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## **Endogenous Mergers and Leadership Acquisition in Cournot Oligopolies**

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# Endogenous Mergers and Leadership Acquisition in Cournot Oligopolies

Walter Ferrarese\*<sup>†</sup>

## Abstract

I set up an endogenous merger model in which, whenever firms agree to join in a coalition, the new entity acquires the leadership in a symmetric Cournot oligopoly. I first explore the case of a single merger and show that, despite being such merger profitable irrespective of the number of participants, only two endogenous equilibria are possible: either a bilateral coalition or an  $n - 1$ -firm coalition. I then allow for multiple coalitions and show that merger waves often occur as a firms' response to the exclusion of monopolization. In other cases, even if monopolization is allowed, the grand coalition does not form and at least one firm prefers to act as a follower. The model provides an explanation of why bilateral mergers are observed in almost every industry, even where synergies are unlikely and why it is possible to observe a single large entity behaving as a market leader. Furthermore, it provides a justification of the strategic nature of merger waves as a response to the exclusion of monopolization. I also check how my results vary with different ex-ante merger policies. Moreover, it is shown that bilateral mergers between identical firms generating no synergies can be beneficial to both consumers and producers.

**Keywords:** endogenous mergers, leadership acquisition, welfare.

**Jel codes:** L11, L13, L22, L41.

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# 1 Introduction

Mergers are mainly driven by two effects: a market power effect, commonly recognized as the reduction in competition due to a lower number of firms and an efficiency effect due to the ability of the merging parties to generate synergies.

In this paper, the focus is on the market power effect of mergers, to which another ingredient is added. Large enough economies of scale or scope or even a simple market power recognition of outside firms may endow a merged entity with a greater advantage than the one of a mere reduction in the number of firms. In particular, I model a scenario in which a merger provides the advantage of becoming the leader in a symmetric Cournot-Nash market.

The leadership acquisition from a static Cournot-Nash market is experimentally confirmed by Huck, Konrad, Muller and Normann (2007, HKMN henceforth), who study the implications of a bilateral merger in a triopoly and in a quadriopoly. HKMN show that, differently from the theoretical predictions, the new entity produces above the Cournot-Nash level and behaves close to a Stackelberg leader. Outside firms, instead, given the supply of the new entity, non cooperatively set their profit maximizing quantity. This result is due to *aspiration levels* (Simon (1955 [a], [b])), namely for fear that the merger turns out to be unprofitable, the new entity increases its own output above the level predicted by the simultaneous play.

The leadership acquisition has also been recently tackled in symmetric Cournot markets by Liu and Wang (2015, LW henceforth) who show that a single leading entity can profitably merge with an arbitrary number of firms. However, LW analyze mergers in an exogenous setting, namely an environment in which the merging firms are randomly selected and imposed to join in a coalition and in which an equilibrium is only dictated by profitability.

This is the way of modeling mergers that dates back to Salant, Switzer and Reynolds (1983, SSR henceforth) who give rise to the well known merger paradox by showing that, in homogenous linear symmetric Cournot-Nash markets, horizontal mergers are profitable only if at least the 80 percent of the market is involved. Moreover, as also suggested by Stigler (1950), even in case of profitable mergers, remaining outside is a better option than participating (free riding issue).

In SSR, unprofitability arises from the fact that the merged entity produces less than the sum of the pre-merger quantities of its members. In contrast, due to Cournot competition in which best responses are downward sloping, the rivals increase their own output. This interaction leads to a lower total output and therefore to a higher market price, which remains high enough only if the outsiders' output increase is weak, namely when the merger involves a big portion of the market.

The paradox comes from the fact that these conclusions do not explain what is commonly empirically observed. In particular Huck, Konrad and Muller (2008, HKM henceforth) report that *“bilateral mergers are observed in virtually all industries, even in industries*

*where cost reductions are unlikely*".

Since then, many different settings have been explored: convex and/or asymmetric costs (Perry and Porter, 1985; Farrell and Shapiro, 1990), differentiated products (Werden and Froeb, 1994), Bertrand competition (Deneckere and Davidson, 1985), non linear inverse demand (Cheung, 1992; Fauli-Oller, 1996; Hennessy, 2000), free entry (Spector, 2003; Davidson and Mukherjee, 2007); dynamic interaction (Dockner and Gaunesdorfer, 2001; Benchekroun, 2003).

These works, however, still assume an exogenous selection of the merging parties. This approach has more recently been set aside by the literature on endogenous mergers, where, according to specific merger formation mechanisms, firms decide whether participating or remaining outside. Thus, an endogenous approach can more reliably determine which market structure could be observed in a given market.

Barros (1998) shows an inverse relation between the cost asymmetry of the merging firms and the pre-merger market concentration and shows that, in a setting á la SSR, stable mergers are impossible. Kamien and Zang (1990, 1991, 1993) study the possibility of monopolization in a simultaneous and sequential bidding game. An interesting result is that under simultaneous acquisition, although merging to monopoly is a profitable option, this can not characterize an endogenous equilibrium. Kwoka (1989) shows a direct relation between the likelihood of mergers and market competition.

Gowrisankaran (1999) constructs a dynamic model in which mergers, entry, exit and investment decisions are all endogenously determined. Possajennikov (2001) and Horn and Persson (2001) study mergers adopting tools from cooperative game theory. Socorro (2004) shows that if firms do not face technological transfer costs among plants, then stable mergers can not occur.

Rodrigues (2001) sets up a merger game in which firms sequentially vote yes or no to enter in a coalition and studies the implications of fixed costs economies. Zhou (2008) adopts the same mechanism under cost uncertainty.

HKM (2004) and Fan and Wolfstetter (2014) study a scenario in which a merger does not imply a reduction in the number of competitors and allows merging firms to exchange information more efficiently. Burguet and Caminal (2015) show that negotiation among firms may lead to the selection of inefficient mergers in terms of aggregate profits and consumer surplus and conclude that authorities should be more uncompromising and reject mergers generating only moderate social benefits.

In this paper, mergers are endogenized in a three stage game. Thus, before market competition, firms decide whether being part of a leading entity or remaining outside. This allows for an endogenous determination of the number of leaders and followers along side the number of firms involved in a merger and departs the paper from the existing literature of mergers in Stackelberg markets (HKM, 2001; Feltovich, 2001; Heywood and McGinthy, 2007-2008; Escrihuela-Villar and Fauli'-Oller, 2010; Chuna and Vasconcelos, 2014), where we find a pre-existing number of leaders and followers.

Before the merger stage, nature attributes to each firm a place in an ordered ranking. In the merger formation process, if at least two firms agree on a merger, the new entity behaves as a leader over the remaining outside firms. The merger formation stage is similar to the one in Qiu and Zhou (2007, QZ henceforth).

First I study a game in which a single merger is allowed. For a given order of play, the first firm can decide to pass or propose a merger to an arbitrary number of firms. If he decides to pass, the player is committed to this choice and can not be the receiver of another merger proposal. If all firms accept, the merger takes place and the new entity behaves as a leader over the remaining firms. If a firm rejects the proposal, the merger stage ends and all firms keep playing a simultaneous Cournot-Nash game.

The first result is that, although LW find that a single merger is profitable for every merger size, only two endogenous equilibria are possible: either a bilateral coalition involving the second and the third firm in the order of play or a coalition involving the last  $n - 1$  firms. Thus, if on the one hand, this second equilibrium seems to confirm that markets point toward monopolization, the first equilibrium shows that it is not always the case, as bilateral mergers are equally likely.

This result is in line with the empirical evidence that bilateral mergers are observed in every market, even when synergies are unlikely (HKM, 2008). Furthermore, it matches the empirical facts that a single large entity is usually observed behaving as a Stackelberg leader (Gollop and Roberts, 1979; Pazo and Jaumandreu, 1999; Ailawadi, Kopalle and Neslin, 2005; de Mello, 2007), and that, in case of large mergers, remaining outside may be more beneficial than participating (Clougherty and Duso, 2008). This latter fact was also theoretically suggested by the seminal works of Stigler (1950), SSR (1983) and Deneckere & Davidson (1985) and experimentally confirmed by Lindqvist and Stennek (2005).

I then allow for multiple heterogenous mergers, in which a firm which has not received a merger proposal can itself become a proposer. The heterogeneity comes from the fact that each leading entity may be the result of the merger of a different number of firms. I first study a model in which monopolization is not allowed and obtain that, in the majority of cases, mergers occur in waves. In the rare cases in which mergers do not occur in waves, I obtain that the first firm in the order of play is the unique follower, while the remaining firms merge in a single entity.

Notice that, differently from QZ, by not restricting attention to bilateral mergers, I obtain that a merger wave can still likely contain bilateral agreements, but larger sizes are reasonable as well.

An interesting feature of the model emerges if one allows for monopolization. In particular, whenever mergers occurred in waves, then the grand coalition forms, while in the rare cases in which the first firm acts as a follower and a unique merged entity forms, then the equilibrium remains the same.

Thus, there seems to be a link between the possibility of monopolization and the endogenous choice of whether forming or not multiple coalitions. As a matter of fact, in most

cases, a merger wave is the optimal response to the removal of monopolization. In rare cases, however, although monopolization is allowed, a unique merger involving all firms still does not characterize an equilibrium.

It is worth noting that merger waves are another stylized fact in the merger literature. An empirical confirmation is provided by Mueller (1989), Resende (1999), Gartner and Albheer (2009), Gugler *et al.* (2012). The theoretical nature of this work makes it closely related clearly to QZ, but also to Fauli-Oller (2000) and Toxvaerd (2008). Fauli-Oller, for example, provides two justifications for the occurrence of merger waves and, similarly to QZ, identifies low levels of demand as a key factor; also, a merger, by lowering the number of competitors, increases the profitability of future mergers and triggers a wave.

A welfare analysis shows that in case of a single bilateral merger, welfare increase, while in all other equilibria welfare reduces, despite measuring welfare with consumer surplus (CS) only or with the sum of CS and industry profit.

This result shows that mergers between symmetric firms generating no synergies are not, in general, detrimental for consumers. This occurs because, although, on the one hand, market power tends to reduce total output, on the other hand, the output increase due to the transition from a static to a sequential market structure is stronger and makes total output (and therefore CS) increase.

I finally provide a robustness check by relaxing the assumption that a rejection implies the end of the merger formation stage. In particular, a rejection implies that the next firm after the original proposer can itself make a proposal. In this new setup, I obtain the same equilibria of the original one.

The paper is structured as follows: in Section 2, I study the implications of a single merger. In Section 3, I allow for multiple heterogeneous leaders, where the heterogeneity is in the number of insiders. In Section 4, I provide an extension of the games in Sections 2 and 3. Section 5 is dedicated to a welfare analysis. Section 6 concludes. All proofs are in the Appendix.

## 2 The merger game with a unique leader

The pre-merger scenario consists of  $n \geq 3$  risk neutral quantity setting firms, producing a homogeneous good at marginal cost  $c$ . The inverse market demand is  $p(Q) = 1 - Q$ , where  $Q = \sum_{h=1}^n q_h$  is total supply; the common marginal cost  $c$  is normalized to zero and there are no fixed costs. Thus, the profit of each firm  $h = 1, \dots, n$  is  $\pi_h = p(Q)q_h$ . I also assume complete information. The unique Cournot-Nash (CN) equilibrium in pure strategies is the one in which each firm produces  $q^{CN}(n) = \frac{1}{n+1}$  and obtains the profit  $\pi^{CN}(n) = \frac{1}{(n+1)^2}$ . The pre-merger aggregate supply is  $\tilde{Q} = \frac{n}{n+1}$ . For the moment, I assume that monopolization is excluded, namely the number of merging firms  $m < n$ .

This assumption is more relevant in the section in which multiple mergers are allowed,

where I discuss the implications on the equilibrium market structure according to whether the grand coalition is allowed to form or not.

Before the start of the game, nature attributes to each of the  $n$  identical firms a place in an ordered ranking  $\rho$ . A firm is denoted by its place in the order of play  $\rho_z$ ,  $z = 1, 2, \dots, n$ .

Mergers are endogenized in a three stage game. In the first stage, for a given order of play, the merger formation process takes place; in the second and third stage, the merger entity/ies compete on quantities with outside firms. Given the post-merger Stackelberg market, two stages are needed to model quantity competition.

*The timing of the game*

**Stage 1)**  $\rho_1$  can propose a merger of size  $m \in [2, n - 1]$  following the order induced by  $\rho$ , or decide to pass. The receivers simultaneously decide whether to accept or reject the proposal. If all the receivers accept, the merger takes place and the game proceeds at stage 2. If one or more receivers reject, the game proceeds at stage 3. If  $\rho_1$  passes,  $\rho_2$  can either propose a merger of size  $m \in [2, n - 1]$  or decide to pass. This process continues until  $\rho_{n-1}$ .

**Stage 2)** The new entity (if one) sets its profit maximizing quantity  $q_I$ .

**Stage 3)** The  $n - m$  outside firms (if any) non cooperatively set their profit maximizing quantity  $q_f$ , given the quantity  $q_I$  set by the leader at stage 2.<sup>1</sup>

Although it is hard to find empirical studies about a merger-induced market leadership, HKMN (2007) provide experimental evidence of the fact that a bilateral merger in a Cournot triopoly and quadriopoly make that the production of the merged entity is larger than the one of a CN firm, while the outsiders, given the production of the new entity, simultaneously set their profit maximizing quantity. In other words, the merger induced a transition from a simultaneous CN industry to a market structure closely related to a Stackelberg industry.

At this point some further definitions and notations are needed. A coalition  $\{\rho_z, \rho_{z+1}, \dots, \rho_{z+w}\}$  for some  $z = 1, \dots, n - 1$  and  $w = 0, \dots, n - 2$ , is a subset of the  $n$  firms including those from the  $z^{th}$  to the  $(z + w)^{th}$  position in the order of play. A coalition structure  $\kappa$  is a partition of the  $n$  firms. The set of coalition structures is denoted by  $\mathbb{K}$ . A subgame perfect Nash equilibrium (SPNE) is a triple  $(\kappa^*, q_I^*(n, m^*), q_f^*(q_I^*(n, m^*))) \in \mathbb{K} \times \mathbb{R}_{++}^2$ , where  $\kappa^*$  is the equilibrium coalition structure,  $q_I^*(n, m^*)$  is the optimal quantity of the merged entity as a function of the pre-merger number of firms  $n$  and the equilibrium size of the leading entity  $m^*$  and  $q_f^*(q_I^*(n, m^*))$  is the quantity set by each follower as a function of the quantity of the leader  $q_I^*(n, m^*)$ . Throughout the paper I equivalently refer to a firm outside the merger as an outsider or a follower and to a merger as a coalition. Let:

$$\pi_{i,z}(n, m) = \frac{\pi_I(n, m)}{m} \quad (1)$$

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<sup>1</sup>Clearly, if no merger occurred at stage 1, then firms are back to a simultaneous Cournot-Nash game, thereby market competition can be described in a single stage.

denote the single insider's payoff in an  $m$ -firm merger.<sup>2</sup>

## 2.1 Equilibrium mergers

In the competition stages, if an  $m$ -firm merger occurred, standard calculations show that the equilibrium quantities are:

$$q_I = \frac{1}{2}; \quad (2)$$

$$q_f(n, m) = \frac{1}{2(n - m + 1)}, \quad (3)$$

and the equilibrium profits are:

$$\pi_I(n, m) = \frac{1}{4(n - m + 1)}; \quad (4)$$

$$\pi_f(n, m) = \frac{1}{4(n - m + 1)^2}. \quad (5)$$

Total supply is  $Q(n, m) = \frac{2n-2m+1}{2(n-m+1)}$  and market price is  $p(n, m) = \frac{1}{2(n-m+1)}$ .

Turning to the merger formation stage, I assume that whenever a player faces two choices providing the same payoff, then he tosses a coin.

In the following proposition, I characterize the equilibria of the game:

**Proposition 1.** *In the three stage game, two equilibria are possible:*

**E1)**  $(\kappa^*, q_I^*(n, m^*), q_f^*(q_I^*(n, m^*))) = ((\{\rho_1\}, \{\rho_2, \dots, \rho_n\}), \frac{1}{2}, \frac{1}{4}),$

*in which the profitable merger involving the last  $n - 1$  firms takes place;*

**E2)**  $(\kappa^*, q_I^*(n, m^*), q_f^*(q_I^*(n, m^*))) = ((\{\rho_1\}, \{\rho_2, \rho_3\}, \{\rho_4\}, \dots, \{\rho_n\}), \frac{1}{2}, \frac{1}{2(n-1)}),$

*in which the profitable bilateral merger  $\{\rho_2, \rho_3\}$  takes place.*

E1 suggests the intuitive feature that markets point toward monopolization. However, in contrast with this view, E2 also justifies bilateral mergers, which, as already pointed out, are empirically observed in almost every market.<sup>3</sup> Notice that, the change in market

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<sup>2</sup>As a rejection implies the end of the merger stage, its structure is similar to an ultimatum game (Guth *et al.*, 1982), whose SPNE entails that the proposer obtains almost all the merger surplus. The experimental evidence, however, shows that agents depart from the equilibrium strategy. In particular, it has been found that proposers are willing to split the pie equally and receivers tend to reject small (profitable) offers (Roth *et al.*, 1991; Slonim and Roth, 1998; Cameron, 1999). This shows that agents are not uniquely motivated by monetary interests, but considerations of fairness or the fear of being rejected, because of very unfair proposals, need to be taken into account. The assumption of an egalitarian split is in line with these findings.

<sup>3</sup>Other hypotheses regarding the attitude towards two options providing the same payoff are clearly possible; for instance, one could envisage that the equilibria are not robust to the removal of the assumption that, in case of indifference, a player tosses a coin. My results are, however, qualitatively robust to this alternative formulation. Being firms symmetric, the main message is about the size of such mergers, not



structure due to the leadership acquisition makes bilateral mergers possible even in absence of cost synergies. It is also worth noting that, differently from HKM (2001), Stadler *et al.* (2006) and QZ (2007), where bilateral agreements are an assumption, here they represent an equilibrium outcome.<sup>4</sup>

I now point out the main drivers of both equilibria. First of all, a proposal is always accepted. The reason is that, not only a merger reduces the number of firms in the market, but also endows the merged entity with the market leadership. These effects makes that the profit in case of rejection can not be larger than the one inside a coalition.

Clearly, however, not all mergers are equally profitable. The profit of a firm involved in a coalition is captured by the  $\pi_{i,z}$  function. This function is U-shaped in  $m$ , attains a minimum at  $m = \frac{n+1}{2}$ , value in which it has a symmetry line. These properties imply that  $\operatorname{argmax}_{2 \leq m < n} \pi_{i,z}(n, m) = \{2, n - 1\}$ . It also implies that only  $\rho_1$  and  $\rho_2$  can maximize their payoff as insiders, by proposing either bilateral coalition or an  $n - 1$ -firm coalition.

Proposition 1 also shows that, in equilibrium, a proposal can only be made by  $\rho_2$ . If a proposal is made by a firm  $\rho_z$ ,  $z \geq 3$ , then  $\pi_{i,z}$  is maximized only in a bilateral coalition. Hence, if both  $\rho_1$  and  $\rho_2$  pass, then  $\rho_3$  merges with  $\rho_4$ ,  $\rho_2$  becomes one of many outsiders and obtains a lower profit than the one in case of proposal. Furthermore,  $\rho_2$  is indifferent between being part of the bilateral coalition  $\{\rho_2, \rho_3\}$  or the one involving  $n - 1$  firms  $\{\rho_2, \dots, \rho_n\}$ . The assumption in the paper is that  $\rho_1$  assigns the same probability to both events, in case he passes. It turns out that  $\rho_1$ 's expected profit is larger than the one in case he proposes when  $n > 3$  and the two are equal if  $n = 3$ .

Proposition 1 also predicts the possibility that a unique large entity behaves as a Stackelberg leader (Gollop and Roberts, 1979; Pazo and Jaumandreu, 1999; Ailawadi, Kopalle and Neslin, 2005; de Mello, 2008).

At this point, one may wonder whether E1 is a consequence of the exclusion of monopolization. The answer is provided by the next proposition:

**Proposition 2.** *Assume monopolization is allowed. When  $n < 7$ , the equilibrium market configuration is with one merger including all firms. When  $n \geq 7$ , the equilibrium market configurations are as in Proposition 1.*

Proposition 2 states that, in markets with at least seven firms, even if  $\rho_1$  is allowed to

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about which firms desire to form a coalition. Indeed, it is possible to show that, for every distribution between the two available options for  $\rho_2$ , namely  $\{\rho_2, \rho_3\}$  and  $\{\rho_2, \rho_3, \dots, \rho_n\}$ , or even if no assumption is made on such distribution, the possible merger sizes are as in Proposition 1. All the details are available from the author upon request. It is also worth to point out that, the fact that *i)* a firm that decided to pass can not be the receiver of a merger proposal and *ii)* a coalition involves firms whose positions follow each other in the order of play, acts as a further refinement over a large set of qualitatively identical equilibria, with respect to those in Proposition 1. If this assumption is relaxed, then, for instance, a situation in which  $\rho_1$  passes,  $\rho_2$  proposes the coalition  $\{\rho_1, \rho_2\}$  or any other coalition  $\{\rho_1, \rho_{z>2}\}$  and the receiving firm  $\rho_{z \neq 2}$  accepts, is an equilibrium.

<sup>4</sup>As a merged entity acquires the leadership despite its size, an extensions of this model may explore the possibility of a stochastic leadership, where the probability of obtaining the first move advantage is an increasing function in the number of insiders.

merge with all the remaining firms, the grand coalition is not an equilibrium. On the one hand, the single firm profit in a monopoly  $\frac{1}{4n}$  is larger than the profit that  $\rho_1$  obtains as a follower if the bilateral merger  $\{\rho_2, \rho_3\}$  takes place; on the other hand, the contribution to  $\rho_1$ 's expected profit from being the unique follower in case  $\{\rho_2, \dots, \rho_n\}$  forms is strong and, if  $\frac{1}{4n}$  is small enough (*i.e.*  $n \geq 7$ ), passing is the best option.

### 2.1.1 Mergers in an exogenous setting

I conclude this section with a single merger by analyzing mergers in an exogenous setting, where a group of firms is randomly selected to join in a coalition and an equilibrium is only dictated by profitability. This is the environment in which seminal papers like SSR (1983) and Perry and Porter (1985) analyze mergers, as well as a large strand of literature since then. The main result is the following:

**Proposition 3.** *If  $m < n$  firms merge in a single entity which acts as a leader over the remaining  $n - m$  outsiders, then:*

- i) the merger is profitable irrespective of the number of participants;*
- ii) provided that  $n \geq 4$  and  $2 \leq m < \frac{n+1}{2}$ , the free riding issue is solved.<sup>5</sup>*

Point *i)* is a special case of Proposition 1 in LW (2015) and states that a single leading entity, due to its increased market power, can profitably merge with an arbitrary number of firms.

Point *ii)* deals with the free riding issue, which was left unexplored by LW, and states that in markets with at least four firms, only if the number of insiders  $m$  is not too large with respect to the pre-merger number of firms  $n$ , participating provides a higher payoff than remaining outside. The larger  $m$ , the larger the profit of the unique leader. This occurs because the followers' production reduces in  $m$ , while the one of the leader is unaffected. The effect on the followers' output can be seen differentiating  $(n - m)q_f$  with respect to  $m$ .<sup>6</sup>

$$\frac{\partial(n - m)q_f}{\partial m} = -\frac{1}{2(n - m + 1)^2} < 0. \quad (6)$$

As a consequence, total output shrinks and the market price rises. Again, this can be checked differentiating  $p$  with respect to  $m$ :

$$\frac{\partial p}{\partial m} = \frac{1}{2(n - m + 1)^2} > 0. \quad (7)$$

Since the quantity of the leader  $q_I = \frac{1}{2}$  is independent from both  $n$  and  $m$ , his profit can increase only through this indirect effect of the followers' quantity on price. However,

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<sup>5</sup>If  $n = 3$  and  $m = 2$  a firm is indifferent between being a follower or part of a bilateral coalition.

<sup>6</sup>Throughout the paper, all variations in both  $n$  and  $m$  do not have an ambiguous sign, so that, although we only consider values in  $\mathbb{N}$ , we can without loss of generality make use of calculus.

if the number of insiders is large (*i.e.*  $m \geq \frac{n+1}{2}$ ), then the price increase is weak and an insider can not obtain a larger profit than the one of an outside firm.

It is worth noting that, in the context of quantity setting mergers, all firms benefit from the reduced number of competitors. In Stackelberg markets, such benefit can make each follower better off with respect to the pre-merger scenario as well. This feature is important, since it is straightforward to show that, in absence of merger, a Stackelberg follower always ends up being worse off than a Cournot-Nash firm. In the classical Stackelberg duopoly, for example, the leader obtains the profit  $\frac{1}{8}$  and the follower obtains the profit  $\frac{1}{16}$ , which are respectively larger and lower than the Cournot-Nash profit  $\frac{1}{9}$ .<sup>7</sup> If the reduction in market power (*i.e.* being a follower) had a greater impact than the benefit from the merger, each follower would end up being worse off than the pre-merger scenario. Were the case, a profitable merger would also imply the resolution of the free riding issue. Since it is not always the case, the search for conditions in which each insider gains more than each outsider is not trivial and deserves its own attention.

From Propositions 1 and 3 the next corollary follows:

**Corollary 1.** *If  $n \geq 4$ , in the endogenous equilibrium in which the last  $n - 1$  firms merge, the free riding issue survives, while in the equilibrium in which  $\{\rho_2, \rho_3\}$  forms, the free riding issue is solved.*

As already pointed out, both insiders and outsiders benefit from the reduced number of competitors and all firms may end up being better off than the pre-merger scenario. Moreover, although turning into a follower, each outsider remains a single firm, while the merged entity has to split his profit among the insiders. It turns out that in E1, the profit inside the coalition is lower than the profit of each independent outsider. This reasoning is reversed in case of bilateral mergers, where the small size of the leading entity makes the profit share of each of the two insiders larger than the profit of a follower.

### 3 The merger game with more heterogeneous leaders

In this section, I allow for multiple mergers. These mergers may be heterogeneous, in the sense that each new entity may be the result of a merger of a different number of firms. Given the possibility of generating multiple independent entities, the pre-merger number of firms  $n \geq 4$ . The game is slightly different than the one described in Section 2 and more closely related to QZ.

*The timing of the game*

**Stage 1)**  $\rho_1$  can propose a merger of size  $m \in [2, n - 1]$  following the order induced by  $\rho$ , or pass. The receivers simultaneously decide whether to accept or the reject the

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<sup>7</sup>More generally, in a game with one leader and  $n - 1$  followers, the leader obtains  $\pi_l = \frac{1}{4n} > \frac{1}{(n+1)^2}$ ,  $\forall n \geq 3$ , while each follower obtains  $\pi_f = \frac{1}{4n^2} < \frac{1}{(n+1)^2}$ ,  $\forall n \geq 3$ .

proposal. If all the receivers accept, the merger takes place. If one or more receivers reject, the merger stage ends and the game proceeds at stage 3. If  $\rho_1$  passes,  $\rho_2$  can either propose a merger of size  $m \in [2, n - 1]$  or pass. If a merger takes place, the first firm in the order of play which has not received a proposal can either propose a merger or pass. If all the receivers accept, a second merger occurs; if one or more receivers reject, the game proceeds at stage 2. This process goes on until  $\rho_{n-1}$ .

**Stage 2)** The merged entities (if any) non cooperatively set their profit maximizing quantity  $q_l$ .

**Stage 3)** The  $n - m$  outside firms (if any) non cooperatively set their profit maximizing quantity  $q_f$  given the quantity  $q_l$  set by the leaders at stage 2.

A SPNE is a triple  $(\kappa^*, q_l^*(n, m^*), q_f^*(q_l^*(n, m^*))) \in \mathbb{K} \times \mathbb{R}_{++}^2$ , where  $\kappa^*$  is the equilibrium coalition structure,  $q_l^*(n, m^*)$  is the optimal quantity set by each merged entity as a function of the pre-merger number of firms  $n$ , the total number of equilibrium merging firms  $m^* \equiv \sum_{l=1}^L m_l^*$ , where  $m_l^*$  is the equilibrium size of leader  $l$ , and  $q_f^*(q_l^*(n, m^*))$  is the optimal quantity set by each follower as a function of the the optimal quantity of the leader/s set at stage 2.

### 3.1 Equilibrium mergers

With multiple mergers, each firm faces the following tradeoffs. A proposer has to realize whether it is more convenient to be part of a single large merger, or being part of a small merger and allow other firms to propose themselves additional mergers. If a proposer foresees that multiple mergers will occur, he might choose to pass and behave as a follower, rather than being one of many insiders. A receiver has to realize whether it is more convenient to accept or, given the possibility of multiple mergers, to reject an offer and remaining outside.

The game is explicitly solved by backward induction. At stage 3, if  $L \geq 1$  coalition/s emerged from stage 2, in the post-merger scenario we are left with  $n - m + L$  firms and each outsider  $f = 1, \dots, n - m$  solves:

$$\max_{q_f} (1 - q_f - Lq_l - q_{-f}) q_f, \quad (8)$$

where  $q_{-f}$  is the total supply of the all followers, but follower  $f$ . Applying symmetry to (8) and solving for  $q_f$  yields:

$$q_f = \frac{1 - Lq_l}{n - m + 1}. \quad (9)$$

At stage 2, each leader  $l = 1, \dots, L$  takes into account the reaction function (9) and solves:

$$\max_{q_l} \left( 1 - q_l - (n - m) \frac{1 - q_l - q_{-l}}{n - m + 1} - q_{-l} \right) q_l, \quad (10)$$

where  $q_{-l}$  is the total supply of all leaders, but leader  $l$ . Applying symmetry to (10), solving for  $q_l$  and substituting in (9), yield the equilibrium quantities:

$$q_l^*(L) = \frac{1}{L+1}, \quad l = 1, \dots, L; \quad (11)$$

$$q_f^*(n, m, L) = \frac{1}{(L+1)(n-m+1)}, \quad f = 1, \dots, n-m. \quad (12)$$

The equilibrium profits are:

$$\pi_l(n, m, L) = \frac{1}{(L+1)^2(n-m+1)}, \quad l = 1, \dots, L; \quad (13)$$

$$\pi_f(n, m, L) = \frac{1}{(L+1)^2(n-m+1)^2}, \quad f = 1, \dots, n-m. \quad (14)$$

As shown in the previous section, in endogenous settings, the fact that a merger is profitable and makes each insider better off than each outsider is not sufficient for an equilibrium.

I now present some useful results for the determination of an equilibrium for an arbitrary pre-merger market size  $n$ . The first lemma regards the profitability of mergers in markets where  $m$  insiders exogenously merge into  $L$  leading entities and shows conditions under which a further merger of a subset of the  $L$  leaders into a single larger coalition is itself profitable.

**Lemma 1.** *Assume a market with  $L \geq 2$  possibly heterogenous leading entities. Assume a merger of  $J \leq L$  leaders. This merger is profitable only if  $\frac{J}{L} > 0.8$ . If the leading entities are identical, then all insiders benefit from the merger.*

Lemma 1 brings back to light the main result of SSR by stating that multiple leading entities can profitably merge only if the new coalition involves at least the 80 percent of the leaders. However, since mergers may be heterogeneous in the number of insiders, a profitable merger does not imply that each one obtains a larger profit than the one before the merger. Consider a situation with two leading entities, of sizes  $m_1 = 2$  and  $m_2 = 3$  respectively, each obtaining an aggregate payoff of 10. Suppose that, after the merger, the payoff of the new larger entity is 21. At this point, a symmetric split of the pie assigns to each insider a profit of 4.2. Thus, each insider of the three firms coalition would find it more profitable forming the five firms coalition, but the insiders of the bilateral coalition would find it unprofitable. If instead each coalition is formed by the same number of firms, then all insiders would obtain a strictly larger payoff than the one as members of independent entities.

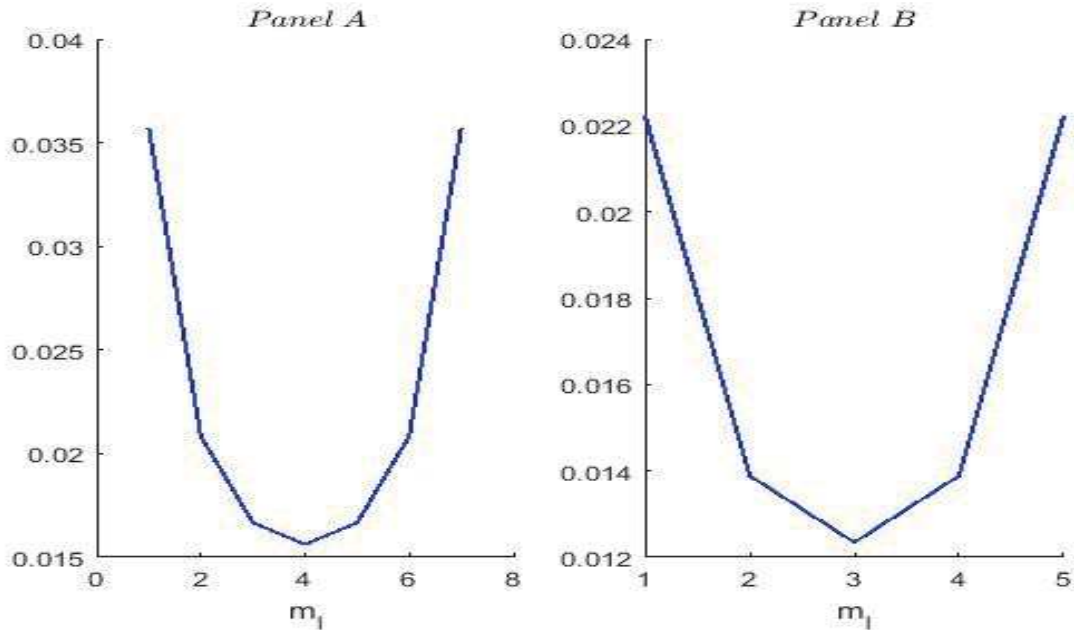
The following lemma, instead, is more strictly related to the three stage game, but still provides only necessary conditions for an equilibrium.

**Lemma 2.** Denote  $\overline{m}_l$  the number of insiders that already merged before  $\rho_z$ . For every firm  $\rho_z$ ,  $z = 1, \dots, n - 1$ :

- i)* if  $\rho_z$  proposes the formation of the additional leading entity  $l$ , then the  $\pi_{i,z}$  function is convex in the merger size  $m_l$ , attains a minimum at  $m_l = \frac{n - \overline{m}_l + 1}{2}$ , value in which it has a symmetry line;
- ii)* if all firms before  $\rho_z$  are members of a coalition, then the  $\pi_{i,z}$  function is maximized by merging with all the remaining firms; if  $\overline{m}_l = z - 2$ , a bilateral merger provides the same per firm payoff of an  $n - z + 1$  firms merger; if  $\overline{m}_l < z - 2$ , a bilateral merger is the best option;
- iii)* it is always more profitable for two firms to join in an additional bilateral coalition rather than acting as followers.

Point *i)* is a generalization of what stated in Subsection 2.1 concerning the shape of the  $\pi_{i,z}$  function. The size of a leading entity  $m_l$  maximizing the single insider's profit is a function of the number of firms that previously formed other coalitions. The case of a single merger is a special one, in which, whenever a proposal is made, other mergers have not occurred (*i.e.*  $\overline{m}_l = 0$ ). A graphical example is provided in Figure 1.

Figure 1: The  $\pi_{i,z}$  function with  $(n, m, m_l, L) = (10, 3, m_l, 1)$  (Panel A) and  $(n, m, m_l, L) = (10, 5, m_l, 2)$  (Panel B).



Point *ii)* provides additional information about the optimal merger size at each position in the order of play, as a function of the number of firms that previously merged into an arbitrary number of coalitions. The most useful part of point *ii)* is that, whenever all firms before  $\rho_z$  decided to be part of a coalition, the optimal merger size for  $\rho_{z+1}$  is the one

involving all the remaining firms. This result is independent from the number of previously formed coalitions, which clearly restricts the number of scenarios that need to be checked to determine the equilibrium coalition structure.

Point *iii)* states that splitting the profit of an additional coalition between two insiders is more advantageous than not forming such coalition and let the two firms behave as followers. This result is useful for the subgame in which  $\rho_{z-1}$  can either propose or pass, and allows to say that, irrespective of what happened before  $\rho_{z-1}$ , this firm will always propose the bilateral coalition  $\{\rho_{z-1}, \rho_z\}$  and  $\rho_z$  will accept.

Lemma 2, however, provides profit maximizing conditions in a non strategic environment. Suppose that all firms before  $\rho_z$  already merged in one or more coalitions and that  $n - z + 1$  is sufficiently large. In this scenario, point *ii)* applies and  $\rho_z$  would like to merge with all the remaining firms. In the strategic environment of the paper, a further split of the  $n - z + 1$  firms could assign to  $\rho_z$  an even larger payoff. This is the other side of the coin of Lemma 1, as a large coalition can be viewed as the union of smaller subcoalitions. If these subcoalitions account for less than the 80 percent of the market, then merging into a larger coalition is not optimal. This suggests the importance of the interplay between Lemmas 1 and 2 to determine the equilibrium coalition structure.

Focusing on the equilibrium of the game, although I do not provide a general result, I show that, even restricting the analysis to  $n = \{4, 5, 6, 7, 8\}$ , it is possible to highlight the relevant features of the model.

**Proposition 4.** *The equilibrium coalition structures are:*

- i)* if  $n = 4$ ,  $\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3, \rho_4\})$ ;
- ii)* if  $n = 5$ ,  $\kappa^* = (\{\rho_1\}, \{\rho_2, \dots, \rho_5\})$ ;
- iii)* if  $n = 6$ ,  $\kappa^* = (\{\rho_1\}, \{\rho_2, \dots, \rho_6\})$ ;
- iv)* if  $n = 7$ ,  $\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3, \rho_4\}, \{\rho_5, \rho_6, \rho_7\})$ ;
- v)* if  $n = 8$ ,  $\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3, \rho_4, \rho_5\}, \{\rho_6, \rho_7, \rho_8\})$ .

Proposition 4 shows that two types of equilibria are possible: in one, the first firm in the order of play decides to pass, predicting that a single merger involving all the remaining firms will take place. Being the profit of both followers and leaders decreasing in  $L$ , the formation of a unique coalition is crucial for  $\rho_1$ 's decision of passing. If, for example, the four firms in case *ii)* would find it even more advantageous splitting into a pair of bilateral subcoalitions, then being the unique follower is not optimal any more. This is what happens in cases *i)*, *iv)* and *v)*.

An important feature of the model is that, differently from QZ, in which firms are constrained to form bilateral coalitions, here I show that, allowing for arbitrary merger sizes, a merger wave can still likely contain bilateral agreements, but larger sizes are possible as well.

### 3.2 The equilibrium with different ex-ante merger policies

Proposition 4 has shown that, if monopolization is not allowed, either a unique merger involving the last  $n - 1$  firms takes place or mergers occur in waves. In line with the analysis in Section 2, I show whether the equilibrium market structure changes as a function of the merger policy tightness. In this case, however, I study, not only the implications of the removal of monopolization, but also the consequences of tighter merger policies. This is because, if the merging parties control or are expected to control after the merger a large market share, then one could reasonably expect even smaller mergers than the one involving all firms to be prohibited. In particular, the EU Horizontal Merger Guidelines states that:

*"The Commission has thus in several cases considered mergers resulting in firms holding market shares between 40% and 50%, and in some cases below 40%, to lead to the creation or the strengthening of a dominant position.*

In this paper, being firms identical, at least in the pre-merger scenario, it is possible to express market shares in terms of the number of firms  $n$  only. Thus, applying a tighter merger policy is equivalent to prohibiting mergers involving a progressively lower number of firms.

The first result concerns the effect of the introduction of monopolization and it is summarized in the next proposition:

**Proposition 5.** *If monopolization is allowed, the equilibrium coalition structures of the three stage game with multiple heterogeneous leaders are:*

*i) if  $n = 4$ ,  $\kappa^* = (\{\rho_1, \dots, \rho_4\})$ ;*

*ii) if  $n = 7$ ,  $\kappa^* = (\{\rho_1, \dots, \rho_7\})$ ;*

*iii) if  $n = 8$ , both monopolization and the merger wave in Proposition 4 are equilibria;*

*iv) as in Proposition 4, if  $n = \{5, 6\}$ .*

Proposition 5 shows two interesting features: if  $\rho_1$  decides to pass in equilibrium, then, even if forming the grand coalition is allowed, being the unique follower is still the best option. Furthermore, and this is probably the most striking feature, whenever a merger wave takes place, the introduction of monopolization makes that the grand coalition forms. The conclusion is that merger waves represent a powerful strategic tool that firms adopt as a response to the exclusion of monopolization. In particular, the merger wave in case *v*), assigns to  $\rho_1$  and  $\rho_2$  the same payoff as the one in a monopoly.

I now assume that the ex-ante merger policy is more severe and also forbids  $n - 1$  and  $n - 2$ -firm mergers. From now on, I only focus on  $n = 5$ . This is because, first, despite monopolization being allowed or not, according to Proposition 5, the equilibrium market structure is the one in which  $\rho_1$  passes and the remaining firms form the coalition



$\{\rho_2, \dots, \rho_5\}$ , thereby it would be interesting to see whether the equilibrium market structure is sensible to tighter policies; second, in terms of pre-merger market shares, the difference between an  $n - 1$ -firm merger and an  $n - 2$ -firm merger, is significant with only five firms.

I can now state the following:

**Proposition 6.** *If  $n = 5$  and:*

*i)  $n - 1$ -firm mergers are forbidden, the equilibrium market structure is*

$$\kappa^* = (\{\rho_1, \rho_2, \rho_3\}, \{\rho_4, \rho_5\});$$

*ii)  $n - 2$ -firm mergers are forbidden, the equilibrium market structure is*

$$\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3\}, \{\rho_4\}, \{\rho_5\})$$

Proposition 6 shows that, if, on the one hand, the equilibrium market structure with five firms of Proposition 4 is not affected by the exclusion of monopolization, it is affected by a tighter merger policy.

In particular, if the coalition  $\{\rho_2, \dots, \rho_5\}$  can not form, then merger waves go back into the picture. It turns out that  $\rho_1$  merges with  $\rho_2$  and  $\rho_3$  and the remaining firms form an additional bilateral coalition. This is because, if  $\rho_1$  passes, then a trilateral merger takes place, and  $\rho_1$  is one of two followers. Thus,  $\rho_1$  can improve his payoff by being part of a coalition rather remaining outside.

When even mergers involving  $n - 2$  firms are forbidden, then a single bilateral merger involving the first two firms in the order of play, takes place. As it will be shown, this relation between the equilibrium market structure and the merger policy tightness can have interesting implications in terms of welfare.

### 3.2.1 Mergers in an exogenous setting

I conclude this section with multiple coalitions, by analyzing mergers in a setting in which there is no strategic approach in the formation of a coalition.

In particular, assuming that at least two mergers occurred, each is profitable if the gain of a leader, given by the difference between its post-merger profit and the pre-merger cumulated profit of its members is positive. Let  $m_l$  denote the number of insiders of leader  $l$ , with  $\sum_l m_l = m$ . Hence the gain can be written as follows:

$$g_l(n, m_l, m, L) = \pi_l - m_l \tilde{\pi}(n) = \frac{(n+1)^2 - m_l(L+1)^2(n-m+1)}{(L+1)^2(n+1)^2(n-m+1)}. \quad (15)$$

Assuming that a leader is formed by at least two firms, the maximum number of insiders of leader  $l$  when  $m < n$  firms decide to merge into  $L \geq 2$  leaders is  $m_l = m - 2(L - 1)$ . Being formed by the largest number of insiders, such leader is the one for which it is harder to obtain a positive post-merger gain. More generally, if leader  $l$  is formed by

$m_l = m - 2(L - 1) - k$  firms, with  $k \in [0, m - 2L]$ , then the remaining  $k$  firms merge into the remaining  $L - 1$  leaders. In case  $m_l = m - 2(L - 1) - k$ , (15) becomes:

$$\tilde{g}_l(n, m, k, L) = \frac{(n + 1)^2 - (m - 2(L - 1) - k)(n - m + 1)(L + 1)^2}{(L + 1)^2(n + 1)^2(n - m + 1)}. \quad (16)$$

I explicitly show equation (16) given its relevance for the next proposition:

**Proposition 7.** *Let:*

$$\bar{k}(n, m, L) \equiv \frac{(m - 2(L - 1))(n - m + 1)(L + 1)^2 - (n + 1)^2}{(n - m + 1)(L + 1)^2}. \quad (17)$$

Define  $k^*$  as the lowest non negative value of  $k$  such that  $\tilde{g}_l(n, m, L, k) > 0$ . The  $k^* : \bar{k}(n, m, L) \rightarrow \mathbb{R}$  functional is such that:

$$k^* = \begin{cases} 0 & \text{if } \bar{k}(n, m, L) \leq 0 \\ \bar{k} & \text{if } \bar{k}(n, m, L) > 0 \end{cases} \quad (18)$$

If  $m < n$  firms merge into  $L \geq 2$  leaders and  $k^* = 0$ , then  $g_l > 0, \forall l = 1, \dots, L$ . If  $k^* = \bar{k}$ , a merger is profitable only if the number of insiders of leader  $l$  is  $m_l \leq \lfloor m - 2(L - 1) - k^* \rfloor$ .

Proposition 7 states that if  $k^* = 0$ , a merger is profitable even for the leader formed by the largest possible number of insiders when  $m < n$  firms merge into  $L \geq 2$  leaders. This implies that all mergers are profitable irrespective to the way in which the  $m$  insiders are spread into the  $L$  leading entities. However, there may exist triples  $(n, m, L)$  for which only some redistributions induce multiple profitable mergers. This occurs when  $k^* = \bar{k}$ , in which a merger is profitable only if the number of insiders is sufficiently small (*i.e.*  $m_l \in [2, \lfloor m - 2(L - 1) - k^* \rfloor]$ ).<sup>8</sup> In other words, a profitable merger requires subtracting at least one unit from the largest possible size  $m - 2(L - 1)$ . This implies that if  $\bar{k} > 0$ , there exist redistributions of the  $m$  merging firms into the  $L$  leaders in which some mergers are profitable and others are not.

As an illustration, consider the triple  $(10, 6, 2)$ . In this case  $\bar{k} = 1.31$  and  $m - 2(L - 1) - \bar{k} = 2.69$ , so that a merger is profitable only if it involves two firms. Adding another merging firm, one obtains that  $\bar{k} = 1.63$  and  $m - 2(L - 1) - \bar{k} = 3.37$ , so that a merger is profitable if it involves at most three firms. For deeper clarity,  $g_l(10, 4, 7, 2) = -0.005$ , while  $g_l(10, 3, 7, 2) = 0.003$ .

The above analysis does not solve the problem that a firm can benefit more from the merger by remaining outside. Hence, I now focus on the resolution of the free riding issue. The profit of insider  $i$ , member of leader  $l$ , of size  $m_l$  in a market with  $L$  leaders and  $m < n$  merging firms is:

$$\pi_i^l(n, m, m_l, L) = \frac{\pi_l}{m_l} = \frac{1}{m_l(L + 1)^2(n - m + 1)}. \quad (19)$$

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<sup>8</sup>The notation  $\lfloor \cdot \rfloor$  means approximating down to the largest integer.

The gain of each insider with respect to each outsider is:

$$g_i^l(n, m_l, m, L) = \pi_i^l - \pi_f = \frac{n - m + 1 - m_l}{m_l(L + 1)^2(n - m + 1)^2}. \quad (20)$$

In line with the reasoning of the profitability analysis, if  $m_l = m - 2(L - 1) - k$ , then (20) becomes:

$$\tilde{g}_i^l(n, m, k, L) = \frac{n - 2(m - L) - 1 + k}{(m - 2(L - 1) - k)(L + 1)^2(n - m + 1)^2}. \quad (21)$$

The main result is the following:

**Proposition 8.** *Let:*

$$\tilde{k}(n, m, L) \equiv 2(m - L) + 1 - n. \quad (22)$$

Define  $k^{**}$  as the lowest non negative value of  $k$  such that  $\tilde{g}_i^l(n, m, L, k) > 0$ . The  $k^{**} : \tilde{k}(n, m, L) \rightarrow \mathbb{R}$  functional is such that:

$$k^{**} = \begin{cases} 0 & \text{if } \tilde{k}(n, m, L) \leq 0; \\ \tilde{k} & \text{if } \tilde{k}(n, m, L) > 0. \end{cases} \quad (23)$$

If  $m < n$  firms merge into  $L \geq 2$  leaders and  $k^{**} = 0$ , then  $g_i^l > 0$ ,  $\forall i = 1, \dots, m_l$ ,  $\forall l = 1, \dots, L$ . If  $k^{**} = \tilde{k}$ , then each insider of leader  $l$  is better off than each outsider only if  $m_l \leq \lfloor m - 2(L - 1) - k^{**} \rfloor$ .

Proposition 8 states that if  $k^{**} = 0$ , every insider in the market is better off than each outsider, irrespective of the way in which the insiders are spread among the leaders. In economic terms, due to the symmetric nature of firms, each leader equally splits the same profit among the insiders, so that, if each member of the leader formed by the largest number of insiders obtains a larger profit than the one of each follower, a fortiori all insiders of all leaders will always benefit more from the merger than the single outsider. When instead  $\tilde{k} > 0$ , the free riding issue is solved only if a firm is part of a merger involving at most  $\lfloor m - 2(L - 1) - k^{**} \rfloor$  firms. Following the reasoning of Proposition 6, if  $\tilde{k} > 0$ , in order to make each insider of leader  $l$  better off than each outsider, at least one unit must be removed from the largest possible merger size  $m - 2(L - 1)$ .

## 4 Extension

So far a rejection implied the end of the merger stage. This assumption may be considered too strong. In this section, I analyze a game in which this assumption is relaxed and show that it provides the same equilibria of the games in Sections 2 and 3. In particular, if  $\rho_z$  proposes a coalition and a receiver rejects, then  $\rho_{z+1}$  can propose a merger. I also assume that if a firm is indifferent between accepting or rejecting, she accepts.

I first focus on a game in which a single merger is allowed and state the following:

**Proposition 9.** *In the merger game with a unique leader, the equilibria characterized in Proposition 1 are robust to the removal of the assumption that a rejection implies the end*

of the merger stage.

I now allow for multiple heterogeneous mergers and, in order to streamline the presentation and make the Appendix more readable, only focus on  $n = \{4, 5, 6\}$ :

**Proposition 10.** *In the merger game with multiple heterogeneous leaders, if  $n = \{4, 5, 6\}$ , the equilibria characterized in Proposition 4 are robust to the removal of the assumption that a rejection implies the end of the merger stage.*

The results in Proposition 9 and 10 are based on the fact that, whenever a firm belongs to an equilibrium coalition structure in Propositions 1 and 4 and a rejection occurs, the rejecting firm can at most tie his payoff by proposing an alternative coalition. This fact, for example, is more obvious in the game with a single leader, where if a rejection from one of the two equilibrium coalition structures occurs, then it is made by  $\rho_z, z \geq 3$ . Each of these firms can at best tie his payoff by merging with the following firm in the order of play.

## 5 Welfare

In this section the welfare implications of mergers are explored, adopting two different measures: consumer surplus (CS) only, and the sum of CS and industry profits  $\Pi$ .<sup>9</sup>

### 5.1 Consumer welfare

Under linear demand,  $CS(Q) = \frac{1}{2}Q^2$ , and the analysis can be carried out in terms of quantity only. From (11) and (12), the post-merger supply is:

$$Q^{post}(n, m) = q_l + (n - m)q_f = \frac{(n - m)(L + 1) + L}{(L + 1)(n - m + 1)}, \quad (24)$$

and the variation between the post and the pre-merger total output is:

$$\Delta Q(n, m) = \frac{(n - m)(L + 1) + L}{(L + 1)(n - m + 1)} - \frac{n}{n + 1} = \frac{L(n - m + 1) - m}{(n + 1)(L + 1)(n - m + 1)}. \quad (25)$$

The following result holds:

**Proposition 11.** *When a single merger is allowed, if the coalition  $\{\rho_2, \rho_3\}$  forms, consumers are better off. When multiple coalitions are allowed, in all cases analyzed in Proposition 4, consumers are worse off.*

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<sup>9</sup>Measuring the welfare effects of mergers through the variation of CS only is the standard approach adopted by the European Commission and the Federal Trade Commission in the US. In this section, I show that, including profits in the social welfare function does not modify the conclusions obtained by looking at the movement of CS only.

Proposition 11 states that in case of a single bilateral merger, CS rises, while in every other equilibrium, the outside firms' production is not large enough to make the post-merger total output increase. Thus, although rarely, bilateral mergers between symmetric firms generating no synergies can be beneficial to consumers. Since the production of the unique leading entity is independent from both  $n$  and  $m$ , the production of the followers' group becomes crucial. In particular, although the leading entity always produces less than the cumulated pre-merger output of its members, in case of a single bilateral coalition, the number of followers is sufficiently high and so is their production. The reverse is true in case of a single merger involving  $n - 1$  firms, and in case of multiple mergers. In both scenarios, the number of competitors shrinks too much.

The fact that it is possible to identify CS increasing mergers in the present setting is interesting, since it is commonly recognized that mergers between identical firms generating no synergies are CS reducing. In this paper, instead, it is shown that the change in the market structure alone can be enough to make consumers better off.

It is worth noting that the second part of Proposition 11 takes care of the welfare effect of a whole merger wave, without considering one merger at the time. However, it is possible to show that, at least for the cases of Proposition 4, once the first (CS enhancing) bilateral merger has occurred, then, given the current market structure with a single leader and  $n - 2$  followers, a further merger is not strictly CS enhancing.<sup>10</sup> Thus, if one includes the authority in the model and only monopolization is excluded, then the equilibrium market structure would be as in Proposition 1. Finally, according to Proposition 6, it follows that CS enhancing mergers can also be driven by a tighter merger policy than a one in which only monopolization is not allowed.

## 5.2 Welfare as the sum of consumer surplus and industry profits

In what follows, I take care of the supply side too and let welfare to be measured by the sum of consumer surplus and industry profits. I will refer to this measure as *social welfare*. The post-merger social welfare writes:

$$W^{post} = \frac{1}{2}(Q^{post})^2 + L\pi_l + (n - m)\pi_f = \frac{[(n - m)(L + 1) + L][(n - m)(L + 1) + (L + 2)]}{2(L + 1)^2(n - m + 1)^2} \quad (26)$$

and the pre-merger social welfare writes:

$$\tilde{W} = \frac{1}{2}\tilde{Q}^2 + n\pi^{CN}(n) = \frac{n(n + 2)}{2(n + 1)^2}. \quad (27)$$

I state the following result:

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<sup>10</sup>With  $n = 7$ , the second bilateral merger, which generates a transition from a market structure with a single leader and five followers to a market structure with two bilateral leading entities and three followers leaves CS unaltered.

**Proposition 12.** *Social welfare follows the same direction of CS.*

Proposition 12 explains that, adding the supply side in the welfare analysis, does not modify the result obtained in Proposition 11. Hence, the direction of the CS variation is sufficient to assess the overall effect of these mergers.

## 6 Conclusions

I endogenized mergers in a scenario in which a merger endows the new entity with the market leadership from a static symmetric Cournot market. I first explored the case of a single merger and shown that, although Liu and Wang (2015) prove that, in an exogenous setting, such merger is profitable irrespective of the number of participants, only two endogenous equilibria are possible: either a bilateral or an  $n - 1$ -firm coalition. Thus, if on the one hand, markets tend to monopolization, on the other hand, bilateral mergers are equally likely. Furthermore, even if monopolization is allowed and a fairly weak condition on the pre-merger number of firms is satisfied, the grand coalition still does not form. I then allowed for multiple heterogeneous mergers in the number of insiders and shown that, whenever an equilibrium is robust to monopolization, at least one firm desires to be a follower. However, when an equilibrium is not robust to monopolization, then mergers occur in waves. This suggests the important strategic role of merger waves as a response to the exclusion of monopolization. In particular, with eight firms, the two players forming the unique bilateral coalition in the wave, obtain the same payoff as the one in a monopoly. Differently from Qiu and Zou (2007), who impose bilateral agreements only, here, by allowing for arbitrary merger sizes, it is shown that merger waves can still likely contain bilateral coalitions, but larger sizes are reasonable as well. I also check how my results vary with different ex-ante merger policies. The welfare analysis shows that a single bilateral merger is socially desirable, while all other equilibria are welfare reducing. This suggests that mergers between identical firms generating no synergies can be welfare increasing through the change in the market structure alone. This result challenges the fact that these type of mergers are usually believed to be welfare decreasing. Finally, I provided a robustness check by showing that a game in which a rejection does not imply the end of the merger stage provides the same equilibria of the original games in which this feature is assumed.

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# Appendix

## Proof of Proposition 1.

The first step is the following lemma:

**Lemma 3.** *A proposal is never rejected.*

*Proof.* A rejection implies going back to a Cournot-Nash game, in which every firm obtains  $\tilde{\pi}(n)$ . From Proposition 1 in LW (2015), a single merger is profitable for every merger size. This implies that, whenever a proposal is made, each insider obtains at least  $\pi^{CN}(n)$ .  $\square$

Since a proposal is never rejected, the profit maximizing action for a proposer is an  $m$  firms merger, where  $m \in \operatorname{argmax}_{2 \leq m < n} \pi_{i,z}(n, m)$ . This function is U-shaped in  $m$ , attains a minimum at  $m = \frac{n+1}{2}$ , value in which it has a symmetry line. This implies that  $\operatorname{argmax}_{2 \leq m < n} \pi_{i,z}(n, m) = \{2, n-1\}$ , which also implies that only  $\rho_1$  and  $\rho_2$  can explore the option of proposing either a bilateral or an  $n-1$  firms coalition. At this point, the analysis begins from the subgame in which  $\rho_{n-1}$  has the option to propose. Were it the case, then all firms before  $\rho_{n-1}$  decided to pass. If  $\rho_{n-1}$  proposes, the only option is to form the bilateral coalition  $\{\rho_{n-1}, \rho_n\}$  and each insider gets  $\frac{1}{8(n-1)}$ . If  $\rho_{n-1}$  passes, all firms are back to a Cournot-Nash game, each obtaining  $\frac{1}{(n+1)^2}$ , where  $\frac{1}{8(n-1)} \geq \frac{1}{(n+1)^2}$ ,  $\forall n \geq 3$ . Hence  $\rho_{n-1}$  proposes. Now notice that each firm  $z \in \{3, n-2\}$  maximizes its post-merger gain in a bilateral coalition. This is because, from  $\rho_3$  forward, a firm can at most merge with  $n-2$  firms (if  $\rho_3$  proposes). Since forming an  $n-1$  firms coalition is not a feasible action, given the already stated properties of the  $\pi_i$  function, a bilateral coalition is the best option. Thus, the analysis of several subgames can be summarized in a single condition. In particular, if  $\rho_{4 \leq z \leq n-2}$  proposes, each outsider obtains  $\frac{1}{4(n-1)^2}$ , while if  $\rho_{3 \leq z \leq n-3}$  proposes, each insider gets  $\frac{1}{8(n-1)}$ , where  $\frac{1}{8(n-1)} \geq \frac{1}{4(n-1)^2}$ ,  $\forall n \geq 3$ . Thus  $\rho_z, z \in \{3, \dots, n-2\}$  proposes. If  $\rho_3$  proposes, each outsider again gets  $\frac{1}{4(n-1)^2}$ . Moving backward, if  $\rho_2$  proposes, he has two options: either forming the bilateral coalition  $\{\rho_2, \rho_3\}$  or the  $n-1$  firms coalition  $\{\rho_2, \dots, \rho_n\}$ , obtaining the profit  $\frac{1}{8(n-1)}$ . This profit, as already seen, is larger than the one as outsider in case  $\rho_3$  proposes. Hence,  $\rho_2$  proposes. I'm left with analyzing the behavior of  $\rho_1$ . If  $\rho_2$  proposes, each outsider gets  $\frac{1}{4(n-1)^2}$  in case of bilateral merger, while in case of an  $n-1$  firms merger, the unique outsider gets  $\frac{1}{16}$ . Since  $\rho_2$  is indifferent between the two options,  $\rho_1$  assigns a probability of  $\frac{1}{2}$  to each event to occur. Hence if  $\rho_1$  passes, his expected profit is  $\mathbb{E}\pi_{\rho_1} = \frac{1}{2} \frac{1}{16} + \frac{1}{2} \frac{1}{4(n-1)^2}$ . If  $\rho_1$  proposes, he can either propose a bilateral merger or an  $n-1$  firms merger, case in which he gets  $\frac{1}{8(n-1)}$ , where  $\frac{1}{2} \frac{1}{16} + \frac{1}{2} \frac{1}{4(n-1)^2} \geq \frac{1}{8(n-1)}$ ,  $\forall n \geq 3$ . Hence  $\rho_1$  should pass and let  $\rho_2$  propose. This implies that the only two possible equilibrium coalition structures are  $(\{\rho_1\}, \{\rho_2, \rho_3\}, \dots, \{\rho_n\})$  and  $(\{\rho_1\}, \{\rho_2, \dots, \rho_n\})$ . ■

## Proof of Proposition 2.

If monopolization is allowed in the game with a unique leader, then  $\rho_1$  has the option of proposing a merger with all the remaining firms, in which he gets  $\frac{1}{4n}$ . The grand coalition does not form if  $\mathbb{E}\pi_{\rho_1} = \frac{1}{2} \frac{1}{16} + \frac{1}{2} \frac{1}{4(n-1)^2} \geq \frac{1}{4n}$ , which holds if  $n \geq 7$ . ■

### Proof of Proposition 3.

For point *i*) see Proposition 1 in LW (2015).

For point *ii*) let:

$$\Delta(n, m) = \frac{\pi_I(n, m)}{m} - \pi_f(n, m) = \frac{n - 2m + 1}{4m(n - m + 1)^2} \quad (\text{A-1})$$

be the gain of each insider with respect to each outsider. Since the denominator of (A-1) is always positive,  $\text{sgn}(\Delta) = \text{sgn}(n - 2m + 1)$ , which is positive if  $m < \frac{n+1}{2}$ . ■

**Proof of Corollary 1** From point *ii*) in Proposition 3, the free riding issue is solved if  $m < \frac{n+1}{2}$ . In E1,  $n - 1$  firms merge, with  $n - 1 > \frac{n+1}{2}$ ,  $\forall n \geq 4$ . In E2, a bilateral coalition occurs, with  $2 < \frac{n+1}{2}$ ,  $\forall n \geq 4$ . ■

**Proof of Lemma 1** Let  $\bar{J}$  denote the set of leaders merging in a single new entity with cardinality  $J \leq L$  and label  $v$  the new entity generated by the merger of the  $J \leq L$  leaders. After this merger, the new number of leaders is  $L^{post} = L - J + 1$ . If all leaders prefer to merge in a single larger entity, it must be that the following superadditivity condition:

$$g_v(n, m = \sum_{j \in \bar{J}} m_j, L^{post}) > \sum_{j \in \bar{J}} g_j(n, m, L) \quad (\text{A-2})$$

holds. Condition (A-2) can be rewritten as:

$$\pi_v(n, m, L^{post}) - \sum_{j \in \bar{J}} m_j \tilde{\pi}(n) > \sum_{j \in \bar{J}} (\pi_j(n, m, L) - m_j \tilde{\pi}(n)), \quad (\text{A-3})$$

where the profit of leader  $v$  writes  $\pi_v(n, m, L^{post}) = \frac{1}{(L-J+2)^2(n-m+1)}$ . Since  $\pi_j(n, m, L)$  is equal for every  $j \in \bar{J}$ , the RHS of (A-3) becomes  $\frac{J}{(L+1)^2(n-m+1)} - \sum_{j \in \bar{J}} m_j \tilde{\pi}(n)$ . After some algebra (A-3) becomes:

$$\frac{(L+1)^2 - J(L-J+2)^2}{(L+1)^2(L-J+2)^2(n-m+1)} > 0. \quad (\text{A-4})$$

Solving (A-4) with respect to  $J$  yields the solution  $J \in \left( \frac{2L+3-\sqrt{4L+5}}{2}, L \right]$ , the result in SSR. Moreover, the solution is not a function of  $m$ . If the leading entities are homogeneous, then the additional gain can symmetrically be split among the insiders, who therefore obtain a strictly larger profit in a single larger coalition. ■

## Proof of Lemma 2

**Point i)** If  $\bar{m}_l$  firms already merged into  $L$  leaders, then the profit of an insider  $i$  of an additional leader of size  $m_l$  is:

$$\frac{\pi_l(n, m = m_l + \bar{m}_l, L + 1)}{m_l} = \frac{1}{m_l(L + 2)^2(n - \bar{m}_l - m_l + 1)}. \quad (\text{A-5})$$

The first derivative of (A-5) w.r.to  $m_l$  writes:

$$\frac{\partial \left( \frac{\pi_l}{m_l} \right)}{\partial m_l} = \frac{-(L + 2)^2(n - \bar{m}_l - 2m_l + 1)}{(m_l(L + 2)^2(n - \bar{m}_l - m_l + 1))^2}, \quad (\text{A-6})$$

which is equal to zero if  $m_l = \frac{n - \bar{m}_l + 1}{2}$ , is negative if  $m_l < \frac{n - \bar{m}_l + 1}{2}$  and positive if  $m_l > \frac{n - \bar{m}_l + 1}{2}$ . The second derivative of (A-5) w.r.to  $m_l$  writes:

$$\frac{\partial^2 \left( \frac{\pi_l}{m_l} \right)}{\partial m_l^2} = \frac{m_l^2(L + 2)}{\left( m_l(L + 2)^2(n - \bar{m}_l - m_l + 1) \right)^4} > 0. \quad (\text{A-7})$$

Thus (A-5) is convex and  $m_l = \frac{n - \bar{m}_l + 1}{2}$  is a minimum.

We are left to show that (A-5) has a symmetry line in  $m_l = \frac{n - \bar{m}_l + 1}{2}$ . In this case, it must be that:

$$\frac{\pi_l(n, m + s, L)}{m_l + s} = \frac{\pi_l(n, m - s, L)}{m_l - s}, \forall s \in \mathbb{R}. \quad (\text{A-8})$$

After some algebra (A-8) reduces to:

$$2s(-n + \bar{m}_l + 2m_l - 1) = 0, \quad (\text{A-9})$$

which holds if  $m_l = \frac{n - \bar{m}_l + 1}{2}$ .

**Point ii)** Pick any firm  $\rho_z$ ,  $z = 1, \dots, n - 1$  which can at most be a member of an  $n - z + 1$  firms coalition. Since (A-5) has a symmetry line in  $m_l = \frac{n - \bar{m}_l + 1}{2}$ , computing the profit maximizing size reduces to checking whether  $n - z + 1 - \frac{n - \bar{m}_l + 1}{2} > (\leq) \frac{n - \bar{m}_l + 1}{2} - 2$ . Previous inequality is positive for  $\bar{m}_l > z - 2$ , which implies that if all firms before  $\rho_z$  merged, then  $\rho_z$  maximizes his payoff as insider of a coalition involving all the remaining firms. If  $\bar{m}_l = z - 2$ , a bilateral merger and an  $n - z + 1$  firms merger provide the same payoff, if  $\bar{m}_l < z - 2$ , a bilateral merger is the best option.

**Point iii)** The profit of an insider of an additional bilateral merger when  $\bar{m}_l$  firms already merged in  $L$  leaders writes:

$$\frac{\pi_l(n, m_l = \bar{m}_l + 2, L + 1)}{2} = \frac{1}{2(L + 2)^2(n - \bar{m}_l - 1)}, \quad (\text{A-10})$$

while the profit if the two insiders turn into followers writes:

$$\pi_f(n, m = \bar{m}_l, L) = \frac{1}{(L+1)^2(n - \bar{m}_l + 1)^2}. \quad (\text{A-11})$$

The difference between (A-10) and (A-11) is positive if:

$$(L+1)^2(n - \bar{m}_l + 1)^2 > 2(L+2)^2(n - \bar{m}_l - 1), \quad (\text{A-12})$$

which is always satisfied. ■

## Proof of Proposition 4

### (four firms)

$\rho_3$  : According to point *iii*) of Lemma 2,  $\rho_3$  proposes the bilateral coalition  $\{\rho_3, \rho_4\}$  and  $\rho_4$  accepts.

$\rho_2$  : This firm is indifferent between the coalitions  $\{\rho_2, \rho_3\}$  and  $\{\rho_2, \rho_3, \rho_4\}$  and in both cases he gets  $\frac{1}{24}$ . If a receiver rejects, he gets  $\pi^{CN}(4) = \frac{1}{25}$ . If  $\rho_2$  passes, the coalition  $\{\rho_3, \rho_4\}$  forms and  $\rho_2$  gets  $\frac{1}{36} < \frac{1}{24}$ . Hence  $\rho_2$  proposes with equal probability  $\{\rho_2, \rho_3\}$  and  $\{\rho_2, \rho_3, \rho_4\}$  and these coalitions form.

$\rho_1$  : If  $\rho_1$  proposes, his best option is the bilateral coalition  $\{\rho_1, \rho_2\}$ . In this case, the additional bilateral coalition  $\{\rho_3, \rho_4\}$  forms and  $\rho_1$  gets  $\frac{1}{18}$ . If  $\rho_2$  rejects, he gets  $\pi^{CN}(4) = \frac{1}{25} < \frac{1}{18}$ . If  $\rho_1$  passes, with equal probability, he becomes the unique follower or one of two followers, and he gets the expected profit  $\mathbb{E}\pi_{\rho_1} = \frac{1}{2} \frac{1}{36} + \frac{1}{2} \frac{1}{16} < \frac{1}{18}$ . Hence the coalition  $\{\rho_1, \rho_2\}$  forms and the equilibrium coalition structure is  $\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3, \rho_4\})$ .

### (five firms)

$\rho_4$  : According to point *iii*) of Lemma 2,  $\rho_4$  proposes the bilateral coalition  $\{\rho_4, \rho_5\}$  and  $\rho_5$  accepts.

$\rho_3$  (**no merger**): If all firms before  $\rho_3$  passed, then  $\rho_3$  prefers the bilateral coalition  $\{\rho_3, \rho_4\}$ , where he gets  $\frac{1}{32}$  to the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$ . If  $\rho_4$  rejects he gets  $\pi^{CN}(5) = \frac{1}{36} < \frac{1}{18}$ . If  $\rho_3$  passes, he obtains  $\frac{1}{64}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \rho_4\}$  and this coalition forms.

$\rho_3$  (**all firms merged**): In this case  $\rho_3$  prefers the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$ , where he gets  $\frac{1}{27}$  to the bilateral coalition  $\{\rho_3, \rho_4\}$ . If a receiver rejects, he gets  $\frac{1}{64}$ . If  $\rho_3$  passes, he gets  $\frac{1}{36}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \rho_4, \rho_5\}$  and this coalition forms.

$\rho_2$  : This firm prefers the four firms coalition  $\{\rho_2, \dots, \rho_5\}$ , where he gets  $\frac{1}{32}$ , to the trilateral coalition  $\{\rho_2, \rho_3, \rho_4\}$ . However,  $\{\rho_2, \dots, \rho_5\}$  can be viewed as the merger of the two bilateral coalitions  $\{\rho_2, \rho_3\}$  and  $\{\rho_4, \rho_5\}$ . These leading entities account for more than the 80 percent of the market, so that Lemma 1 applies and  $\rho_2$  is better off in the four firms coalition. If a receiver rejects  $\{\rho_2, \dots, \rho_5\}$ , he gets  $\pi^{CN}(5) = \frac{1}{36} < \frac{1}{32}$ . If  $\rho_2$  passes,  $\{\rho_3, \rho_4\}$  forms and  $\rho_2$  gets  $\frac{1}{64}$ . Hence  $\rho_2$  proposes  $\{\rho_2, \dots, \rho_5\}$  and this coalition forms.

$\rho_1$  : This firm prefers the four firms coalition  $\{\rho_1, \dots, \rho_4\}$ , where he gets  $\frac{1}{32}$ , to the trilateral coalition  $\{\rho_1, \rho_2, \rho_3\}$ . However, if  $\{\rho_1, \rho_2, \rho_3\}$  forms, then  $\{\rho_4, \rho_5\}$  forms as well and  $\rho_1$  gets  $\frac{1}{27}$ . If  $\{\rho_1, \rho_2\}$  forms, then  $\{\rho_3, \rho_4, \rho_5\}$  forms as well and  $\rho_1$  gets  $\frac{1}{18}$ . If  $\rho_2$  rejects, he gets  $\pi^{CN}(5) = \frac{1}{36} < \frac{1}{18}$ . If  $\rho_1$  passes, he becomes the unique follower and gets  $\frac{1}{16}$ , which is his best option. Hence the equilibrium coalition structure is  $\kappa^* = (\{\rho_1\}, \{\rho_2, \dots, \rho_5\})$ .

**(six firms)**

$\rho_5$  : According to point *iii*) of Lemma 2,  $\rho_5$  proposes the bilateral coalition  $\{\rho_5, \rho_6\}$  and  $\rho_6$  accepts.

$\rho_4$  (**no merger**): If all firms before  $\rho_4$  passed, then  $\rho_4$  prefers the bilateral coalition  $\{\rho_4, \rho_5\}$ , where he gets  $\frac{1}{40}$  to the trilateral coalition  $\{\rho_4, \rho_5, \rho_6\}$ . If a receiver rejects, he gets  $\pi^{CN}(6) = \frac{1}{49} < \frac{1}{40}$ . If  $\rho_4$  passes, he obtains  $\frac{1}{100}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5\}$  and this coalition forms.

$\rho_4$  (**all firms merged**): In this case  $\rho_4$  prefers the trilateral coalition  $\{\rho_4, \rho_5, \rho_6\}$ , where he gets  $\frac{1}{27}$  to the bilateral coalition  $\{\rho_4, \rho_5\}$ . If a receiver rejects, he gets  $\frac{1}{64}$ . If  $\rho_4$  passes, he gets  $\frac{1}{36}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5, \rho_6\}$  and this coalition forms.

$\rho_4$  (**two firms merged**): In this case  $\rho_4$  is indifferent between the trilateral coalition  $\{\rho_4, \rho_5, \rho_6\}$  and the bilateral coalition  $\{\rho_4, \rho_5\}$ ; in both cases he gets  $\frac{1}{54}$ . If a receiver rejects, he gets  $\frac{1}{144}$ . If  $\rho_4$  passes, he gets  $\frac{1}{81}$ . Hence  $\rho_4$  proposes with equal probability  $\{\rho_4, \rho_5\}$  and  $\{\rho_4, \rho_5, \rho_6\}$  and these coalitions form.

$\rho_3$  (**no merger**): In this case  $\rho_3$  is indifferent between the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$  and the four firms coalition  $\{\rho_3, \dots, \rho_6\}$ ; in both cases he gets  $\frac{1}{48}$ . According to Lemma 1, this option is better than  $\{\rho_3, \rho_4\}$ , where  $\{\rho_5, \rho_6\}$  forms as well. If a receiver rejects he gets  $\pi^{CN}(6) = \frac{1}{49} < \frac{1}{48}$ . If  $\rho_3$  passes, he gets  $\frac{1}{100}$ . Hence  $\rho_3$  proposes with equal probability  $\{\rho_3, \rho_4, \rho_5\}$  and  $\{\rho_3, \dots, \rho_6\}$  and these coalitions form.

$\rho_3$  (**all firms merged**): In this case  $\rho_3$  prefers the four firms coalition  $\{\rho_3, \dots, \rho_6\}$ , where he gets  $\frac{1}{36}$  to the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$ . However, we also need to consider that  $\{\rho_3, \dots, \rho_6\}$  can be viewed as the merger of the two bilateral coalitions  $\{\rho_3, \rho_4\}$  and  $\{\rho_5, \rho_6\}$ . In this case, these two coalitions account for less than the 80 percent of the market, and according to Lemma 1,  $\rho_3$  is better off as a member of the bilateral coalition  $\{\rho_3, \rho_4\}$ , where he gets  $\frac{1}{32}$ . If  $\rho_4$  rejects, he gets  $\frac{1}{144}$ . If  $\rho_3$  passes, with equal probability he becomes the unique follower or one of two followers and obtains the expected profit  $\mathbb{E}\pi_{\rho_3} = \frac{1}{2} \frac{1}{36} + \frac{1}{2} \frac{1}{81} < \frac{1}{32}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \rho_4\}$  and this coalition forms.

$\rho_2$  : In this case  $\rho_2$  prefers the five firms coalition  $\{\rho_2, \dots, \rho_6\}$ , where he gets  $\frac{1}{40}$  to the four firms coalition  $\{\rho_2, \dots, \rho_5\}$ . However, we also need to consider that  $\{\rho_2, \dots, \rho_6\}$  can be viewed as the merger of a bilateral coalition and a trilateral coalition. Due to the heterogeneity in the number of insiders, point *iii*) of Lemma 2 does not apply and we explicitly check the various possibilities. If the coalition  $\{\rho_2, \rho_3\}$  forms, then with equal probability  $\{\rho_4, \rho_5\}$  and  $\{\rho_4, \rho_5, \rho_6\}$  form as well and  $\rho_2$  has the expected profit  $\mathbb{E}\pi_{\rho_2} = \frac{1}{2} \frac{1}{54} + \frac{1}{2} \frac{1}{36} < \frac{1}{40}$ . If the coalition  $\{\rho_2, \rho_3, \rho_4\}$  forms, then  $\{\rho_5, \rho_6\}$  forms as well and  $\rho_2$  gets  $\frac{1}{54}$ . If he passes, with equal probability  $\{\rho_3, \dots, \rho_6\}$  and  $\{\rho_3, \rho_4, \rho_5\}$  form and  $\rho_2$  has an expected profit  $\mathbb{E}\pi_{\rho_2} = \frac{1}{2} \frac{1}{36} + \frac{1}{2} \frac{1}{64} < \frac{1}{40}$ . If a receiver rejects  $\{\rho_2, \dots, \rho_6\}$ , he gets  $\pi^{CN}(6) = \frac{1}{49} < \frac{1}{40}$ . Hence  $\rho_2$  proposes  $\{\rho_2, \dots, \rho_6\}$  and this coalition forms.

$\rho_1$  : In this case if  $\rho_1$  proposes  $\{\rho_1, \dots, \rho_5\}$ , he gets  $\frac{1}{40}$ . If the coalition  $\{\rho_1, \dots, \rho_4\}$  forms, the coalition  $\{\rho_5, \rho_6\}$  forms as well and  $\rho_1$  gets  $\frac{1}{36}$ . If the coalition  $\{\rho_1, \rho_2, \rho_3\}$  forms, the coalition  $\{\rho_4, \rho_5, \rho_6\}$  forms as well and  $\rho_1$  gets  $\frac{1}{27}$ . If the coalition  $\{\rho_1, \rho_2\}$  forms, the coalitions  $\{\rho_3, \rho_4\}$  and  $\{\rho_5, \rho_6\}$  form as well and  $\rho_1$  gets  $\frac{1}{32}$ . If  $\rho_1$  passes,  $\{\rho_2, \dots, \rho_6\}$  forms and  $\rho_1$  gets  $\frac{1}{16}$ , which is his best option. Hence, the equilibrium coalition structure is  $\kappa^* = (\{\rho_1\}, \{\rho_2, \dots, \rho_6\})$ .

(**seven firms**)

$\rho_6$  : According to point *iii*) of Lemma 2,  $\rho_5$  proposes the bilateral coalition  $\{\rho_6, \rho_7\}$  and  $\rho_7$  accepts.

$\rho_5$  (**no merger**): If all firms before  $\rho_5$  passed, then  $\rho_5$  prefers the bilateral coalition  $\{\rho_5, \rho_6\}$ , where he gets  $\frac{1}{54}$  to the trilateral coalition  $\{\rho_5, \rho_6, \rho_7\}$ . If  $\rho_6$  rejects, he gets  $\pi^{CN}(7) = \frac{1}{64} < \frac{1}{54}$ . If  $\rho_5$  passes, he obtains  $\frac{1}{144}$ . Hence  $\rho_5$  proposes  $\{\rho_5, \rho_6\}$  and this coalition forms.

$\rho_5$  (**all firms merged**): In this case, the four firms before  $\rho_5$  could have merged in a four firms coalition or in two bilateral coalitions. However, from point *i*) of Lemma 2, the optimal choice for  $\rho_5$  is independent from the number of already formed leading



entities. Thus, without loss of generality, we focus on the case in which a four firms coalition formed. In this case  $\rho_5$  prefers the trilateral coalition  $\{\rho_5, \rho_6, \rho_7\}$ , where he gets  $\frac{1}{27}$  to the bilateral coalition  $\{\rho_5, \rho_6\}$ . If a receiver rejects, he gets  $\frac{1}{64}$ . If  $\rho_5$  passes, he gets  $\frac{1}{36}$ . Hence  $\rho_5$  proposes  $\{\rho_5, \rho_6, \rho_7\}$  and this coalition forms.

$\rho_5$  (**three firms merged**): In this case  $\rho_5$  is indifferent between the bilateral coalition  $\{\rho_5, \rho_6\}$  and the trilateral coalition  $\{\rho_5, \rho_6, \rho_7\}$ ; in both cases he gets  $\frac{1}{54}$ . If a receiver rejects, he gets  $\frac{1}{100}$ . If he passes, he gets  $\frac{1}{81}$ . Hence  $\rho_5$  proposes with equal probability  $\{\rho_5, \rho_6\}$  and  $\{\rho_5, \rho_6, \rho_7\}$  and these coalitions form.

$\rho_5$  (**two firms merged**): In this case  $\rho_5$  prefers the bilateral coalition  $\{\rho_5, \rho_6\}$ , where he gets  $\frac{1}{54}$ , to the trilateral coalition  $\{\rho_5, \rho_6, \rho_7\}$ . If a receiver rejects, he gets  $\frac{1}{144}$ . If  $\rho_5$  passes, he gets  $\frac{1}{144}$ . Hence  $\rho_5$  proposes  $\{\rho_5, \rho_6\}$  and this coalition forms.

$\rho_4$  (**no merger**): If all firms before  $\rho_4$  passed, then  $\rho_4$  prefers the trilateral coalition  $\{\rho_4, \rho_5, \rho_6\}$ , where he gets  $\frac{1}{60}$  to the four firms coalition  $\{\rho_4, \dots, \rho_7\}$ . According Lemma 1,  $\{\rho_4, \rho_5, \rho_6\}$  is better than  $\{\rho_4, \rho_5\}$ , where  $\{\rho_5, \rho_6\}$  forms as well. If a receiver rejects  $\{\rho_4, \rho_5, \rho_6\}$  he gets  $\pi^{CN}(7) = \frac{1}{64} < \frac{1}{60}$ . If  $\rho_4$  passes, he obtains  $\frac{1}{144}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5, \rho_6\}$  and this coalition forms.

$\rho_4$  (**all firms merged**): In this case  $\rho_4$  prefers the four firms coalition  $\{\rho_4, \dots, \rho_7\}$ , where he gets  $\frac{1}{36}$  to the trilateral coalition  $\{\rho_4, \rho_5, \rho_6, \}$ . However,  $\{\rho_4, \dots, \rho_7\}$  can be viewed as the merger of the two bilateral coalitions  $\{\rho_4, \rho_5\}$  and  $\{\rho_6, \rho_7\}$ , which account for less than the 80 percent of the market. Thus, according to Lemma 1, in case of  $\{\rho_4, \rho_5\}$ ,  $\rho_4$  gets  $\frac{1}{32}$ . If  $\rho_4$  rejects, he gets  $\frac{1}{100}$ . If  $\rho_4$  passes, with equal probability, the coalitions  $\{\rho_5, \rho_6\}$  and  $\{\rho_5, \rho_6, \rho_7\}$  form and  $\rho_4$  gets the expected profit  $\mathbb{E}\pi_{\rho_4} = \frac{1}{2} \frac{1}{36} + \frac{1}{2} \frac{1}{81} < \frac{1}{32}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5\}$  and this coalition forms.

$\rho_4$  (**two firms merged**): In this case  $\rho_4$  prefers the four firms coalition  $\{\rho_4, \dots, \rho_7\}$ , where he gets  $\frac{1}{72}$  to the trilateral coalition  $\{\rho_4, \rho_5, \rho_6, \}$ . At this point we apply the same reasoning of previous point and  $\{\rho_4, \rho_5\}$  is an even better option for  $\rho_4$ , where he gets  $\frac{1}{64}$ . If  $\rho_5$  rejects, he gets  $\frac{1}{144}$ . If  $\rho_4$  passes, he gets  $\frac{1}{144}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5\}$  and this coalition forms.

$\rho_3$  (**no merger**): In this case  $\rho_3$  prefers the five firms coalition  $\{\rho_3, \dots, \rho_7\}$ , where he gets  $\frac{1}{60}$ , to the four firms coalition  $\{\rho_3, \dots, \rho_6\}$ . If  $\rho_3$  proposes  $\{\rho_3, \rho_4\}$ , the coalition  $\{\rho_5, \rho_6\}$  forms as well and  $\rho_3$  gets  $\frac{1}{72}$ . If  $\rho_3$  proposes  $\{\rho_3, \rho_4, \rho_5\}$ , the coalition  $\{\rho_6, \rho_7\}$  forms as well and  $\rho_3$  gets  $\frac{1}{81}$ . If a receiver rejects  $\{\rho_3, \dots, \rho_7\}$ , he gets  $\pi^{CN}(7) = \frac{1}{64} < \frac{1}{60}$ . If  $\rho_3$  passes, he gets  $\frac{1}{100}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \dots, \rho_7\}$  and this coalition forms.

$\rho_3$  (**all firms merged**): In this case  $\rho_3$  prefers the five firms coalition  $\{\rho_3, \dots, \rho_7\}$ , where he gets  $\frac{1}{45}$ , to the four firms coalition  $\{\rho_3, \dots, \rho_6\}$ . If  $\rho_3$  proposes  $\{\rho_3, \rho_4\}$ , the coalition  $\{\rho_5, \rho_6, \rho_7\}$  forms as well and  $\rho_3$  gets  $\frac{1}{32}$ . This option is better than  $\{\rho_3, \rho_4, \rho_5\}$ , where in the end all firms merge again, but  $\rho_3$  is part of a larger coalition. If  $\rho_4$  rejects, he gets  $\frac{1}{144}$ . If  $\rho_3$  passes, he gets  $\frac{1}{64}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \rho_4\}$  and this coalition forms.

$\rho_2$  : This firm prefers the six firms coalition  $\{\rho_2, \dots, \rho_7\}$ , where he gets  $\frac{1}{48}$ , to the five firms coalition  $\{\rho_2, \dots, \rho_6\}$ . If  $\{\rho_2, \dots, \rho_5\}$  forms, then  $\{\rho_6, \rho_7\}$  forms as well and  $\rho_2$  gets  $\frac{1}{72}$ . If  $\{\rho_2, \rho_3, \rho_4\}$  forms, then with equal probability, the coalitions  $\{\rho_5, \rho_6\}$  and  $\{\rho_5, \rho_6, \rho_7\}$  form and  $\rho_2$  has the expected profit  $\mathbb{E}\pi_{\rho_2} = \frac{1}{2} \frac{1}{54} + \frac{1}{2} \frac{1}{81} < \frac{1}{48}$ . If  $\{\rho_2, \rho_3\}$  forms, the coalitions  $\{\rho_4, \rho_5\}$  and  $\{\rho_6, \rho_7\}$  form as well and  $\rho_2$  gets  $\frac{1}{64}$ . If he passes, he gets  $\frac{1}{36}$ , which is his best option.

$\rho_1$  : If  $\{\rho_1, \dots, \rho_6\}$  forms,  $\rho_1$  gets  $\frac{1}{48}$ . If  $\{\rho_1, \dots, \rho_5\}$  forms, the coalition  $\{\rho_6, \rho_7\}$  forms as well and  $\rho_1$  gets  $\frac{1}{45}$ . If  $\{\rho_1, \dots, \rho_4\}$  forms, the coalition  $\{\rho_5, \rho_6, \rho_7\}$  forms as well and  $\rho_1$  gets  $\frac{1}{36}$ . If  $\{\rho_1, \rho_2, \rho_3\}$  forms, then the coalitions  $\{\rho_4, \rho_5\}$  and  $\{\rho_6, \rho_7\}$  form as well and  $\rho_1$  gets  $\frac{1}{48}$ . If  $\{\rho_1, \rho_2\}$  forms, the coalitions  $\{\rho_3, \rho_4\}$  and  $\{\rho_5, \rho_6, \rho_7\}$  form as well and  $\rho_1$  gets  $\frac{1}{32}$ . If he passes, he gets  $\frac{1}{36}$ . If  $\rho_2$  rejects, he gets  $\pi^{CN}(7) = \frac{1}{64} < \frac{1}{32}$ . Hence,  $\rho_1$  proposes  $\{\rho_1, \rho_2\}$  and this coalition forms. Thus, the equilibrium structure is  $\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3, \rho_4\}, \{\rho_5, \rho_6, \rho_7\})$ .

### (eight firms)

$\rho_7$  : According to point *iii*) of lemma 2,  $\rho_7$  proposes the bilateral coalition  $\{\rho_7, \rho_8\}$  and  $\rho_8$  accepts.

$\rho_6$  (**no merger**): In this case  $\rho_6$  prefers the bilateral coalition  $\{\rho_6, \rho_7\}$ , where he gets  $\frac{1}{56}$ , to the trilateral coalition  $\{\rho_6, \rho_7, \rho_8\}$ . If  $\rho_7$  rejects, he gets  $\pi^{CN}(8) = \frac{1}{81} < \frac{1}{56}$ . If  $\rho_6$  passes, he gets  $\frac{1}{196}$ . Hence,  $\rho_6$  proposes  $\{\rho_6, \rho_7\}$  and this coalition forms.

$\rho_6$  (**all firms merged**): Without loss of generality, I focus on the case in which a single five firms coalition formed. In this case  $\rho_6$  prefers the trilateral coalition  $\{\rho_6, \rho_7, \rho_8\}$ , where he gets  $\frac{1}{27}$ , to the bilateral coalition  $\{\rho_6, \rho_7\}$ . If a receiver rejects, he gets  $\frac{1}{64}$ . If  $\rho_6$  passes, he gets  $\frac{1}{36}$ . Hence  $\rho_6$  proposes  $\{\rho_6, \rho_7, \rho_8\}$  and this coalition forms.

$\rho_6$  (**four firms merged**): Without loss of generality, I focus on the case in which a single four firms coalition formed. In this case  $\rho_6$  is indifferent between the bilateral coalition  $\{\rho_6, \rho_7\}$  and the trilateral coalition  $\{\rho_6, \rho_7, \rho_8\}$ ; in both cases he gets  $\frac{1}{54}$ . If a receiver rejects, he gets  $\frac{1}{100}$ . If  $\rho_6$  passes, he gets  $\frac{1}{81}$ . Hence  $\rho_6$  proposes with equal probability  $\{\rho_6, \rho_7\}$  and  $\{\rho_6, \rho_7, \rho_8\}$  and these coalitions form.

$\rho_6$  (**three firms merged**): In this case  $\rho_6$  prefers the bilateral coalition  $\{\rho_6, \rho_7\}$ , where he gets  $\frac{1}{72}$ , to the trilateral coalition  $\{\rho_6, \rho_7, \rho_8\}$ . If  $\rho_7$  rejects, he gets  $\frac{1}{144}$ . If  $\rho_6$  passes, he gets  $\frac{1}{144}$ . Hence  $\rho_6$  proposes  $\{\rho_6, \rho_7\}$  and this coalition forms.

$\rho_6$  (**two firms merged**): In this case  $\rho_6$  prefers the bilateral coalition  $\{\rho_6, \rho_7\}$ , where he gets  $\frac{1}{90}$ , to the trilateral coalition  $\{\rho_6, \rho_7, \rho_8\}$ . If  $\rho_7$  rejects, he gets  $\frac{1}{196}$ . If  $\rho_6$  passes, he gets  $\frac{1}{225}$ . Hence  $\rho_6$  proposes  $\{\rho_6, \rho_7\}$  and this coalition forms.

$\rho_5$  (**no merger**): In this case  $\rho_5$  prefers the trilateral coalition  $\{\rho_5, \rho_6, \rho_7\}$ , where he gets  $\frac{1}{72}$ , to the four firms coalition  $\{\rho_5, \dots, \rho_8\}$ , which, according to Lemma 2, is, in turn, better than  $\{\rho_5, \rho_6\}$ , where the coalition  $\{\rho_7, \rho_8\}$  forms as well. If a receiver rejects, he gets  $\pi^{CN}(8) = \frac{1}{81} < \frac{1}{72}$ . If  $\rho_5$  passes, he gets  $\frac{1}{196}$ . Hence  $\rho_5$  proposes  $\{\rho_5, \rho_6, \rho_7\}$  and this coalition forms.

$\rho_5$  (**all firms merged**): Without loss of generality, I focus on the case in which a single five firms coalition formed. In this case  $\rho_5$  prefers the four firms coalition  $\{\rho_5, \dots, \rho_8\}$ , where he gets  $\frac{1}{36}$ , to the trilateral coalition  $\{\rho_5, \rho_6, \rho_7\}$ . If  $\{\rho_5, \rho_6\}$  forms, the coalition  $\{\rho_7, \rho_8\}$  forms as well and, according to Lemma 1,  $\rho_5$  prefers this option to  $\{\rho_5, \dots, \rho_8\}$ , since he gets  $\frac{1}{32}$ . If  $\rho_6$  rejects, he gets  $\frac{1}{100}$ . If  $\rho_5$  passes, he gets  $\frac{1}{36}$ . Hence  $\rho_5$  proposes  $\{\rho_5, \rho_6\}$  and this coalition forms.

$\rho_5$  (**three firms merged**): In this case  $\rho_5$  prefers the four firms coalition  $\{\rho_5, \dots, \rho_8\}$ , where he gets  $\frac{1}{72}$ , to the trilateral coalition  $\{\rho_5, \rho_6, \rho_7\}$ . If  $\{\rho_5, \rho_6\}$  forms, the coalition  $\{\rho_7, \rho_8\}$  forms as well and  $\rho_5$  gets  $\frac{1}{64}$ . If  $\rho_6$  rejects, he gets  $\frac{1}{144}$ . If  $\rho_5$  passes, he gets  $\frac{1}{144}$ . Hence  $\rho_5$  proposes  $\{\rho_5, \rho_6\}$  and this coalition forms.

$\rho_5$  (**two firms merged**): In this case  $\rho_5$  is indifferent between the four firms coalition  $\{\rho_5, \dots, \rho_8\}$  and the trilateral coalition  $\{\rho_5, \rho_6, \rho_7\}$ ; in both cases he gets  $\frac{1}{108}$ . If  $\{\rho_5, \rho_6\}$  forms, then  $\{\rho_7, \rho_8\}$  forms as well and  $\rho_5$  gets  $\frac{1}{96}$ . If  $\rho_6$  rejects, he gets  $\frac{1}{196}$ . If  $\rho_5$  passes, he gets  $\frac{1}{225}$ . Hence  $\rho_5$  proposes  $\{\rho_5, \rho_6\}$  and this coalition forms.

$\rho_4$  (**no merger**): In this case  $\rho_4$  is indifferent between the five firms coalition  $\{\rho_4, \dots, \rho_8\}$  and the four firms coalition  $\{\rho_4, \dots, \rho_7\}$ ; in both cases he gets  $\frac{1}{80}$ . If  $\{\rho_4, \rho_5, \rho_6\}$  forms, then  $\{\rho_7, \rho_8\}$  forms as well and  $\rho_4$  gets  $\frac{1}{108}$ . If  $\{\rho_4, \rho_5\}$  forms, then the coalition  $\{\rho_6, \rho_7\}$  forms as well and  $\rho_4$  gets  $\frac{1}{90}$ . If a receiver rejects, he gets  $\pi^{CN}(8) = \frac{1}{81} < \frac{1}{80}$ . If he passes, he gets  $\frac{1}{144}$ . Hence  $\rho_4$  proposes with equal probability  $\{\rho_4, \dots, \rho_8\}$  and  $\{\rho_4, \dots, \rho_7\}$  and these coalitions form.

$\rho_4$  (**all firms merged**): In this case  $\rho_4$  prefers the five firms coalition  $\{\rho_4, \dots, \rho_8\}$ , where he gets  $\frac{1}{45}$ , to the four firms coalition  $\{\rho_4, \dots, \rho_7\}$ . If  $\{\rho_4, \rho_5\}$  forms, the coalition  $\{\rho_6, \rho_7, \rho_8\}$  forms as well and  $\rho_4$  gets  $\frac{1}{32}$ . This option is better than  $\{\rho_4, \rho_5, \rho_6\}$ , where

all firms merge as well, but  $\rho_4$  is part of a larger coalition. If  $\rho_5$  rejects, he gets  $\frac{1}{144}$ . If he passes, he gets  $\frac{1}{64}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5\}$  and this coalition forms.

$\rho_4$  (**two firms merged**): In this case  $\rho_4$  prefers the five firms coalition  $\{\rho_4, \dots, \rho_8\}$ , where he gets  $\frac{1}{90}$ , to the four firms coalition  $\{\rho_4, \dots, \rho_7\}$ . If  $\{\rho_4, \rho_5, \rho_6\}$  forms, the coalition  $\{\rho_7, \rho_8\}$  forms as well and  $\rho_4$  gets  $\frac{1}{96}$ . If  $\{\rho_4, \rho_5\}$  forms, then with equal probability the coalitions  $\{\rho_6, \rho_7\}$  and  $\{\rho_6, \rho_7, \rho_8\}$  form and  $\rho_4$  has the expected profit  $\mathbb{E}\pi_{\rho_4} = \frac{1}{2} \frac{1}{64} + \frac{1}{2} \frac{1}{96} > \frac{1}{90}$ . If  $\rho_5$  rejects, he gets  $\frac{1}{196}$ . If  $\rho_4$  passes, he gets  $\frac{1}{144}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5\}$  and this coalition forms.

$\rho_3$  (**no merger**): In this case  $\rho_3$  prefers the six firms coalition  $\{\rho_3, \dots, \rho_8\}$ , where he gets  $\frac{1}{72}$ , to the five firms coalition  $\{\rho_3, \dots, \rho_7\}$ . If  $\{\rho_3, \dots, \rho_6\}$  forms, the coalition  $\{\rho_7, \rho_8\}$  forms as well and  $\rho_3$  gets  $\frac{1}{108}$ . If  $\{\rho_3, \rho_4, \rho_5\}$  forms, the coalition  $\{\rho_6, \rho_7\}$  forms as well and  $\rho_3$  gets  $\frac{1}{108}$ . If  $\{\rho_3, \rho_4\}$  forms, the coalitions  $\{\rho_5, \rho_6\}$  and  $\{\rho_7, \rho_8\}$  form as well and  $\rho_3$  gets  $\frac{1}{96}$ . If he passes, with equal probability, the coalitions  $\{\rho_4, \dots, \rho_7\}$  and  $\{\rho_4, \dots, \rho_8\}$  form and  $\rho_3$  has the expected profit  $\mathbb{E}\pi_{\rho_3} = \frac{1}{2} \frac{1}{64} + \frac{1}{2} \frac{1}{100} < \frac{1}{72}$ . If a receiver rejects  $\{\rho_3, \dots, \rho_8\}$ , he gets  $\pi^{CN}(8) = \frac{1}{81} < \frac{1}{72}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \dots, \rho_8\}$  and this coalition forms.

$\rho_3$  (**all firms merged**): In this case  $\rho_3$  prefers the six firms coalition  $\{\rho_3, \dots, \rho_8\}$ , where he gets  $\frac{1}{54}$ , to the five firms coalition  $\{\rho_3, \dots, \rho_7\}$ . If  $\{\rho_3, \dots, \rho_6\}$  forms, the coalition  $\{\rho_7, \rho_8\}$  forms as well and  $\rho_3$  gets  $\frac{1}{64}$ . If  $\{\rho_3, \rho_4, \rho_5\}$  forms, the coalition  $\{\rho_6, \rho_7, \rho_8\}$  forms as well and  $\rho_3$  gets  $\frac{1}{48}$ . If  $\{\rho_3, \rho_4\}$  forms, the coalitions  $\{\rho_5, \rho_6\}$  and  $\{\rho_7, \rho_8\}$  form as well and  $\rho_3$  gets  $\frac{1}{50}$ . If a receiver rejects  $\{\rho_3, \rho_4, \rho_5\}$ , he gets  $\frac{1}{196}$ . If  $\rho_3$  passes, then  $\{\rho_4, \rho_5\}$  forms and then, with equal probability, the coalitions  $\{\rho_6, \rho_7\}$  and  $\{\rho_6, \rho_7, \rho_8\}$  form as well and  $\rho_3$  has the expected profit  $\mathbb{E}\pi_{\rho_3} = \frac{1}{2} \frac{1}{64} + \frac{1}{2} \frac{1}{144} < \frac{1}{48}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \rho_4, \rho_5\}$  and this coalition forms.

$\rho_2$  : In this case  $\rho_2$  prefers the seven firms coalition  $\{\rho_2, \dots, \rho_8\}$ , where he gets  $\frac{1}{56}$ , to the six firms coalition  $\{\rho_2, \dots, \rho_7\}$ . If  $\{\rho_2, \dots, \rho_6\}$  forms, the coalition  $\{\rho_7, \rho_8\}$  forms as well and  $\rho_2$  gets  $\frac{1}{90}$ . If  $\{\rho_2, \dots, \rho_5\}$  forms, then with equal probability, the coalitions  $\{\rho_6, \rho_7\}$  and  $\{\rho_6, \rho_7, \rho_8\}$  form as well and  $\rho_2$  has the expected profit  $\mathbb{E}\pi_{\rho_2} = \frac{1}{2} \frac{1}{72} + \frac{1}{2} \frac{1}{108} < \frac{1}{48}$ . If  $\{\rho_2, \rho_3, \rho_4\}$  forms, the coalitions  $\{\rho_5, \rho_6\}$  and  $\{\rho_7, \rho_8\}$  form as well and  $\rho_2$  gets  $\frac{1}{96}$ . If  $\{\rho_2, \rho_3\}$  forms, the coalition  $\{\rho_4, \rho_5\}$  forms and, with equal probability, the coalitions  $\{\rho_6, \rho_7\}$  and  $\{\rho_6, \rho_7, \rho_8\}$  form as well and  $\rho_2$  has the expected profit  $\mathbb{E}\pi_{\rho_2} = \frac{1}{2} \frac{1}{72} + \frac{1}{2} \frac{1}{96} < \frac{1}{48}$ . If he passes, he gets  $\frac{1}{36}$ , which is his best option.

$\rho_1$  : If  $\{\rho_1, \dots, \rho_7\}$  forms,  $\rho_1$  gets  $\frac{1}{48}$ . If  $\{\rho_1, \dots, \rho_6\}$  forms, the coalition  $\{\rho_7, \rho_8\}$  forms as well and  $\rho_1$  gets  $\frac{1}{54}$ . If  $\{\rho_1, \dots, \rho_5\}$  forms, the coalition  $\{\rho_6, \rho_7, \rho_8\}$  forms as well and  $\rho_1$  gets  $\frac{1}{45}$ . If  $\{\rho_1, \dots, \rho_4\}$  forms, the coalitions  $\{\rho_5, \rho_6\}$  and  $\{\rho_7, \rho_8\}$  form as well

and  $\rho_1$  gets  $\frac{1}{64}$ . If  $\{\rho_1, \rho_2, \rho_3\}$  forms, the coalitions  $\{\rho_4, \rho_5\}$  and  $\{\rho_6, \rho_7, \rho_8\}$  form as well and  $\rho_1$  gets  $\frac{1}{48}$ . If  $\{\rho_1, \rho_2\}$  forms, the coalitions  $\{\rho_3, \rho_4, \rho_5\}$  and  $\{\rho_6, \rho_7, \rho_8\}$  form as well and  $\rho_1$  gets  $\frac{1}{32}$ . If  $\rho_2$  rejects, he gets  $\pi^{CN}(8) = \frac{1}{81} < \frac{1}{32}$ . If  $\rho_1$  passes, he gets  $\frac{1}{36}$ . Hence,  $\rho_1$  proposes  $\{\rho_1, \rho_2\}$  and the equilibrium coalition structure is  $\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3, \rho_4, \rho_5\}, \{\rho_6, \rho_7, \rho_8\})$ .

**Proof of Proposition 5** If  $n = \{5, 6\}$ ,  $\rho_1$  obtains the profit  $\frac{1}{16}$  as the unique follower. If the grand coalition forms  $\rho_1$  gets  $\frac{1}{20}$  and  $\frac{1}{24}$  respectively.

If  $n = 4$ ,  $\rho_1$  obtains the profit  $\frac{1}{18}$ , which is lower than the one as a member of the grand coalition  $\frac{1}{16}$ ; if  $n = \{7, 8\}$ ,  $\rho_1$  obtains the profit  $\frac{1}{32}$ . This profit is lower than the one as a member of the grand coalition  $\frac{1}{28}$  if  $n = 7$  and the two are equal if  $n = 8$ . ■

### Proof of Proposition 6

$\rho_4$  : According to point *iii*) of Lemma 2,  $\rho_4$  proposes the bilateral coalition  $\{\rho_4, \rho_5\}$  and  $\rho_5$  accepts.

$\rho_3$  (**no merger**): If all firms before  $\rho_3$  passed, then, in both cases where  $n-1$  and  $n-2$ -firm mergers are forbidden,  $\rho_3$  prefers the bilateral coalition  $\{\rho_3, \rho_4\}$ , where he gets  $\frac{1}{32}$  to the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$ . If  $\rho_4$  rejects he gets  $\pi^{CN}(5) = \frac{1}{36} < \frac{1}{18}$ . If  $\rho_3$  he passes, he obtains  $\frac{1}{64}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \rho_4\}$  and this coalition forms.

$\rho_3$  (**all firms merged**): In this case, if an  $n-2$ -firm merger is forbidden, then  $\rho_{z=\{3,4,5\}}$  act as followers and the equilibrium coalition structure is  $\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3\}, \{\rho_4\}, \{\rho_5\})$ .

If an  $n-1$ -firm merger is forbidden, then  $\rho_3$  is indifferent between passing and letting  $\{\rho_4, \rho_5\}$  form and forming the bilateral coalition  $\{\rho_3, \rho_4\}$ , since in both cases he gets  $\frac{1}{36}$ .

$\rho_2$  : As the coalition  $\{\rho_2, \dots, \rho_5\}$  is forbidden, then  $\rho_1$  is indifferent between being part of the bilateral coalition  $\{\rho_2, \rho_3\}$  and letting  $\{\rho_4, \rho_5\}$  form or being part of the trilateral coalition  $\{\rho_2, \rho_3, \rho_4\}$ . In both cases he gets  $\frac{1}{36}$ .

$\rho_1$  : As the coalition  $\{\rho_1, \dots, \rho_4\}$  is forbidden, then, if  $\{\rho_1, \rho_2, \rho_3\}$  forms, then  $\{\rho_4, \rho_5\}$  forms as well and  $\rho_1$  gets  $\frac{1}{27}$ . If  $\{\rho_1, \rho_2\}$  forms, then  $\{\rho_3, \rho_4\}$  forms as well and  $\rho_1$  gets  $\frac{1}{36}$ . If  $\rho_1$  passes, then, with equal probability, either two bilateral coalitions or a single trilateral coalition forms. In both cases,  $\rho_1$  gets  $\frac{1}{36}$ . Thus, the equilibrium coalition structure is  $\kappa^* = (\{\rho_1, \rho_2, \rho_3\}, \{\rho_4, \rho_5\})$ . ■

**Proof of Proposition 7** Setting (16) equal to zero, multiplying both sides by  $(L+1)^2(n+1)^2(n-m+1)$  and solving w.r.to  $k$  yields  $\bar{k}$ . Thus  $\bar{k}$  represents that value of  $k$  such that

(19) is exactly zero. Since  $\frac{\partial \tilde{g}_l}{\partial k} = \frac{(n-m+1)(L+1)^2}{(L+1)^2(n+1)^2(n-m+1)} > 0$ , then (16) is positive if  $k > \bar{k}$ . Thus, if  $\bar{k} < 0$ , it means that (16) is positive even if  $k = 0$ , namely when a leader is formed by the largest number of insiders  $m - 2(L - 1)$ . When  $\bar{k} > 0$ , it means that (16) is not positive for every value of  $k$ , but it is restricted to be larger than  $\bar{k}$ . Thus, when defining  $k^*$  as the smallest non negative value of  $k$  such that (16) is positive, it takes the functional form as in Proposition 6. At this point let:

$$\mathbb{M} \subset \mathbb{K} = \{\kappa \in \mathbb{K} \mid 4 \leq m < n, L \geq 2\}$$

be the set of coalition structures where at least two bilateral mergers occur and pick any element  $\kappa_m \in \mathbb{M}$ . All mergers are profitable only if:

$$g_l(n, m_l, m, L) = \pi_l(n, m, L) - m_l \tilde{\pi}(n) = \frac{(n+1)^2 - m_l(L+1)^2(n-m+1)}{(L+1)^2(n+1)^2(n-m+1)} > 0, \forall l = 1, \dots, L. \quad (\text{A-13})$$

Being  $\pi_l$  the same for all leaders  $l = 1, \dots, L$ ,  $\tilde{\pi}(n)$  fixed and  $g_l$  decreasing in  $m_l$ , then if  $\min_{l=1, \dots, L} \{g_l\} > 0$  for some  $\kappa_m \in \mathbb{M}$ , then  $(g_1, \dots, g_l, \dots, g_L) \gg 0$ . As already pointed out in the paper, apart from which of the  $n$  identical firms act as insiders, the coalition structure in which it is harder for all mergers to be profitable when  $m$  out of  $n$  firms merge into  $L \geq 2$  leaders is the one in which  $k^* = 0$ , that is a leader is formed by  $m - 2(L - 1)$  insiders and the other mergers involve two firms. Formally:

$$g_l(n, m, m_l = m - 2(L - 1), L) = \min_{\kappa_m \in \mathbb{M}} \left\{ \min_{l=1, \dots, L} \{g_l\} \right\}.$$

It follows that if  $g_l(n, m_l = m - 2(L - 1), m, L) > 0$ , then all mergers are profitable in the originally picked distribution and in all other distributions as well. Conversely, if  $k^* > 0$  a merger is profitable only if a leader is formed by at most  $\lfloor m - 2(L - 1) - k^* \rfloor$  insiders, namely if at least one unit is removed from the largest size  $m - 2(L - 1)$ . ■

**Proof of Proposition 8** This proof follows the same reasoning of Proposition 6 and it is therefore omitted. ■

**Proof of Proposition 9** In the original game,  $\rho_1$  decides to pass, anticipating that  $\rho_2$  either proposes the bilateral coalition  $\{\rho_2, \rho_3\}$  or the one involving the last  $n - 1$  firms  $\{\rho_2, \rho_3, \dots, \rho_n\}$ . These two equilibria are robust to the possibility that, whenever a merger proposal by  $\rho_z$  is rejected, then  $\rho_{z+1}$  can propose another merger. The main point to show this result is that, in both equilibria, none of the firms can profitably deviate by rejecting.

In case  $\rho_2$  proposes the bilateral coalition  $\{\rho_2, \rho_3\}$ , then  $\rho_3$  can at most obtain the same payoff by proposing the bilateral coalition  $\{\rho_3, \rho_4\}$ . In this case I assume that he accepts the original offer of  $\rho_2$ . The same is true in the other coalition, where each firm  $\rho_z$ ,  $z \geq 3$  can at most tie his payoff by proposing a bilateral coalition. ■

**Proof of Proposition 10**  
(four firms)

$\rho_3$  : This firm always proposes the coalition  $\{\rho_3, \rho_4\}$  and according to point *iii*) of Lemma 2,  $\rho_4$  accepts.

$\rho_2$  : This firm is indifferent between  $\{\rho_2, \rho_3\}$  and  $\{\rho_2, \rho_3, \rho_4\}$ ; in both cases he gets  $\frac{1}{24}$ . If he proposes  $\{\rho_2, \rho_3\}$  and  $\rho_3$  rejects, then  $\{\rho_3, \rho_4\}$  forms and  $\rho_3$  gets  $\frac{1}{24}$  again. In this case we assume that he accepts. The same reasoning is valid for both  $\rho_3$  and  $\rho_4$ , in case  $\rho_2$  proposes  $\{\rho_2, \rho_3, \rho_4\}$ . If he passes,  $\{\rho_3, \rho_4\}$  forms and  $\rho_2$  gets  $\frac{1}{36}$ . Hence  $\rho_2$  proposes with equal probability  $\{\rho_2, \rho_3\}$  and  $\{\rho_2, \rho_3, \rho_4\}$  and these coalitions form.

$\rho_1$  : If he proposes  $\{\rho_1, \rho_2\}$  and  $\rho_2$  rejects, he gets  $\pi^{CN}(4) = \frac{1}{25}$ ; if he accepts, the coalition  $\{\rho_3, \rho_4\}$  forms as well and  $\rho_2$  gets  $\frac{1}{18}$ . Hence  $\rho_2$  accepts. The same reasoning is valid for both  $\rho_2$  and  $\rho_3$ , in case  $\rho_1$  proposes  $\{\rho_1, \rho_2, \rho_3\}$ . In this case each insider gets  $\frac{1}{24}$ . If  $\rho_1$  passes, then with equal probability, the coalitions  $\{\rho_2, \rho_3\}$  and  $\{\rho_2, \rho_3, \rho_4\}$  form and  $\rho_1$  has the expected profit  $\mathbb{E}\pi_{\rho_1} = \frac{1}{2} \frac{1}{36} + \frac{1}{2} \frac{1}{16} < \frac{1}{18}$ . Hence  $\rho_1$  proposes  $\{\rho_1, \rho_2\}$  and the equilibrium coalition structure is  $\kappa^* = (\{\rho_1, \rho_2\}, \{\rho_3, \rho_4\})$ .

**(five firms)**

$\rho_4$  : This firm always proposes the coalition  $\{\rho_4, \rho_5\}$  and according to point *iii*) in Lemma 2,  $\rho_5$  accepts.

$\rho_3$  (**no merger**): In this case  $\rho_3$  prefers the bilateral coalition  $\{\rho_3, \rho_4\}$ , where he gets  $\frac{1}{32}$ , to the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$  and  $\rho_4$  accepts, since he can not improve his payoff by forming  $\{\rho_4, \rho_5\}$ . If  $\rho_3$  passes, he gets  $\frac{1}{36}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \rho_4\}$  and this coalition forms.

$\rho_3$  (**all firms merged**): In this case  $\rho_3$  prefers the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$ , where he gets  $\frac{1}{27}$ , to the bilateral coalition  $\{\rho_3, \rho_4\}$ . Both  $\rho_4$  and  $\rho_5$  accept, since they can not improve their payoff by forming  $\{\rho_4, \rho_5\}$ . If  $\rho_3$  passes, he gets  $\frac{1}{36}$ . Hence  $\rho_3$  proposes  $\{\rho_3, \rho_4, \rho_5\}$  and this coalition forms.

$\rho_2$  : If  $\rho_2$  proposes  $\{\rho_2, \dots, \rho_5\}$ , each insider gets  $\frac{1}{32}$ . If  $\rho_3$  rejects, the coalition  $\{\rho_3, \rho_4\}$  forms and  $\rho_3$  gets  $\frac{1}{32}$ . Thus he accepts. The same reasoning is valid for  $\rho_4$ . If  $\rho_5$  rejects,  $\{\rho_3, \rho_4\}$  forms and he gets  $\frac{1}{64}$ . Since  $\rho_2$  obtains a larger profit in this four firms coalition than in the trilateral coalition  $\{\rho_2, \rho_3, \rho_4\}$ , this proposal does not occur. If  $\rho_2$  proposes  $\{\rho_2, \rho_3\}$  and  $\rho_3$  rejects, then  $\{\rho_3, \rho_4\}$  forms and  $\rho_3$  gets  $\frac{1}{32}$ . Hence  $\rho_3$  accepts. If he accepts, the coalition  $\{\rho_4, \rho_5\}$  forms and  $\rho_3$  gets  $\frac{1}{36}$ . If  $\rho_2$  passes, the coalition  $\{\rho_3, \rho_4\}$  forms and  $\rho_2$  gets  $\frac{1}{64}$ . Hence  $\rho_2$  proposes  $\{\rho_2, \dots, \rho_5\}$  and this coalition forms.

$\rho_1$  : The largest payoff in case of proposal is obtained in  $\{\rho_1, \dots, \rho_4\}$  and equals  $\frac{1}{32}$ . If a receiver rejects, he ties his payoff as a member of  $\{\rho_2, \dots, \rho_5\}$ . Hence the receivers accept. If  $\rho_1$  passes, he gets  $\frac{1}{16}$ . Hence, the equilibrium coalition structure is  $\kappa^* = (\{\rho_1\}, \{\rho_2, \dots, \rho_5\})$ .

(six firms)

$\rho_5$  : This firm always proposes the coalition  $\{\rho_5, \rho_6\}$  and according to point *iii*) of Lemma 2,  $\rho_6$  accepts.

$\rho_4$  (**no merger**): In this case  $\rho_4$  prefers the bilateral coalition  $\{\rho_4, \rho_5\}$ , where he gets  $\frac{1}{40}$ , to the trilateral coalition  $\{\rho_4, \rho_5, \rho_6\}$ . If  $\rho_5$  rejects, then  $\{\rho_5, \rho_6\}$  forms and  $\rho_5$  gets  $\frac{1}{40}$ . Hence  $\rho_5$  accepts. If  $\rho_4$  passes, he gets  $\frac{1}{81}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5\}$  and this coalition forms.

$\rho_4$  (**all firms merged**): In this case  $\rho_4$  prefers the trilateral coalition  $\{\rho_4, \rho_5, \rho_6\}$ , where he gets  $\frac{1}{27}$ , to the bilateral coalition  $\{\rho_3, \rho_4\}$ . Both  $\rho_5$  and  $\rho_6$  accept, since they can not improve their payoff forming  $\{\rho_5, \rho_6\}$ . If  $\rho_4$  passes, he gets  $\frac{1}{36}$ . Hence  $\rho_4$  proposes  $\{\rho_4, \rho_5, \rho_6\}$  and this coalition forms.

$\rho_4$  (**two firms merged**): This firm is indifferent between  $\{\rho_4, \rho_5\}$  and  $\{\rho_4, \rho_5, \rho_6\}$ ; in both cases he gets  $\frac{1}{54}$ . Both  $\rho_5$  and  $\rho_6$  accept, since they can not improve their payoff forming  $\{\rho_5, \rho_6\}$ . If  $\rho_4$  passes, he gets  $\frac{1}{81}$ . Hence  $\rho_4$  proposes with equal probability  $\{\rho_4, \rho_5\}$  and  $\{\rho_4, \rho_5, \rho_6\}$  and these coalitions form.

$\rho_3$  (**no merger**): This firm is indifferent between the four firms coalition  $\{\rho_3, \dots, \rho_6\}$  and the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$ ; in both cases he gets  $\frac{1}{48}$ . These options are better than  $\{\rho_3, \rho_4\}$ , where  $\{\rho_5, \rho_6\}$  forms as well and  $\rho_3$  gets  $\frac{1}{54}$ . However, if  $\rho_4$  rejects, then  $\{\rho_4, \rho_5\}$  forms and  $\rho_4$  gets  $\frac{1}{40}$ . It follows that every proposal by  $\rho_3$  is rejected.

$\rho_3$  (**all firms merged**): In this case  $\rho_3$  prefers the four firms coalition  $\{\rho_3, \dots, \rho_6\}$ , where he gets  $\frac{1}{36}$ , to the trilateral coalition  $\{\rho_3, \rho_4, \rho_5\}$ . However, he is even better in  $\{\rho_3, \rho_4\}$ , predicting that  $\{\rho_5, \rho_6\}$  will form as well. In this case he gets  $\frac{1}{32}$ . If  $\rho_4$  rejects, then, with equal probability, the coalitions  $\{\rho_4, \rho_5\}$  and  $\{\rho_4, \rho_5, \rho_6\}$  form and  $\rho_4$  gets  $\frac{1}{54}$ . Hence  $\rho_4$  accepts. If  $\rho_3$  passes, then with equal probability, the coalitions  $\{\rho_4, \rho_5\}$  and  $\{\rho_4, \rho_5, \rho_6\}$  form and  $\rho_3$  has the expected profit  $\mathbb{E}\pi_{\rho_3} = \frac{1}{2} \frac{1}{36} + \frac{1}{2} \frac{1}{81} < \frac{1}{32}$ . Hence,  $\rho_3$  proposes  $\{\rho_3, \rho_4\}$  and this coalition forms.

$\rho_2$  : In this case  $\rho_2$  prefers the five firms coalition  $\{\rho_2, \dots, \rho_6\}$ , where he gets  $\frac{1}{40}$ , to the four firms coalition  $\{\rho_2, \dots, \rho_5\}$ . If  $\{\rho_2, \rho_3\}$  forms, then with equal probability  $\{\rho_4, \rho_5\}$  and  $\{\rho_4, \rho_5, \rho_6\}$  form as well and  $\rho_2$  has the expected profit  $\mathbb{E}\pi_{\rho_2} = \frac{1}{2} \frac{1}{36} + \frac{1}{2} \frac{1}{54} < \frac{1}{40}$ . If



$\{\rho_2, \rho_3, \rho_4\}$  forms, then  $\{\rho_5, \rho_6\}$  forms as well and  $\rho_2$  gets  $\frac{1}{54}$ . If  $\rho_3$  rejects  $\{\rho_2, \dots, \rho_6\}$ , then every proposal from  $\rho_3$  is rejected as well,  $\{\rho_4, \rho_5\}$  forms and  $\rho_3$  gets  $\frac{1}{100}$ . If  $\rho_4$  rejects, then he further rejects every proposal from  $\rho_3$ ,  $\{\rho_4, \rho_5\}$  forms and  $\rho_4$  gets  $\frac{1}{40}$ . Hence  $\rho_4$  accepts. The same is valid for  $\rho_5$ . If  $\rho_6$  rejects, then every proposal from  $\rho_3$  is rejected as well,  $\{\rho_4, \rho_5\}$  forms,  $\rho_6$  is one of four followers, obtaining a profit of  $\frac{1}{100}$ . Hence  $\rho_6$  accepts,  $\rho_2$  proposes  $\{\rho_2, \dots, \rho_6\}$  and this coalition forms.

$\rho_1$  : The best option if he proposes is  $\{\rho_1, \rho_2, \rho_3\}$ , since  $\{\rho_4, \rho_5, \rho_6\}$  forms as well and he gets  $\frac{1}{27}$ . If he passes, then  $\{\rho_2, \dots, \rho_6\}$  forms and  $\rho_1$  gets  $\frac{1}{16}$ . Hence, the equilibrium coalition structure is  $\kappa^* = (\{\rho_1\}, \{\rho_2, \dots, \rho_6\})$ .

**Proof of Proposition 11** Equation (25) is positive if  $L > \frac{m}{n-m+1}$ . In case of a single merger, previous condition becomes  $m < \frac{n+1}{2}$ . If  $m = 2$ , namely a single bilateral condition occurs (E1), previous inequality holds, while it does not hold if  $m = n - 1$  (E2).

If  $L \geq 2$ , in the cases of Proposition 4,  $L > \frac{m}{n-m+1}$  never holds. ■

**Proof of Proposition 12** The difference between the post and the pre-merger social welfare is:

$$\Delta W = \frac{[(n-m)(L+1)+L][(n-m)(L+1)+(L+2)]}{2(L+1)^2(n-m+1)^2} - \frac{n(n+2)}{2(n+1)^2}. \quad (\text{A-14})$$

If  $L = 1$ , (A-14) becomes:

$$\frac{1}{2} \left( \frac{4(n-m+1)^2 - (n+1)^2}{4(n+1)^2(n-m+1)^2} \right). \quad (\text{A-15})$$

(A-15) is positive if  $4(n-m+1)^2 > (n+1)^2$ , which simplifies to  $m < \frac{n+1}{2}$ . Thus, social welfare follows the same direction of CS. If  $L \geq 2$ , (A-14) is never positive in the cases of Proposition 4. ■

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