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Abstract

This paper shows that, in a pure currency economy with heterogeneous agents and multiple commodities, a pecuniary externality plays a key role in making the equilibrium allocation constrained inefficient. Monetary policy intervention can help improve matters.

Keywords: Money, Heterogeneity, Pecuniary Externality, Monetary Policy
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1 Introduction

In his famous essay on the optimum quantity of money, Milton Friedman (1969) argued that the social optimum can be achieved by a monetary policy that equates the cost of holding fiat money to its benefit. In pure currency environments in which economic agents hold money to transfer their income over time and discount future payoffs relative to current ones, the optimal policy – known as the *Friedman rule* - takes the form of a contraction of the money stock at the agents' discount rate, rewarding impatient agents who hold an asset that carries no intrinsic return - i.e. money, with implicit interest, via a deflation induced by the contraction (Truman Bewley (1980) and Robert Townsend (1980)). Subsequently, David Levine (1991)

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has argued that in pure currency economies in which agents hold money to self-insure against uncertain trading prospects, a monetary expansion inducing an inflation may benefit unlucky traders who have run out of cash. Recently, Neil Wallace (2014) has conjectured that the interplay of these two forces should make active monetary intervention of either the contractionary or expansionary type socially optimal in any pure currency economy - except possibly in knife-edge, degenerate cases in which the monetary authorities could instead refrain from intervening.

This paper presents a pure currency environment in which money is essential as a medium of exchange, and shows that, in such an economy, a fairly general force is at work, whereby an asset reallocation among agents induces a change in their purchasing power, which, in turn, alters the relative price between commodities, with knock-on effects on the the equilibrium allocation. This force, known in the literature as the pecuniary externality, may invite intervention by monetary authorities, sometimes of the expansionary type, even in the absence of any insurance role for money. With complete markets the externality is correctly priced, while in borrowing constrained - or, more generally, incomplete markets- economies, it may not be fully internalized by the agents. Although necessary, borrowing constraints are not sufficient for the externality to emerge. The environment should also feature: *i.* multiple commodities, otherwise there would be no relative price to speak of; *ii.* a less than perfectly elastic supply of the liquidity constrained commodity, otherwise its price would not change; and *iii.* some agents' heterogeneity, otherwise the asset reallocation would have symmetric effects on all of them, silencing the effects of the externality. Uncertainty is not crucial.

We exhibit the emergence of this force in the standard microfounded monetary economy by Lagos and Wright (2005), which is suitable for our purposes, since it has borrowing constraints and multiple commodities. We extend the model introducing ex-ante agents' heterogeneity, giving rise to a non-degenerate, but still manageable equilibrium distribution of money holdings, preserving the tractability of the framework, while allowing for asymmetric effects of changes in monetary conditions. The monetary authority is subject to the same informational limitations as the rest of the economic actors and, hence, the set of policy instruments is restricted to incentive feasible ones. We show that the pecuniary externality emerges generically in the space of cost functions, affects the efficiency of equilibrium, and is relevant for policy analysis.

First, we characterize the equilibrium, static and dynamic, proving existence and uniqueness, and showing some comparative statics results. Then, we move on to the question of efficiency and optimal policy. The equilibrium is always inefficient relative to first-best, for any feasible monetary policy that might be adopted by public authorities. The equilibrium is also generically *second-best inefficient* in the absence of active policy intervention. Hence, some monetary intervention, expansionary or contractionary, is required if authorities are concerned with constrained efficiency. In general, by exploiting the presence of the pecuniary externality, public authorities can improve upon the equilibrium allocation, altering the distribution of cash among different agents and, hence, their purchasing power, thus, modifying the relative price between the good acquired with money and the other commodities. This option is not available if there is no ex-ante heterogeneity or if the supply of the good acquired with money is perfectly elastic, due to a linear cost of production. The effect is so pervasive that there are robust examples in which the equilibrium remains second-best inefficient under any feasible monetary policy intervention. Finally, we ask whether monetary authorities should intervene through contractionary or expansionary policies when the objective is to improve welfare. Expansionary policy helps when the elasticity of supply of the good acquired with money is low, and, hence, the pecuniary externality is sufficiently strong.

There is a vast literature on the optimum quantity of money in pure currencies economies, that has formalized Friedman's intuition first in models where money was assumed to enter the economy via the utility function (Brock (1974)) or via exogenously imposed cash-in-advance constraints (Grandmont and Younes (1973)), and, then, subsequently in models in which money served as a store of value (Bewley (1980), Townsend (1980)). Friedman's idea applies also to the Lagos and Wright (2005) environment, where money plays the role of a medium of exchange. As mentioned before, Levine (1991) has proposed an alternative channel, whereby a steady monetary expansion is beneficial since it redistributes resources from rich sellers to poor buyers. Scheinkman and Weiss (1986) have made a similar point, regarding one time monetary expansions and Lippi, Ragni and Trachter (2015) extended it to time varying monetary policy. Kehoe, Levine and Woodford (1992) analyzed an environment in which both the rate of return dimension considered by Friedman and the redistributive rationale of Levine are present and monetary expansions have both the positive effect of providing insurance for unlucky buyers and the negative consequence of reducing the value of money. Specific assumptions on fundamentals can make one or the other prevail. Bhattacharya, Haslag and Martin (2005) have shown that, in several models, including Lagos and Wright (2005), with agents' heterogeneity, redistributive effects may make inflationary policies beneficial away from the Friedman rule. Boel and Waller (2015) have extended the Lagos and Wright (2005) model in a way which is akin to ours to analyze the efficacy of quantitative easing at the zero lower bound. The generic second-best inefficiency of equilibrium allocations when markets are incomplete was first pointed out by Hart (1975), and subsequently formalized by Geanakoplos and Polemarchakis (1986). These are economies in which there is only a limited number of instruments to transfer purchasing power across states of uncertainty. Our model is one where, instead, there is no uncertainty, but only one instrument, i.e. money, to finance expenditures in the presence of active liquidity constraints. To the best of our knowledge, the pecuniary externality has not so far been pointed out to play a significant role in monetary models in which money is essential. A similar effect has been pointed out to play a role in non-monetary, credit constrained economies, by Lorenzoni (2008), in an economy with uncertainty, and by Moore (2013), in two examples without uncertainty. The present model easily lends itself to a comparison with a version of Levine (1991) in which the agents' types alternate deterministically over time and exchange multiple goods. Absent non-trivial uncertainty, the distributive effect of Levine (1991) plays no role. With more than one good traded, though, the relative price effect, i.e. the pecuniary externality, is at work.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the equilibrium. Section 4 analyzes monetary policy. Section 5 concludes. The derivation of equilibrium and the proofs are in the Appendix.

2 Model

Fundamentals The model builds on a version of Lagos and Wright (2005) with competitive markets. Time is discrete and continues forever. Each time period is divided into two sub-periods, day and night, in which two different goods are produced, traded and consumed by a continuum of infinitely-lived agents. There are two types of agents, indexed by $i = 1, 2$, with equal mass. During the day, agents can trade a perishable consumption good, x, and face randomness in their preferences and production possibilities. With equal probability, an agent may turn out to be in a position to consume but unable to produce, i.e. a buyer, or viceversa, a seller. Consumption yiels utility $u(\cdot)$, with $u'(\cdot) > 0$ and $u''(\cdot) < 0$. Production entails a utility cost $c(\cdot)$, with $c'(\cdot) > 0$ and $c''(\cdot) \geq 0$. Usual Inada conditions are assumed. During the night, agents can produce, trade and consume another perishable good, X, which serves as the numeraire of the economy. In contrast to the first sub-period, there is no randomness in the second sub-period. Agents can consume and produce the night-time good with linear utility and linear cost of effort. We depart from Lagos and Wright (2005) simply by assuming that current period payoffs have different weights for the two types, δ_i. We also assume that each type i weighs current payoffs alternately over time, one period with δ_i and the next with δ_h, $h \neq i$, starting with δ_i. As we will see later on, this is sufficient to obtain a non degenerate but tractable distribution of money holdings accross agents. Agents discount future payoffs at a positive rate $\beta < \delta_i$ for all i.

Exchange of goods Exchange of x during the day is anonymous and happens at a competitive price p in units of X. The market for the night-time good is walrasian with price normalized to unity.

Money An intrinsically worthless, perfectly divisible and storable object called fiat money is available in the economy, which can be used to trade goods. Its supply is M, and its value in units of X is ϕ.

Government The Government can alter the money supply using policy tools, τ, denominated in units of X and operated at night. The Government does not observe agents' preferences. Hence, the policy scheme cannot be directly conditioned on the agents' type and participation in it is voluntary.

3 Monetary Equilibrium

We construct symmetric[1] equilibria with valued money.[2] The sequence of trades within a period is as follows. During the day, after the realization of uncertainty, the buyers spend money to purchase the consumption good, x, in a competitive and anonymous market. The sellers produce and trade the good in exchange for money. During the night, all agents consume and produce the other good, X, and acquire cash for the following period. The Government operates the transfers at the end of the night.

[1] i.e. where ex-ante identical agents are treated identically.

[2] There is always an equilibrium without trade in which the value of money is nil.

3.1 Individual Behavior

We describe, first, the decision problem of individual agents taking prices as given, starting with the decisions taken during the day, after the realization of uncertainty, and, then, moving to the decisions taken during the night. The derivation of the optimality conditions can be found in the Appendix.

Day-time We consider, first, the decision problem of a buyer, then, of a seller. A buyer of type i chooses consumption x_i to solve

$$V_i^b(m_h) = Max \;\; \delta_i u(x_i) + W_i^b\left(\overline{m}_i^b\right),$$

with $h \neq i$, subject to the constraint, whose non-negative multiplier appears in square brackets,

$$px_i \leq \phi m_h, \qquad [\lambda_i] \tag{1}$$

which reflects the purchase of the consumption good with cash, limited by the value of the amount held at the beginning of the period, m_h.[3] The function $W_i^b\left(\overline{m}_i^b\right)$ represents the value of operating in the night market with unspent money holdings $\overline{m}_i^b = m_h - \frac{px_i}{\phi}$. A seller of type i chooses an amount of the good y to solve

$$V_i^s(m_h) = Max \;\; -\delta_i c(y) + W_i^s\left(\overline{m}_i^s\right),$$

where $W_i^s\left(\overline{m}_i^s\right)$ represents the value of operating in the night market with money holdings $\overline{m}_i^s = m_h + \frac{py}{\phi}$, comprising the initial amount held at the beginning of the period and the amount acquired selling the day-time good. Given that the cost function is the same for all types, and trade occurs in a single market at a uniform competitive price, production will be uniform across types. Hence, we have dropped the sub-script for y. The expected value of entering any given period, before the realization of uncertainty, for an agent of type i is $V_i\left(m_h\right) = \frac{1}{2}V_i^b(m_h) + \frac{1}{2}V_i^s(m_h)$,

[3] The sub-script keeps track of the fact that the payoff of the agent had a different δ in the previous period.

since he or she begins the day with asset holdings m_h, and there is an equal probability of being a buyer or a seller.

Night-time At the beginning of the night, an agent who was of type $j = b, s$ during the day faces the choice over consumption X_i^j, effort e_i^j and money holdings for the future, $m_{i,+1}$, to solve

$$W_i^j\left(\overline{m}_i^j\right) = Max\ \delta_i\left(X_i^j - e_i^j\right) + \beta V_h\left(m_{i,+1}\right),$$

where $V_h\left(m_{i,+1}\right)$ represents the expected value of operating in the following day market with money holdings $m_{i,+1}$. The maximization is subject to the budget constraint,

$$X_i^j + \phi m_{i,+1} = e_i^j + \phi\overline{m}_i^j + \tau, \tag{2}$$

which states that the effort, the real value of current money holdings and Government transfers can be used to acquire night-time consumption and money for the future. We have incorporated the idea, which is standard in the Lagos and Wright (2005) framework, that these decisions are the same for all the agents of the same type. This is due to the linearity of the night-time payoff, which allows to separate the decisions about future asset holdings from current holdings.

3.2 Government Policy

The term τ in the budget constraint (2) represents the policy intervention of the Government at night, after trade has occurred. The environment places restrictions on the type of intervention that is feasible for the Government. Since the agents' type is private information, the policy cannot be tailored to the agents' identities. However, the Government can exploit the potential heterogeneity in money holdings of the agents, providing them with the incentive to reveal such holdings. Consider a scheme that responds to the agents' real money holdings at the end of the night, in a linear way, $\tau = \tau\left(e_i^j - X_i^j + \phi\overline{m}_i^j\right) =$

$$a + b\left(e_i^j - X_i^j + \phi\overline{m}_i^j\right), \tag{3}$$

with type independent parameters a and b, due to the agents' anonymity. Under policy (3) with $b \geq 0$, the agents receive transfers that are non-decreasing in money holdings, thus, providing the incentive to reveal them to the Government. Agents may also receive a constant payment or be asked to pay a constant fee, a. However, coercive taxation is infeasible and participation in the transfer scheme must be voluntary (Andolfatto (2010)). Therefore, the overall amount (3) should be non-negative, if the Government wants agents to participate. This two-part transfer scheme - taken from Wallace (2014)- avoids strategic manipulation issues. We will restrict attention to the case in which the Government makes sure that both types have the incentive to participate, and, hence, (3) is non-negative for both types. Therefore, the budget constraint of the Government is

$$\phi M_{+1} = \phi M + a + b \sum_i \frac{e_i^j - X_i^j + \phi \overline{m}_i^j}{2}. \tag{4}$$

A transfer scheme with $a = 0$ and $b > 0$, has the only effect of augmenting in equal proportion the money holdings of both types, without any other real effect. We will refer to this type of policy as neutral intervention. A transfer scheme with $a > 0$ tends to favor cash-poor types, and with $a < 0$, rich types. We will refer to them as progressive and regressive intervention, respectively. We simplify Government policy, reducing it to the choice of one variable, which we define as $\gamma \equiv \frac{M_{+1}}{M(1+b)}$, the gross rate of money supply, rescaled by b. Using the definition of γ, (4), (3) and the fact that the two types hold the entire stock of money at any given time, we obtain

$$\frac{a}{1+b} = \phi M (\gamma - 1). \tag{5}$$

Policy will, then, be summarized by the parameter γ, with a lower bound that depends on b. In particular, if $b > 0$, the lower bound on γ is strictly below unity, hence, the constant component of the transfer, a, may become negative. The parameter b acts only as a scaling factor and will not appear explicitly henceforth. Notice that, by (5), neutral intervention, $a = 0$, corresponds to $\gamma = 1$, while $\gamma > 1$, corresponds to a progressive policy that tends to redistribute resources toward cash-poor types, $a > 0$, and $\gamma < 1$, a regressive policy that tends to redistribute resources toward cash-rich types, $a < 0$.

3.3 Agents' Optimality

Taking Government policy and prices as given, the agents' optimization leads to an Euler condition for each type,

$$\frac{\phi}{1+b} = \frac{\beta}{2\alpha_h}\phi_{+1}\left[\frac{u'(x_{i,+1})}{p_{+1}} + 1\right], \tag{6}$$

where $\alpha_h = \frac{\delta_h}{\delta_i}$, for $h \neq i$. Equation (6) governs the intertemporal decision of a type i agent to accumulate cash. Optimality in production leads to a condition that equates the marginal cost of production to the price of the day-time good,

$$c'(y) = p, \tag{7}$$

since the day-time market is competitive. These conditions determine the demand and supply of the agents for the day-time good as a function of its price. The real demand for money is, then, determined by the binding constraint, (1)

$$px_i = \phi m_h. \tag{8}$$

The prices are determined by the market clearing conditions which are stated next.

3.4 Market Clearing

Market clearing for the day-time good requires equality of aggregate demand and supply,

$$\sum_i \frac{x_i}{2} = y, \tag{9}$$

and the money market clearing condition is

$$\sum_i \frac{m_i}{2} = M. \tag{10}$$

Since the night market for good X clears whenever the other markets do by Walras Law, we omit its market clearing condition.

3.5 Equilibrium

Optimality and market clearing conditions together give rise to the equilibrium system, whose solution -through a fixed point argument- delivers the equilibrium allocation and the prices. Using the definition of γ, (8), (9) and (10), we obtain for every type

$$py = \frac{\beta}{2\gamma\alpha_h} p_{+1} y_{+1} \left[\frac{u'\left(x_{i,+1}\right)}{p_{+1}} + 1 \right]. \tag{11}$$

Hence, we can reduce the fixed point problem to four equations, (11) for each type, (7) and (9), in four unknowns, namely, the demand of the day-time good for each type, the suppy and its price. We now state the definition of a monetary equilibrium. Let x be the vector of demands of the day-time good for each type.

Definition 1 *A monetary equilibrium (ME) is a vector* (x, y, p)*, satisfying (11), (7) and (9), for any admissible* γ*. A stationary monetary equilibrium (SME) is a time invariant ME.*

Monetary equilibria will be parameterized by the policy instrument, γ, which affects the ME through the Euler conditions, (11). First, we analyze stationary or steady state equilibria, then, we look at dynamic equilibria.

3.6 Steady State

At an SME, the equilibrium system can be reduced to an equation that determines the demand of the day-time good as a function of its price,

$$x_i = u'^{-1}\left(\frac{2\gamma - \beta/\alpha_h}{\beta/\alpha_h} p \right), \tag{12}$$

for each type i and $h \neq i$, the supply of the day-time good as a function of its price,

$$y = c'^{-1}\left(p\right), \tag{13}$$

which we can substitute into the market clearing condition for the day-time good, to obtain

$$F(p) \equiv \sum_i u'^{-1}\left(\frac{2\gamma - \beta/\alpha_i}{\beta/\alpha_i}p\right) - 2c'^{-1}(p) = 0. \tag{14}$$

Showing that an equilibrium exists amounts to proving that (14) admits a positive solution in p. Once the equilibrium price is determined, (12) and (13) give the equilibrium values of demand and supply for the day-time good, $\phi = \frac{py}{M}$ and $m_i = \frac{px_i}{\phi}$, using (10) and (1).

Existence and uniqueness The first Proposition establishes the existence and uniqueness of SME.

Proposition 1 *An SME exists and is unique, for any admissible* γ.

Henceforth, variables with a tilda will indicate equilibrium values. Given the assumptions on fundamentals, the equilibrium allocation and price are continuously differentiable in the policy parameter.

Distribution of money In this model, the only dimension in which the agents may differ is their weight on current payoffs, δ. The next Proposition shows that heterogeneity in payoffs is necessary and sufficient to induce the agents to hold different amounts of money at equilibrium.

Proposition 2 *At the SME, the money distribution is non-degenerate iff* $\delta_i \neq \delta_h$ *for every* i.

With heterogeneous agents, the distribution of money holdings is two-point, at equilibrium. As we can see from (11), $x_{i,+1}$ is decreasing in $\alpha_h = \delta_h/\delta_i$ which implies that the type with a current low and future high δ acquires more money and will consume more in the next period. Notice that in this model, since agents' types alternate over time, aggregate variables remain constant at the steady state: hence the distribution of money holdings remains stationary although non degenerate. When agents are homogeneous, i.e. $\delta_i = \delta_h$ for every i, the model reduces to the Lagos and Wright (2005) environment, which gives rise to a degenerate, one-point equilibrium distribution of money holdings. Henceforth, we will maintain the assumption that the agents are heterogeneous.

Comparative statics Since the equilibrium is unique for any value of the policy parameter γ and the equilibrium allocation and price vary smoothly in γ, we can perform comparative statics exercises with respect to policy. A property that will turn out to play an important role in what follows is the change, induced by policy, on the price of the day-time good in terms of the night-time good, p. Differentiating (12) with respect to the policy parameter, at the SME, one obtains the effect of policy on day-time consumption for each type,

$$\frac{d\widetilde{x}_i}{d\gamma} = \frac{\widetilde{x}_i}{\gamma}\left(\frac{u'\left(\widetilde{x}_i\right)}{u''\left(\widetilde{x}_i\right)\widetilde{x}_i}\right)\left[\frac{2\gamma}{2\gamma-\beta/\alpha_h}+\left(\frac{d\widetilde{p}}{d\gamma}\frac{\gamma}{\widetilde{p}}\right)\right], \tag{15}$$

where the second term in brackets is the effect on the price of a change in policy, sometimes called, in the incomplete markets literature, the *pecuniary externality*. A change in policy alters the amount of money held by the two types. This portfolio reallocation, in turn, alters the demand and supply of the day-time good, affecting the price of the good. In turn, the change in price has asymmetric effects on different agents. Differentiating (13) with respect to the policy parameter, at the SME, one obtains the effect of policy on day-time production,

$$\frac{d\widetilde{y}}{d\gamma} = \frac{\widetilde{y}}{\gamma}\left(\frac{c'\left(\widetilde{y}\right)}{c''\left(\widetilde{y}\right)\widetilde{y}}\right)\left(\frac{d\widetilde{p}}{d\gamma}\frac{\gamma}{\widetilde{p}}\right). \tag{16}$$

which is unambiguously determined by the price effect. Finally, the effect of policy on the price can be computed differentiating (14) with respect to the policy parameter, at the SME, obtaining

$$\frac{d\widetilde{p}}{d\gamma} = -\frac{\widetilde{p}}{\gamma}\frac{\sum_i \widetilde{x}_i\left(\frac{u'(\widetilde{x}_i)}{u''(\widetilde{x}_i)\widetilde{x}_i}\right)\frac{2\gamma}{2\gamma-\beta/\alpha_i}}{\sum_i \widetilde{x}_i\left(\frac{u'(\widetilde{x}_i)}{u''(\widetilde{x}_i)\widetilde{x}_i}\right)-2\widetilde{y}\left(\frac{c'(\widetilde{y})}{c''(\widetilde{y})\widetilde{y}}\right)}, \tag{17}$$

which has a negative sign and may therefore be able to alter the overall effect of policy on day-time consumption, through (15). Hence, policy has potentially two effects. First, by altering the value of money, it affects directly the demand for the good. Second, by inducing a portfolio reallocation, it indirectly changes the relative price between day-time and night-time consumption. The latter effect may be sufficently strong to overturn the former at least for some agents. It also has an unambiguously negative effect on the supply of the good. Define the relative risk aversion functions of utility and cost as $\rho = \rho(x_i) \equiv -\frac{u''(x_i)x_i}{u'(x_i)}$ and $\eta = \eta(y) \equiv \frac{c''(y)y}{c'(y)}$, respectively. Define also $A_i \equiv \frac{\widetilde{x}_i}{\rho(\widetilde{x}_i)(2\gamma - \beta/\alpha_i)}$. Substitute (17) into (15), obtaining an explicit expression for the effect of policy changes on the consumption of each agent,

$$\frac{d\widetilde{x}_i}{d\gamma} = -\frac{2}{\eta \sum_i \frac{\widetilde{x}_i}{\rho(\widetilde{x}_i)} + \sum_i \widetilde{x}_i} \left[A_i \sum_i \widetilde{x}_i + \eta \left(\delta_i^2 - \delta_h^2\right) \beta \prod_i \frac{A_i}{\delta_i} \right], \tag{18}$$

for each type, where $h \neq i$. From (18), we can see clearly that there is a positive effect of policy changes on consumption only on the type with the higher current δ, who is also the type who currently accumulates less liquid resources and will therefore consume less. The effect is stronger the higher the curvature of the cost function, η. A constant marginal cost of day-time production, instead, would cancel the effect making the sign of (15) always negative. Indeed, the pecuniary externality cannot be operative when the supply is perfectly elastic. Another instance in which the sign of (18) is unambiguously negative for all agents is when they are homogeneous, namely when $\delta_i = \delta_h$ and the model reduces to the Lagos and Wright (2005) environment.

3.7 Dynamics

We now consider briefly dynamic equilibria. The Euler conditions, (11), together with (7), can be used to give

$$x_{i,+1} = u'^{-1} \left(\frac{2\gamma}{\beta/\alpha_h} \frac{pc'^{-1}(p)}{c'^{-1}(p_{+1})} - p_{+1} \right), \tag{19}$$

for each type i and $h \neq i$. Using the equations (19), (13) and the market clearing condition (9), the dynamic system can be reduced to a single dynamic equation in the price of the day-time good,

$$G\left(p_{+1},p\right)\equiv\sum\nolimits_{i}u'^{-1}\left(\frac{2\gamma}{\beta/\alpha_{i}}\frac{pc'^{-1}\left(p\right)}{c'^{-1}\left(p_{+1}\right)}-p_{+1}\right)-2c'^{-1}\left(p_{+1}\right)=0. \tag{20}$$

Hence, the dynamics of the model is conveniently described by a single dynamic equation connecting the current and future price of the day-time good. With standard bifurcation techniques, cycles of period two and of higher order and sunspot equilibria can be shown to exist in this case, when the curvature of the utility function is sufficiently high. Mathematically, the slope of the function $p_{+1}=g\left(p\right)$, implicitly defined by (20), can be altered by changing the relative risk aversion of the objective function, giving rise, in some cases, to an inverted relationship between current and future prices. Economically, the ordinary relationship between current and future prices can be altered making the supply function more or less elastic to changes in the price, using the curvature of the cost function. Such cycles are expectations driven. Intuitively, when the agents expect the price to be, say, high in the future, they will plan to demand less of the good and will therefore hold less money to finance lower consumption. This, however, will put a downward pressure on the value of money in the future, inducing a low price of the good in the future, and so on. Viceversa, when a low price is expected. For this phenomenon to occur, the elasticity of demand to price changes should be low and the elasticity of supply should cross a threshold to become sufficiently high. The expectations just mentioned are self-fulfilling. This can be seen considering sunspot events, in the tradition of Cass and Shell (1983). Exploiting the no-trade equilibrium, which exists always, global cycles and even chaotic trajectories can also be shown to exist, for some values of the risk aversion.

4 Optimal Monetary Policy

We have seen above that in this framework there is a pecuniary externality at work. In this section, we confirm that the equilibrium allocation is not only first-best, but also second-best inefficient, precisely because of the price externality. Finally, we ask what may be the optimal policy should the Government want to maximize social welfare, weighing the agents' payoffs equally.

First-best efficiency The equilibrium allocation, when the agents are heterogeneous, is always inefficient relative to first-best, independently of the monetary policy adopted by the Government. For an allocation to be unconstrained Pareto efficient, it has to satisfy the following necessary condition,

$$\sum\nolimits_i \theta_i \delta_i \left[u'\left(x_i\right) - c'\left(y\right)\right] = 0, \tag{21}$$

together with feasibility $\sum_i x_i = 2y$, and Pareto weights $\theta_i \in (0,1)$ for both i, such that $\sum_i \theta_i = 1$. Since at the SME the liquidity constraint (1) is binding for at least one of the agents, the allocation fails to be efficient for any monetary policy choice by the Government.

Proposition 3 *The SME is inefficient, for any admissible γ.*

With homogeneous agents, the equilibrium allocation can be efficient provided the policy parameter is set appropriately by the Government to achieve the so-called Friedman rule, namely γ equal to the discount factor of the agents. With heterogeneous payoffs, instead, the Friedman rule is not even defined, since there is no single (adjusted) discount factor for all the agents. Here, the allocation is inefficient if policy cannot discriminate between types, because of the underlying anonymity of the agents.

Second-best efficiency The natural next question concerns second-best efficiency, or, as it has been called in the incomplete markets literature, constrained Pareto efficiency. We will confine attention to Government intervention through the transfer scheme (3), since Government policy should be subject to the same constraints the environment imposes on private agents. Hence, the Government picks γ and, then, the price and allocation adjusts to the new equilibrium. The aim is to increase the utility of every agent. Agent i's utility evaluated at the SME, is

$$V_i\left(\widetilde{x},\widetilde{y}\right)=\frac{1}{1-\beta^2}\sum_{\iota=i,h}\delta_\iota\beta^{|i-\iota|}\left\{\frac{1}{2}\left[u\left(\widetilde{x}_\iota\right)-c\left(\widetilde{y}\right)\right]+c'\left(\widetilde{y}\right)\left(\widetilde{y}-\widetilde{x}_\iota\right)\left(\gamma-\frac{1}{2}\right)\right\}, \quad (22)$$

where the night-time budget constraint has been substituted into the objective function. The derivative of (22) with respect to γ evaluated at the SME is given by $\frac{dV_i(\widetilde{x},\widetilde{y})}{d\gamma}=\frac{1}{2}\frac{c'(\widetilde{y})}{1-\beta^2}\times$

$$\sum_{\iota=i,h}\delta_\iota\beta^{|i-\iota|}\left\{\frac{\widetilde{\lambda}_\iota}{\delta_\iota}\frac{d\widetilde{x}_\iota}{d\gamma}+2\left(1-\gamma\right)\left(\frac{d\widetilde{x}_\iota}{d\gamma}-\frac{d\widetilde{y}}{d\gamma}\right)+\left(\widetilde{x}_{\iota'}-\widetilde{x}_\iota\right)\left(1+\frac{2\gamma-1}{2\gamma}\frac{d\widetilde{p}}{d\gamma}\frac{\gamma}{\widetilde{p}}\right)\right\}, \quad (23)$$

for each type i and $\iota'\neq\iota$. Thus, monetary policy has three effects on the utility of agents. First, it has an effect on consumption, weighed by the shadow value of the liquidity constraint; second, an effect on net demand, whose sign depends on whether policy is regressive or progressive: third, a redistributive effect between the two types, the cash-rich and cash-poor. The first question we ask is whether $\gamma=1$, may be optimal, or in other words whether the SME may be constrained efficient without active Government intervention. Evaluating (23) at $\gamma=1$, we obtain

$$\frac{1}{2}\frac{c'\left(\widetilde{y}\right)}{1-\beta^2}\sum_{\iota=i,h}\delta_\iota\beta^{|i-\iota|}\left\{\frac{\widetilde{\lambda}_\iota}{\delta_\iota}\frac{d\widetilde{x}_\iota}{d\gamma}+\left(\widetilde{x}_{\iota'}-\widetilde{x}_\iota\right)\left(1+\frac{1}{2}\frac{d\widetilde{p}}{d\gamma}\frac{\gamma}{\widetilde{p}}\right)\right\}, \quad (24)$$

where the derivatives are evaluated at neutral intervention. It can be observed that only the first and third effect mentioned earlier survive at neutral intervention. The following Proposition establishes that, except in knife-edge cases, the SME with neutral intervention is not second-best efficient. A property holds generically in some space, if it holds in an open and dense subset of such a space. In turn, a set is open and dense in some space, if its complement in such a space is closed and has empty interior.

Proposition 4 *Generically in the space of cost functions, the SME is constrained inefficient at $\gamma=1$.*

The price effect mentioned earlier has an asymmetric impact on the two types who evaluate the effect of income changes with different implicit prices, due to the incompleteness of markets with binding liquidity constraints. In general, the Government can improve the situation relative to the allocation attained with neutral intervention, reallocating the monetary asset and exploiting the asymmetric price effect on the demands of the two different agents for the day-time good, acting through the pecuniary externality. Next, we ask whether the SME is constrained inefficient for any policy intervention. We did not find a general answer to this question, but we can prove that there exist robust examples, in which the SME is generically constrained inefficient for any admissible γ. As before, ρ and η denote the value of the relative risk aversion of the utility and cost functions, respectively, evaluated at the SME.

Proposition 5 *For ρ sufficiently small and η sufficiently large, the SME is generically constrained inefficient for all admissible γ.*

In this Proposition, genericity holds in the subset of cost functions that satisfy the assumed restrictions on the curvatures. The restiction to the region of the low risk aversion of the utility function and high risk aversion of the cost function, insures that γ approaching the highest discount factor does not achieve constrained optimality for any of the types. This result shows that the presence of the pecuniary externality may have unavoidable consequences for the constrained efficiency of the allocation, which policy can try to mitigate but will not be able to offset completely, at least in some robust cases. The Proposition does not imply that outside the region there are examples in which the equilibrium allocation is constrained efficient for some policy intervention nor it excludes this possibility. An unambiguous case, in which there exists a policy that makes the SME constrained efficient is when the cost function is linear and, thus, the pecuniary externality is absent. In this case, constrained efficiency is obtained with γ approaching the highest of the two discount factors. This case however is not robust to quadratic perturbations of the cost function.

Welfare Given the constrained inefficiency of the equilibrium allocation for all interventions, in some case, and always at neutral intervention, the reader may be interested in knowing whether active monetary policy intervention by the Government can improve the allocation for the agents according to a welfare criterion, at least relative to neutral intervention, and, in case, whether γ should be greater or smaller than 1. Since in the present model utility is transferable, due to the linearity of the night-time payoff, an admissible criterion to evaluate policy is the weighted sum of ex-ante utilities of the types,

$$W(\widetilde{x}, \widetilde{y}) = \sum\nolimits_i V_i(\widetilde{x}, \widetilde{y}), \tag{25}$$

using their (equal) mass as weights. As noted above, the equilibrium allocation and price are all potentially sensitive to monetary policy, hence, a change in γ will have both direct and indirect effects. The derivative of (25) with respect to γ evaluated at neutral intervention is the sum over types of (24), i.e.

$$\frac{1}{2}\frac{c'(\widetilde{y})}{1-\beta}\sum\nolimits_i \widetilde{\lambda}_i \left.\frac{d\widetilde{x}_i}{d\gamma}\right|_{\gamma=1}. \tag{26}$$

The next result will provide an answer to the question whether it is desirable for a Government interested in the overall welfare, to marginally change policy from neutral intervention, and, in case, whether the change should be toward progressive or regressive policies.

Proposition 6 *There exists a unique value $\overline{\eta} > 0$, such that:*

i. if $\eta < \overline{\eta}$, policy should marginally change toward $\gamma < 1$;

ii. if $\eta > \overline{\eta}$, policy should marginally change toward $\gamma > 1$;

iii. only if $\eta = \overline{\eta}$, policy can remain neutral.

As seen from (18), the only effect of monetary policy on the ex-ante welfare that tends to favor progressive policy is the pecuniary externality. For economies in which the supply is not too elastic with respect to price changes, the price effect tends to favor the type with smaller cash holdings and, thus, progressive policies. Since global concavity of the objective function (25) in γ is not guaranteed in general, while local second order conditions are easily verified, the statement of Proposition 8 has been restricted to local properties, with marginal changes around neutral intervention. An example in which global concavity of (25) in γ is guaranteed is when the utility function is $u(x_i) = \zeta x_i - (\omega/2)(x_i)^2$,[4] the cost function $c(y) = y^2$, with ζ sufficiently large and ω small to guarantee that utility is increasing over the relevant range. In this case, we can find values of parameters so that the global maximum of (25), which turns out to be a quadratic function of γ, is achieved at a progressive policy.

Hence, in this economy with heterogeneous agents there is a first order effect on the welfare of agents arising from the pecuniary externality that is typical of an incomplete markets setting. Since monetary economies in which money is not a veil, but plays an essential role, will have to be incomplete markets economies in which liquidity constraints play a major role, this effect is bound to emerge in a robust way, unless silenced by assumptions making either the agents completely homogeneous or the supply of goods perfectly elastic.

5 Conclusion

We have used an extension of the Lagos and Wright (2005) framework that makes room for a non-degenerate disrtibution of money holdings at equilibrium to show the presence of an effect, akin to the pecuniary externality of the incomplete markets literature, that makes monetary policy intervention, in some circumstances of the expansionary type, beneficial. Non-trivial uncertainty, generating the effect explored in Levine (1991), can be introduced in the model, at the cost of sacrificing analytical tractability, for instance assuming that the agents can enter the night-time market

[4] This utility function violates the assumptions made earlier on fundamentals. Nevertheless, an equilibrium can be shown to exist and be unique, over some range of parameters values.

only stochastically. Our results can be obtained in other monetary environments in which agents hold money for transaction purposes as well, for instance, in a Lucas island model with heterogeneous agents. Trading protocols other than competitive trade would give rise to similar results. Bargaining or price posting protocols, for instance, would complicate the analysis, by increasing the set of relative prices that should be determined in the decentralized market, but would not, in general, sterilyze the pecuniary externality effect. An interesting question, which we leave for future research, is to what extent the result continues to hold in economies that rather than being pure-currency ones, have access to other trading instruments, possibly with credit markets or intermediaries that allow to reshuffle liquidity when needed.

References

[1] David Andolfatto (2010), Essential Interest-Bearing Money, Journal of Economic Theory, 145, 1495-1507

[2] Truman Bewley (1980), The Optimum Quantity of Money, in *Models of Monetary Economies*, Kareken and Wallace (eds.), Minneapolis Fed

[3] Joydeep Bhattacharya, Joseph Haslag and Antoine Martin (2005), Heterogeneity, Redistribution, and the Friedman Rule, *International Economic Review,* 46, 305-729

[4] Paola Boel and Christopher Waller (2015), On the Theoretical Efficacy of Quantitative Easing at the Zero Lower Bound, Sveriges Riskbank Working Paper Series, n. 310

[5] William Brock (1974), Money and Growth: the case of Long-run Perfect Foresight, *International Economic Review*, 15, 750-777.

[6] David Cass and Karl Shell (1983), Do Sunspot Matter?, *Journal of Political Economy*, 91, 193-227

[7] Milton Friedman (1969), *The Optimum Quantity of Money*, Macmillan

[8] John Geanakoplos and Herakles Polemarchakis (1986), Existence, regularity and constrained suboptimality of competitive allocations when the asset market is incomplete, in *Uncertainty, Information and Communication: Essays in Honour of K. J. Arrow*, Heller, Starr, and Starrett (eds.), Cambridge University Press

[9] Jean-Michel Grandmont and Yves Younes (1973), On the Efficiency of a Monetary Equilibrium, *Review of Economic Stidies*, 40, 149-165

[10] Oliver Hart (1975), On the optimality of equilibrium when the market structure is incomplete, *Journal of Economic Theory*, 11, 418–443

[11] Timothy Kehoe, David Levine and Michael Woodford (1992), The Optimum Quantity of Money Revisited, in *The Economic Analysis of Markets and Games: Essays in Honor of Frank Hahn*, Dasgupta, Gale, Hart, and Maskin (eds.), Cambridge, MA

[12] Ricardo Lagos and Randall Wright (2005), A Unified Framework for Monetary Theory and Policy Analysis, *Journal of Political Economy*, 113, 463-484

[13] David Levine (1991), Asset Trading Mechanisms and Expansionary Policy, *Journal of Economic Theory*, 54, 148-164

[14] Francesco Lippi, Stefania Ragni, and Nicholas Trachter (2015), Optimal Monetary Policy with Heterogeneous Money Holdings, *Journal of Economic Theory*, 159, 339-368

[15] Guido Lorenzoni (2008), Inefficient Credit Booms, *Review of Economic Stidies*, 75, 809-833

[16] John H. Moore (2013), Pecuniary Externality through Credit Constraints: Two Examples without Uncertainty, mimeo

[17] Jose Scheinkman and Laurence Weiss (1986), Borrowing Constraints and Aggregate Economic Activity, *Econometrica*, 23-45

[18] Robert Townsend (1980), Models of Money With Spatially Separated Agents, in *Models of Monetary Economies*, Kareken and Wallace (eds.), Minneapolis Fed

[19] Neil Wallace (2014), Optimal Money-creation in Pure-currency Economies: a Conjecture, *Quarterly Journal of Economics*, 129, 259-274

6 Appendix

We derive, here, explicitly the equilibrium conditions, starting with the optimality conditions for the agents. Then, we prove the results stated in the text.

Equilibrium. The first order conditions for a buyer during the day for the optimal consumption decision, x_i, is

$$\delta_i u'(x_i) - p\lambda_i - p\frac{W_i^{b\prime}\left(\widetilde{m}_i^b\right)}{\phi} = 0. \tag{27}$$

The first order conditions for a seller during the day for the good, y, is

$$-\delta_i c'\left(y\right) + p\frac{W_i^{s\prime}\left(\widetilde{m}_i^s\right)}{\phi} = 0. \tag{28}$$

The envelope condition for cash between day and night is

$$W_i^{j\prime}\left(\widetilde{m}_i^j\right) = \delta_i \phi, \tag{29}$$

for all i and j. Combining (27) and (29), we obtain the multiplier

$$\lambda_i = \delta_i \left[\frac{u'\left(x_i\right)}{p} - 1\right] \geq 0, \tag{30}$$

for the shadow value of consumption. From (28) and (29), we obtain

$$c'\left(y\right) = p, \tag{31}$$

which is the standard condition equating price and marginal cost under perfect competition. Given that the cost function is the same for all agents, production is uniform across types. Using (3) and (5), rewrite the budget constraint (2) as $X_i^j + \frac{\phi m_{i,+1}}{1+b} = e_i^j + \phi \widetilde{m}_i^j + \phi M (\gamma - 1)$. Substitute for $X_i^j - e_i^j$ into the objective function at night, reducing the problem to the choice of $m_{i,+1}$. The first order condition for $m_{i,+1}$ is

$$-\frac{\delta_i \phi}{1+b} + \beta V_h' (m_{i,+1}) = 0. \tag{32}$$

The envelope condition between night and day is

$$V_i' (m_h) = \frac{\phi (\delta_i + \lambda_i)}{2} + \frac{\phi \delta_i}{2}. \tag{33}$$

Consider (33) for type h, delay it one period and insert it into (32), to obtain the Euler equation for money

$$\phi = \frac{\delta_h \beta}{\delta_i} \phi_{+1} (1+b) \left[\frac{1}{2} \left(\frac{\lambda_{h,+1}}{\delta_h} + 1 \right) + \frac{1}{2} \right]. \tag{34}$$

In sum, the optimality conditions are: (30), (31), (34), and the complementary slackness condition for the constraint, (1). These conditions give the optimal demand and supply for all the items traded in all the markets, taking prices as given. Next, we determine the values of the multipliers. Using $\alpha_i = \frac{\delta_i}{\delta_h}$ for both i and $h \neq i$, rewrite (34) as

$$\lambda_{h,+1} = 2 (\beta / \delta_i)^{-1} \left(\frac{\phi}{\phi_{+1} (1+b)} - \beta / \alpha_i \right). \tag{35}$$

Optimization requires the multiplier (35) to be non-negative for both types. Henceforth, we will concentrate on situations in which (35) is strictly positive for both types. Thus, we will concentrate on situations in which the constraint (1) is binding for all types. Delay (30) one period and use it together with (35) to obtain (6) for each i. The other conditions are (31), (1) at equality and market clearing, (9) and (10). At steady state, real variables are time invariant, hence, $\phi_{+1} M_{+1} = \phi M$. By definition, $\gamma \equiv \frac{M_{+1}}{M(1+b)}$. Thus, equations (30) and (35) give $u' (x_i) = (\beta / \alpha_h)^{-1} (2\gamma - \beta / \alpha_h) p$.

Proofs. Next, we prove the Propositions in the text.

Proof of Proposition 1. The function (14) is continuous in p. By the Inada conditions, $F(0) = +\infty$, $F(\infty) = -\infty$, for all admissible γ. By the Intermediate Value Theorem a finite $\widetilde{p} > 0$ exists such that $F(\widetilde{p}) = 0$ for all admissible γ. Since $\frac{\partial F(p,\gamma)}{\partial p} < 0$, $\widetilde{p}$ is unique for any admissible γ. Given the equilibrium price, the rest of the system determines uniquely the other equilibrium variables.■

Proof of Proposition 2. By (12), $x_1 \neq x_2$ iff $\alpha_1 \neq \alpha_2 \Leftrightarrow \delta_1 \neq \delta_2$. By the binding (1), $m_h = \frac{p}{\phi} x_i$ for all i and $h \neq i$. Hence, $m_1 \neq m_2$ iff $x_1 \neq x_2$.■

Proof of Proposition 3. At an SME, $u'(x_i) = (\beta/\alpha_i)^{-1}(2\gamma - \beta/\alpha_i)\, c'(y)$. Since $\gamma \geq \overline{\beta}$ is required for the SME to exist, $u'(x_i) > c'(y)$ at least for the agent with $\beta/\alpha_i < \overline{\beta}$, while for the other type $u'(x_i) \geq c'(y)$. Hence, the necessary condition (21) for efficiency is violated.■

Proof of Proposition 4. It is necessary to guarantee constrained efficiency to have both derivatives (24) equal to zero, i.e.

$$\widetilde{\lambda}_i \frac{d\widetilde{x}_i}{d\gamma} = (\widetilde{x}_i - \widetilde{x}_h)\left(1 + \frac{1}{2}\frac{d\widetilde{p}}{d\gamma}\frac{\gamma}{\widetilde{p}}\right), \tag{36}$$

for both i, which implies

$$\sum\nolimits_i \widetilde{\lambda}_i \frac{d\widetilde{x}_i}{d\gamma} = 0. \tag{37}$$

Substitute (35) and (15) into (37), to obtain, after some algebra,

$$\eta(\widetilde{y}) = \frac{\sum_i \widetilde{x}_i \sum_i \frac{\widetilde{x}_i}{\beta/\alpha_i \rho(\widetilde{x}_i)} \frac{1-\beta/\alpha_i}{2-\beta/\alpha_i} \prod_i \rho(\widetilde{x}_i)(2 - \rho(x_i))}{(\alpha_2 - \alpha_1)^2 \prod_i \widetilde{x}_i}. \tag{38}$$

Since (37) is necessary for (36) to hold, and (37) holds if only if (38) holds, we have that (38) is necessary for (36), which are in turn necessary conditions for constrained optimality. Next, we show that (38) holds at most for a set of cost functions with empty interior. The cost functions belong to the space of C^2 functions endowed with the weak topology. Consider the following quadratic perturbation of the cost function, $c_\epsilon(y) = c(y) + \epsilon\,[c(y) - c'(\widetilde{y})\, y]$, with $\epsilon \geq 0$, which alters the curvature of the cost function without affecting the equilibrium. With such a perturbation, the relative risk aversion is $\eta_\epsilon(\widetilde{y}) = (1+\epsilon)\,\eta(\widetilde{y})$. Suppose (38) holds. Take a small but positive

ϵ and perturb the cost function. Then, the RHS of (38) is unaffected, since the RHS is independent of δ and the equilibrium is unchanged, while the LHS is larger, hence, equality cannot be maintained. Therefore, any small perturbation of the cost function makes (38) not verified, which implies that the the necessary conditions for constrained optimality is violated. Given the uniqueness of SME for any γ, (38) holds at most for a finite set of values of η (in fact, at most one value). Since the present model has uncertainty, payoffs are identified uniquely up to affine transformations, which do not affect the risk aversion. Therefore, for any given η we can associate a unique (up to affine transformations) cost function. Thus, we conclude that the set of cost functions such that constrained optimality may obtain has empty interior. It is also closed, being finite. Hence, its complement is open and dense. The statement of the Proposition follows.■

Proof of Proposition 5. Define

$$A_\kappa \equiv \frac{\widetilde{x}_\kappa}{\rho\left(\widetilde{x}_\kappa\right)\left(2\overline{\beta} - \beta/\alpha_\kappa\right)},$$

for $\kappa = i, h$, and

$$B \equiv 4\overline{\beta}\left(1 - \overline{\beta}\right)\left(\alpha_h - \alpha_i\right)\prod\nolimits_i \frac{A_i}{\alpha_i} + \left(x_i - x_h\right)\sum\nolimits_i A_i\left(1 - \beta/\alpha_i\right).$$

Consider the type with the highest β/α_i. Let $\gamma \to \overline{\beta}$. Inserting (15), (16) and (17) into (23), we obtain an expression for the derivative that is proportional to and has the same sign as $\overline{\Gamma}\left(\eta, \overline{\beta}\right) \equiv$

$$\left[2\left(1 - \overline{\beta}\right)A_h - \left(x_i - x_h\right)\right]2\widetilde{y} - B\eta,$$

with $h \neq i$. If ρ is not too high, $\overline{\Gamma}\left(\eta, \overline{\beta}\right) > 0$ and, hence, this type benefits from a small increase in γ away from $\overline{\beta}$. If η is sufficiently high, also the type with the lowest discount factor benefits from a small increase in γ away from $\overline{\beta}$. Thus, $\gamma = \overline{\beta}$ cannot be optimal in this case. For an interior value of γ to be optimal it has to be that (23) is equal to zero for both i. This implies $\sum_i \widetilde{\lambda}_i \frac{d\widetilde{x}_i}{d\gamma} = 0$, since $\sum_i \frac{d\widetilde{x}_i}{d\gamma} = \sum_i \frac{d\widetilde{y}}{d\gamma}$ by market clearing in the goods market. Hence, it is necessary for optimality to have

$$\eta = \frac{\sum_i \widetilde{x}_i \sum_i \frac{\widetilde{x}_i(1-\beta/\alpha_i)}{\beta/\alpha\rho(\widetilde{x}_i)(2\gamma-\beta/\alpha)} \prod_i \rho\left(\widetilde{x}_i\right)\left(2\gamma - \rho\left(x_i\right)\right)}{\left(\alpha_2 - \alpha_1\right)^2 \prod_i \widetilde{x}_i},$$

at any given $\gamma > \overline{\beta}$. The same perturbation argument used in the proof of the previous Proposition shows that such a condition holds at most for a subset of cost functions that is closed and with empty interior. Therefore, generically in the subspace of cost functions satisfying the restriction $\underline{\eta} < \eta < \overline{\eta}$, the SME is constrained sub-optimal for any $\gamma \geq \overline{\beta}$.■

Proof of Proposition 6. The derivative of ex-ante welfare evaluated at neutral intervention (26) is proportional to and has the same sign as

$$\Psi\left(\eta\right) \equiv 2\eta\left(\alpha_2 - \alpha_1\right)^2 \prod_i A_i - \left(\widetilde{x}_1 - \widetilde{x}_2\right)\left(\alpha_2 - \alpha_1\right) \sum_i \widetilde{x}_i,$$

where $\left(\widetilde{x}_1 - \widetilde{x}_2\right)\left(\alpha_2 - \alpha_1\right) > 0$. Hence, there exist a unique $\overline{\eta} > 0$ s.t. $\Psi\left(\overline{\eta}\right) = 0$, and $\left(\eta - \overline{\eta}\right)\Psi\left(\eta\right) > 0$, for all $\eta \neq \overline{\eta}$.■

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