10^{-5}	0.882
$\hat{\gamma}_0$	-2.344 (0.140)	0.023	5.158	-1.675 (0.057)	0.004	2.951	-0.975 (0.030)	0.002	3.237	-0.276 (0.045)	0.005	16.608
$\hat{\gamma}_1$	0.902 (0.163)	0.036	15.660	0.939 (0.085)	0.011	8.564	0.934 (0.050)	0.007	7.054	0.885 (0.077)	0.019	11.536
$\hat{\sigma}_v$	0.233 (0.005)	3.1×10^{-4}	6.739	0.229 (0.006)	4.6×10^{-4}	8.278	0.216 (0.007)	0.001	13.497	0.159 (0.051)	0.011	36.219
$MSE(\hat{u}_{it})$	0.009 (6.2×10^{-4})			0.022 (9.2×10^{-4})			0.042 (0.002)			0.074 (0.011)		
ρ	0.657 (0.456)			0.819 (0.576)			0.920 (0.729)			0.965 (0.847)		
Conv. Rate	0.279			0.865			0.998			0.586		

Table 6: Summary Statistics and brief definition for variables considered in our Hospital Technical Efficiency application. The data set consist of the population of Lazio Hospital (N=111) over the 2000-2005 period (for a total of 621 observations).

Variable	Definition	Mean	Std. Dev.
Output variables:			
Y_1	Sum of DRG weights related to Complex Surgery	979.15	2846.44
Y_2	Sum of DRG weights related to Emergency Room Treatments	189.87	260.08
Y_3	Sum of DRG weights related to Cancers and HIV	1170.59	2693.84
Y_4	Sum of DRG weights related to General Surgery	3506.41	4382.72
Y_5	Sum of DRG weights related to General Medicine	5358.47	8163.07
Input variables:			
X_1	No. of Physicians	111.45	189.11
X_2	No. of Nurses	223.45	388.37
X_3	No. of other staff	216.06	424.61
X_4	No. of beds	216.77	315.99
Inefficiency factors:			
Gini index		0.671	0.13
Nurses/Beds	No. of Nurses / No. of Beds ratio	0.881	0.574
Phys/Beds	No. of Physicians / No. of Beds ratio	0.47	0.333
Public Hospitals	Dummy variable for fully Public Hospitals	0.469	0.50
Not-for-profit Hospitals	Dummy variable for Not-for-profit Hospitals	0.163	0.37
For-profit Hospitals	Dummy variable for For-profit Hospitals	0.369	0.48
Rome	Dummy variable for Hospitals located in the Rome area	0.723	0.45
Viterbo	Dummy variable for Hospitals located in the Viterbo area	0.068	0.25
Rieti	Dummy variable for Hospitals located in the Rieti area	0.029	0.17
Latina	Dummy variable Hospitals located in the Latina area	0.081	0.27
Frosinone	Dummy variable for Hospitals located in the Frosinone area	0.100	0.30
Year 2000	Dummy variable for year 2000	0.169	0.37
Year 2001	Dummy variable for year 2001	0.166	0.37
Year 2002	Dummy variable for year 2002	0.169	0.37
Year 2003	Dummy variable for year 2003	0.172	0.38
Year 2004	Dummy variable for year 2004	0.167	0.37
Year 2005	Dummy variable for year 2005	0.156	0.36

Table 7: Scale and output distance elasticities evaluated at the average values. Comparison between PDE and Pooled PDE. $\epsilon_{Y,X}$ denotes return to scale estimated by $\sum_{i=1}^4 \epsilon_{Y,X_i}$.

	Pooled PDE			PDE	
	Estimate	St Error		Estimate	St Error
$\epsilon_{Y,X}$	1.041	0.025	$\epsilon_{Y,X}$	1.103	0.027
ϵ_{Y,X_1}	0.099	0.048	ϵ_{Y,X_1}	0.166	0.017
ϵ_{Y,X_2}	0.195	0.061	ϵ_{Y,X_2}	0.265	0.018
ϵ_{Y,X_3}	0.001	0.037	ϵ_{Y,X_3}	-0.120	0.013
ϵ_{Y,X_4}	0.746	0.066	ϵ_{Y,X_4}	0.791	0.028
ϵ_{Y,Y_1}	-0.009	0.012	ϵ_{Y,Y_1}	0.023	0.007
ϵ_{Y,Y_2}	0.086	0.018	ϵ_{Y,Y_2}	0.116	0.010
ϵ_{Y,Y_3}	0.096	0.015	ϵ_{Y,Y_3}	0.099	0.012
ϵ_{Y,Y_4}	0.216	0.030	ϵ_{Y,Y_4}	0.162	0.012
ϵ_{Y,Y_5}	0.612	0.026	ϵ_{Y,Y_5}	0.601	0.017

Table 8: Inefficiency effects. Comparison between PDE and “Pooled” PDE. The dependent variable in the output distance model is the inverse of the sum of the General Medicine DRG weights. $\bar{\sigma} = \frac{1}{N} \exp(Z'\gamma)$. Significance levels: “*”: $p < 10\%$, “**”: $p < 5\%$, “***”: $p < 1\%$.

	Pooled PDE	True FE PDE
2 nd quartile of beds	0.039 (0.07)	-0.718 *** (0.12)
3 rd quartile of beds	-0.096 (0.09)	-1.250 *** (0.15)
4 th quartile of beds	-0.124 (0.11)	-1.728 *** (0.19)
Gini	0.130 (0.39)	-0.979 * (0.51)
Nurses/beds	-0.247 (0.15)	-0.400 ** (0.16)
Physicians/beds	0.050 (0.18)	0.444 ** (0.21)
Private	-0.084 (0.11)	-0.190 (0.15)
Not-for-profit	0.279 *** (0.08)	-0.030 (0.14)
Year 2001	0.001 (0.07)	-0.227 ** (0.11)
Year 2002	-0.043 (0.07)	-0.355 *** (0.11)
Year 2003	-0.018 (0.08)	-0.159 (0.11)
Year 2004	0.070 (0.09)	0.058 (0.11)
Year 2005	0.066 (0.08)	0.303 *** (0.12)
Viterbo	0.064 (0.09)	-1.224 *** (0.22)
Latina	0.118 (0.11)	-1.084 *** (0.20)
Rieti	-0.016 (0.11)	-0.639 ** (0.32)
Frosinone	-0.029 (0.11)	-0.741 *** (0.16)
Constant	-0.968 *** (0.31)	-0.142 (0.43)
σ	0.34	0.13
τ	0.04	0.03
σ/τ	8.1	4.8

Figure 1: Signal-to-noise ratio “induced” bias on inefficiency effects and variance parameters. The blue triangular, the red square and the green circle lines respectively refer to γ_0 , γ_1 and τ .

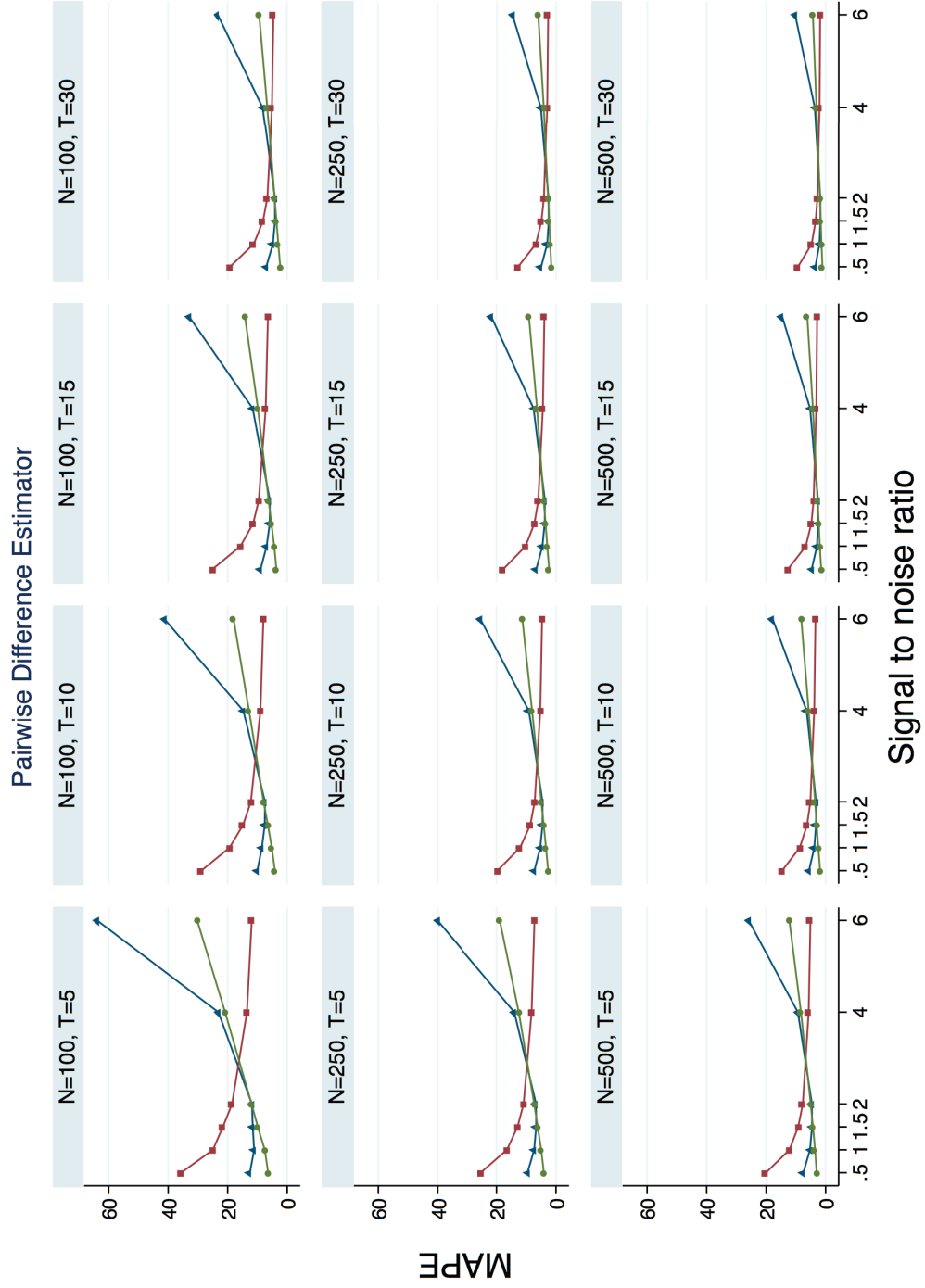


Figure 2: Signal-to-noise ratio “induced” bias on inefficiency effects and variance parameters. The blue triangular, the red square and the green circle lines respectively refer to γ_0 , γ_1 and τ .

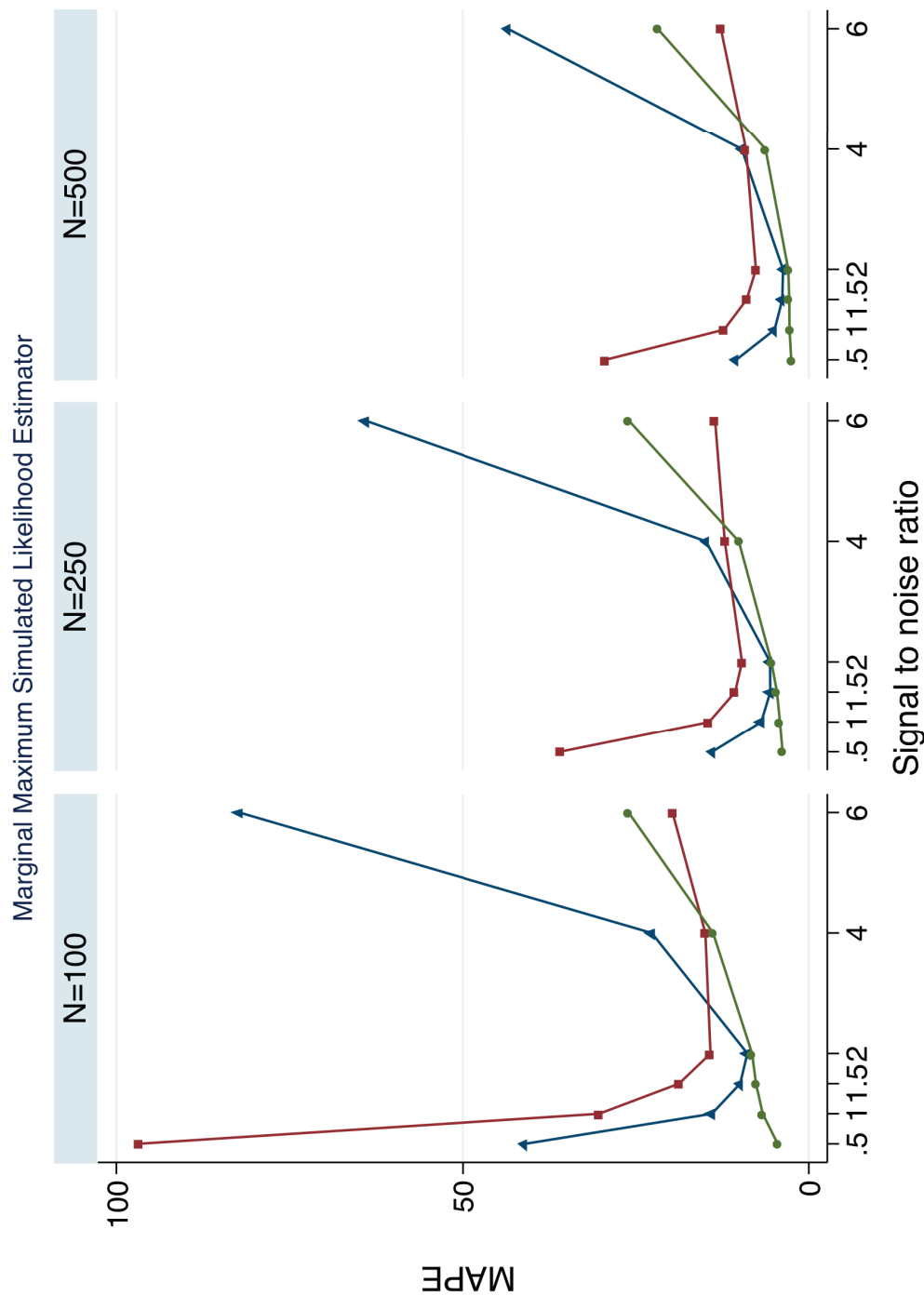


Figure 3: Signal-to-noise ratio “induced” bias on inefficiency effects and variance parameters. The blue triangular, the red square and the green circle lines respectively refer to γ_0 , γ_1 and τ .

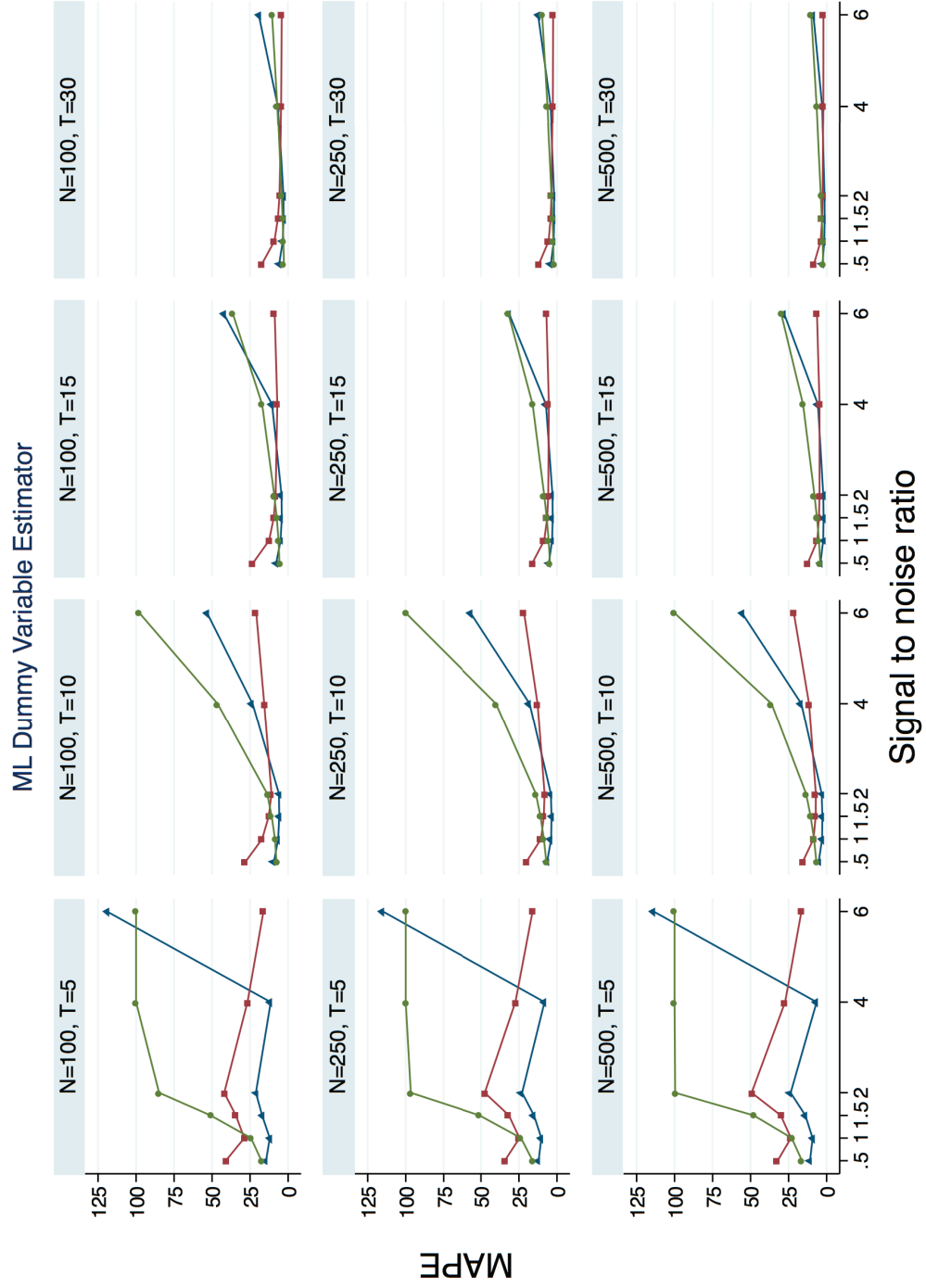


Figure 4: PD & MMSL vs MLDV estimator: convergence rates comparison. Rates are computed over 1,000 MC converged replications. The blue triangular, the red square and the green circle lines respectively refer to the PD, MMSL and MLDV estimators.

