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Abstract

Many economic variables are characterized by changes in their conditional mean and volatility, and time-varying Vector Autoregressive Models are often used to handle such complexity. Unfortunately, as the number of series grows, they present increasing estimation and interpretation issues. This paper tries to address this problem by proposing a Multivariate Autoregressive Index model that features time-varying mean and volatility. Technically, we develop a new estimation methodology that mixes switching algorithms with the forgetting factors strategy of Koop and Korobilis (2012). This substantially reduces the computational burden and allows one to select or weigh the number of common components, and other data features, in real-time without additional computational costs. Using US macroeconomic data, we provide a forecast exercise that shows the feasibility and usefulness of this model.

Keywords: Large Vector Autoregressive Models, Multivariate Autoregressive Index Models, Time-Varying Parameter Models, Bayesian Vector Autoregressive Models.

1 Introduction

The availability of real-time datasets and the economic instability have changed the nature of economic models, calling for the development of new methodologies that capture the ever-changing economic environment. One notable example is Cogley and Sargent (2002) where they use a small time-varying parameter Vector Autoregressive model (TVP-VAR) to detect drifts in inflation-unemployment dynamics. Other important contributions include Cogley et al. (2005), Cogley and Sargent (2005), Primiceri (2005) and d'Agostino et al. (2013). These models capture a wide range of structural instabilities and consistently outperform standard homoskedastic approaches in terms of forecast accuracy. However, their practical applications are primarily limited to small-scale systems that involve only a few variables.

The influential paper of Bańbura et al. (2010) shows that large VARs improve forecast accuracy and provide a more sensible impulse-response analysis. This has sparked interest in TVP-VARs for large datasets, as shown by Carriero et al. (2009), Koop (2013), Bańbura et al. (2013), Carriero et al. (2011), Ellahie and Ricco (2017), and Morley and Wong (2020).

Several papers develop large constant coefficient VARs with time-varying volatility (VAR-SV) see, among others, Carriero et al. (2016, 2022b) and Kastner and Huber (2020). In particular, Chan and Eisenstat (2018) show that the VAR-SV forecasts are better than the regime-switching VARs. Other papers develop large VARs with time-varying coefficients and volatility (TVP-VAR-SV), see, among others, Koop and Korobilis (2013) and Kapetanios et al. (2019). In both cases, large VARs feature a huge amount of parameters to be estimated and require dimension reduction approaches, examples are Chan et al. (2020b) where they use a factor-like structure to estimate a TVP-VAR with many variables and Chan et al. (2020a) where they propose a composite likelihood for large VARs to efficiently estimate the parameters. Chan (2023), in order to avoid overparameterization problems, introduced a large hybrid TVP-VAR in which only a few equations have time-varying coefficients, while the coefficients are constant in others. This approach requires the definition of a structural form parametrization that raises the issue of variable ordering that the authors partially solve with a data-driven approach.

The estimation of the aforementioned approaches is mainly based on Markov chain Monte Carlo (MCMC) methods. As the number of variables increases, this slows the estimation because thousands of latent states and parameters have to be simulated. Motivated by this issue, Koop and Korobilis (2013) developed a computationally efficient methodology to estimate large TVP-VAR-SV, using the forgetting factor approach for the time-varying mean parameters and the exponential weighted moving average (EWMA) for the time-varying error covariance matrix. Forgetting factors, also known as discount factors, have a long history in state space models, see Raftery et al. (2010). They do not require the use of MCMC methods and are useful in economic and financial applications, see Dangl and Halling (2012) and Grassi et al. (2017).

Recently, Koop and Korobilis (2014) extended the methodology to time-varying parameter factor augmented VAR with time-varying volatility (TVP-FAVAR-SV). Although TVP-VAR-SV are generally easier to handle than TVP-FAVAR-SV in terms of online estimation, it remains an open question whether a small number of common components can efficiently summarize the data for forecasting or economic analysis.

The paper proposes a new model that bridges TVP-VAR-SV and TVP-FAVAR-SV with a new estimation strategy based on Koop and Korobilis (2013) to avoid MCMC. Specifically, to reduce the dimensionality, we draw from the recent developments in Multivariate Autoregressive Index (MAI) models, see Carriero et al. (2016), Cubadda et al. (2017), Cubadda and Guardabascio (2019), Carriero et al. (2020), Cubadda and Hecq (2022a), and Cubadda and Mazzali (2023) among others. The MAI model, originally introduced by Reinsel (1983), is a bridge between reduced rank VARs (Cubadda and Hecq, 2022b) and the dynamic factor model (DFM) (Stock and Watson, 2016, and Lippi, 2019). On the one hand, it reduces dimension by imposing a sort of reduced rank structure to the VAR; on the other hand, it allows the identification of a few linear combinations of the variables that are labeled indexes, whose lags are entirely responsible for the dynamics of the system.

Although the mathematical formulation of the MAI is similar to that of the DFM, an advantage of the former for classical inference is that it does not require that the dimension of the system diverges to infinity to achieve identification,[1] therefore, MAI

[1] We remark that this condition is not necessary for Bayesian inference on DFMs.

can also be applied to small or medium VARs. In addition, the factor structure can be tested for and not simply imposed as in the DFM, and the estimation error of the indexes is explicitly taken into account, see Cubadda and Guardabascio (2019) for further details.

The contribution of the paper is twofold. The first is to propose a MAI with time-varying mean and volatility (TVP-MAI-SV). As Carriero et al. (2016) and Carriero et al. (2020) show, the MAI estimation is computationally intensive due to nonlinearity in the parameters. To overcome this problem, the second contribution of the paper is to develop approximate estimation methods for TVP-MAI-SV which do not involve the use of MCMC and simplify the estimation of nonlinear parameters using a switching algorithm (Cubadda et al., 2017) with forgetting factors (Koop and Korobilis, 2014). This substantially reduces the computational burden and allows one to select or weight, in real-time, the number of common components and other features of the data using Dynamic Model Selection (DMS) or Dynamic Model Averaging (DMA) without further computational cost.

The empirical application uses 25 US quarterly time series for forecasting three key macroeconomic variables: real gross domestic product (GDP), consumer price index (CPI) and effective federal funds rate (FFR). Point and density forecast evaluation show that the TVP-MAI-SV model has very promising out-of-sample properties compared to a set of univariate and multivariate competitors.

The remainder of this paper proceeds as follows. Section 2 presents the MAI model and introduces the new TVP-MAI-SV. Section 3 discusses the new estimation approach. Section 4 contains the empirical application. Finally, Section 5 draws some conclusions. All the derivations are reported in Appendices A and B.

2 From the MAI model to the TVP-MAI-SV

Let $\mathbf{y}_t \equiv (y_{1,t}, \ldots, y_{N,t})'$ denote the N-vector of the time series of interest. In the fixed parameter framework, variables $\mathbf{y}_t$ are assumed to be generated by a VAR of order p (VAR(p)):

$$\mathbf{y}_t = \Phi(L)\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t, \quad t = 1, 2, \ldots, T, \tag{1}$$

where $\Phi(L) = \sum_{h=0}^{p-1} \Phi_h L^h$, and $\boldsymbol{\varepsilon}_t$ are i.i.d. innovations with $\mathrm{E}(\boldsymbol{\varepsilon}_t) = 0$, $\mathrm{E}(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t') = \mathbf{H}$ (positive definite).

To reduce the number of model parameters in equation (1), Reinsel (1983) imposed the following set of restrictions on the mean parameters of a stationary VAR:

$$\Phi(L) = \beta(L)\boldsymbol{\omega}', \tag{2}$$

where $\boldsymbol{\omega}$ is full-rank $N \times q$ matrix with $q < N$, $\beta(L) = \sum_{h=0}^{p-1} \beta_h L^h$, and β_h is a $N \times q$ matrix for $h = 1, \ldots, p$.

The rationale underlying assumption in equation (2) is that the unrestricted VAR foresees N linearly independent mechanisms by which past information is transmitted to the system. However, since it is generally believed that few common shocks generate most macroeconomic fluctuations, it is reasonable to assume that there are a reduced number of channels through which variables are influenced by their past. In words, this is exactly what equation (2) implies (see Carriero et al., 2016 and Cubadda and Guardabascio, 2019 for more details).

Notice that the assumption in equation (2) is equivalent to postulating the following structure for series $\mathbf{y}_t$:

$$\mathbf{y}_t = \beta(L)\mathbf{f}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{3}$$

where $\mathbf{f}_t = \boldsymbol{\omega}'\mathbf{y}_t$. Reinsel (1983) defines the q-dimensional series $\mathbf{f}_t = (f_{1,t}, \ldots, f_{q,t})$ as index variables and labels equation (3) as the MAI model.

An interesting property of MAI is that the indexes themselves have a VAR(p) representation. Indeed, if we premultiply by $\boldsymbol{\omega}'$ both sides of equation (3) we get:

$$\mathbf{f}_t = \alpha(L)\mathbf{f}_{t-1} + \boldsymbol{\epsilon}_t,$$

where $\alpha(L) = \boldsymbol{\omega}'\beta(L)$ and $\boldsymbol{\epsilon}_t = \boldsymbol{\omega}'\boldsymbol{\varepsilon}_t$. This feature is in sharp contrast with reduced rank VAR models, where linear combinations of the variables generally do not admit a finite-order VAR representation, see Cubadda et al. (2009), and highlights the analogy between the role of indexes in the MAI and factors in the DFMs.

Moreover, using the decomposition of the identity matrix as in Centoni and

Cubadda (2003)

$$\mathbf{H}\boldsymbol{\omega}(\boldsymbol{\omega}'\mathbf{H}\boldsymbol{\omega})^{-1}\boldsymbol{\omega}' + \boldsymbol{\omega}_\perp(\boldsymbol{\omega}'_\perp\mathbf{H}^{-1}\boldsymbol{\omega}_\perp)^{-1}\boldsymbol{\omega}'_\perp\mathbf{H}^{-1} = \mathrm{I}_N,$$

where $\boldsymbol{\omega}_\perp$ is a full-rank $(n-q)\times n$ matrix such that $\boldsymbol{\omega}'_\perp\boldsymbol{\omega} = 0$, we can decompose variables $\mathbf{y}_t$ as:

$$\mathbf{y}_t = \chi_t + \iota_t,$$

where $\chi_t = \mathbf{H}\boldsymbol{\omega}(\boldsymbol{\omega}'\mathbf{H}\boldsymbol{\omega})^{-1}\mathbf{f}_t$, $\iota_t = \boldsymbol{\omega}_\perp(\boldsymbol{\omega}'_\perp\mathbf{H}^{-1}\boldsymbol{\omega}_\perp)^{-1}\mathbf{u}_t$, and $\mathbf{u}_t = \boldsymbol{\omega}'_\perp\mathbf{H}^{-1}\mathbf{y}_t$.

Since it is easy to see that the innovations of $\mathbf{f}_t$ and $\mathbf{u}_t$ are uncorrelated, the components χ_t and ι_t are in turn uncorrelated at all lags and leads. Therefore, the component χ_t has an analogous interpretation as the common component in DFMs.

Recently, there has been renewed interest in MAI, Carriero et al. (2016) derived classical and Bayesian estimation of large MAIs and applied this model to structural analysis, Cubadda et al. (2017) proposed a multivariate realized volatility model with an index structure, Cubadda and Guardabascio (2019) extended the model by allowing individual AR structures, Carriero et al. (2020) studied a MAI with stochastic volatility and provided MCMC estimation, Cubadda and Hecq (2022a) combined the MAI with reduced rank regression to achieve a dimension reduction in large VARs, while Cubadda and Mazzali (2023) endowed the cointegrated VAR with an index structure.

We extend the traditional MAI model, allowing variation in both the mean and variance equation, the TVP-MAI-SV takes the form:

$$\mathbf{y}_t = \sum_{h=1}^{p} \beta_{h,t} \mathbf{f}_{t-h} + \boldsymbol{\varepsilon}_t, \tag{4}$$

where $\boldsymbol{\varepsilon}_t \sim \mathcal{N}(0, \mathbf{H}_t)$, and $\beta_{h,t}$ is an $N \times q$ matrix of time-varying coefficients that evolve as random walks.

Notice that, similarly as in the TVP-FAVAR literature, it is assumed that the index loadings vary over time, while the index weights $\boldsymbol{\omega}$ remain stable. Furthermore, $\boldsymbol{\varepsilon}_t$ has a time-varying covariance matrix $\mathbf{H}_t$.

The model given in equation (4) is difficult to estimate with existing methods due to $\boldsymbol{\omega}$ restrictions that enter the model nonlinearly. To address this issue, Carriero et al. (2016) and Carriero et al. (2020) propose to include a random walk Metropolis step in the Gibbs sampling algorithm. This step aims to target the posterior kernel of each element of $\boldsymbol{\omega}$ and provides accurate results. However, it comes with a significant computational burden. To address this issue and reduce computational time, we develop a novel hybrid algorithm, which will be detailed in the following Section.

3 Estimation

The estimation of the model in equation (4) is based on a fast two-step algorithm that significantly reduces the computational burden. Subsection 3.1 presents the state space representation of TVP-MAI-SV and briefly explains the related estimation issues. Subsection 3.2 presents the new hybrid switching algorithm used to estimate TVP-MAI-SV. Subsection 3.3 describes the model selection. All the derivations are reported in Appendix A.

3.1 Bayesian Estimation

The model in equation (4) can be written in state space form as follows:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{Z}_t \boldsymbol{\beta}_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(0, \mathbf{H}_t), \\ \boldsymbol{\beta}_t &= \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \sim \mathcal{N}(0, \mathbf{Q}_t), \end{aligned} \tag{5}$$

where $\mathbf{Z}_t = [\mathbf{f}'_{t-1}, \dots, \mathbf{f}'_{t-p}]' \otimes \mathrm{I}_N$, which depends on the unknown matrix $\boldsymbol{\omega}$, and $\boldsymbol{\beta}_t = \mathrm{Vec}\left([\beta'_{1,t}, \dots, \beta'_{p,t}]'\right)$ is a Nqp vector containing the time-varying coefficients (states), which are assumed to follow a multivariate random walk dynamics. Finally, the errors $\boldsymbol{\varepsilon}_t$ and β_t are assumed to be mutually independent at all leads and lags, and $\mathbf{H}_t$ features time-varying volatility.

The model in equation (5) is usually estimated with a classical or Bayesian approach. In the first case, the likelihood is calculated with the Kalman filter (KF) see Durbin and Koopman (2012), and the time-varying parameters are filtered as latent state variables once $\mathbf{H}_t$ and $\mathbf{Q}_t$ are estimated. In the second case, simulation methods such as MCMC require the specification of $\mathbf{H}_t$ and $\mathbf{Q}_t$ together with the initial condition ($\boldsymbol{\beta}_{0|0}$) of the model parameters, see Koop (2003). Although Bayesian algorithms are reliable in this context, as discussed in Carriero et al. (2018), they become computationally intensive as the number of parameters increases and unfeasible when many models have to be estimated.

To solve this problem, we propose a new hybrid algorithm to estimate both static ($\boldsymbol{\omega}$) and dynamic ($\boldsymbol{\beta}_t$) parameters of equation (5).

3.2 Hybrid algorithm for TVP-MAI-SV

Following Koop and Korobilis (2014) and Cubadda et al. (2017) we combine the discount factor methodology with the switching algorithm to estimate the time-varying index loadings ($\boldsymbol{\beta}_t$) and the index weights ($\boldsymbol{\omega}$). The model also features time-varying volatility ($\mathbf{H}_t$), estimated using two versions of the EWMA filter. The estimation starts from the same algorithm described in Cubadda et al. (2017) and introduces the time-varying parameters as follows:

Given an initial estimate of $\boldsymbol{\omega}_\mathbf{0}$ and $\mathbf{H}_0$:

1) Run an approximation of the KF (using forgetting factors) for model in equation (5) to estimate the latent states $\hat{\boldsymbol{\beta}}_t$, see Appendix A for details;

2) Given the previous values of $\boldsymbol{\beta}_t$ and $\boldsymbol{\omega}$, estimate the measurement error covariance matrix ($\mathbf{H}_t$) using either the classical EWMA formula:

$$\hat{\mathbf{H}}_t = \kappa\hat{\mathbf{H}}_{t-1} + (1-\kappa)\hat{\boldsymbol{\varepsilon}}_t\hat{\boldsymbol{\varepsilon}}_t^{'},$$

where $\hat{\boldsymbol{\varepsilon}}_t$ is produced by the KF, or its Dynamic Conditional Correlation variant

(DCC-EWMA) suggested by Johansson et al. (2023) given by:

$$\hat{\mathbf{H}}_t = \hat{\mathbf{D}}_t \hat{\mathbf{R}}_t \hat{\mathbf{D}}_t,$$

with two discount factors: κ_1 for variances ($\hat{\mathbf{D}}_t$) and κ_2 for correlation ($\hat{\mathbf{R}}_t$), see Appendix A for details. In both cases, the decay factor(s) must be selected, we discuss this issue in Section 4.

3) Premultiply by $\hat{\mathbf{H}}_t^{-1/2}$ and apply the Vec$(\cdot)$ operator to both sides of equation (4), under the property Vec(ABC) = (C$'$ $\otimes$ A)Vec(B), we get:

$$\mathrm{Vec}\left(\hat{\mathbf{H}}_t^{-1/2}\mathbf{y}_t\right) = \sum_{h=1}^{p}\left(\mathbf{y}'_{t-h} \otimes \hat{\mathbf{H}}_t^{-1/2}\hat{\beta}_{h,t}\right)\mathrm{Vec}\left(\boldsymbol{\omega}'\right) + \mathrm{Vec}\left(\hat{\mathbf{H}}_t^{-1/2}\boldsymbol{\varepsilon}_t\right). \qquad (6)$$

Given $\hat{\boldsymbol{\beta}}_t$ and $\hat{\mathbf{H}}_t$, estimate Vec$(\boldsymbol{\omega}')$ with OLS in equation (6).[2]

4) Repeat steps 1), 2), and 3) until numerical convergence occurs.

A few comments are in order. The above algorithm offers several advantages over the available alternatives: it is computationally simple and fast avoiding the bottleneck

[2]As correctly noted by a referee, more parameters than the free ones in $\boldsymbol{\omega}$ are estimated in step 3). This approach is valid when the goal is forecasting, but not if the interest lies in estimation of the model parameters. Following Cubadda et al. (2017), this issue can be solved by using the normalization $\boldsymbol{\omega}' = [I_q, \boldsymbol{\varpi}']$, where $\boldsymbol{\varpi}$ is a $(N-q)\times q$ matrix, partitioning the variables conformably as $\mathbf{y}_t = [\mathbf{y}'_{1,t}, \mathbf{y}'_{2,t}]'$, and replacing Equation (6) with the following:

$$\mathrm{Vec}\left[\hat{\mathbf{H}}_t^{-1/2}\left(\mathbf{y}_t - \sum_{h=1}^{p}\hat{\beta}_{h,t}\mathbf{y}'_{1,t-h}\right)\right] = \left(\sum_{h=1}^{p}\mathbf{y}'_{2,t-h} \otimes \hat{\mathbf{H}}_t^{-1/2}\hat{\beta}_{h,t}\right)\mathrm{Vec}\left(\boldsymbol{\varpi}'\right) + \mathrm{Vec}\left(\hat{\mathbf{H}}_t^{-1/2}\boldsymbol{\varepsilon}_t\right),$$

from which one can estimate by OLS the $\boldsymbol{\varpi}$ coefficients.

in the estimation of $\boldsymbol{\omega}$ due to its nonlinearity, see Carriero et al. (2016); it does not need a normalization condition for the parameters $\boldsymbol{\omega}$ and over-identifying restrictions can be easily imposed on $\boldsymbol{\omega}$.

To speed up numerical convergence, it is important to make the appropriate choices regarding the various hyperparameters and initial conditions. As in Koop and Korobilis (2014), we choose fairly non-informative priors. The initial conditions for the time-varying parameters ($\boldsymbol{\beta}_t$) and the time-varying covariance ($\mathbf{H}_t$) are set as follows: $\boldsymbol{\beta}_0 \sim \mathcal{N}(0_{Nqp\times 1}, 4\mathrm{I}_{Nqp})$, $\hat{\mathbf{H}}_0 = \mathrm{I}_N$ for both the EWMA formulations. The initial conditions for $\boldsymbol{\omega}$ are obtained from the eigenvectors that are associated with the first q principal components of the series $\mathbf{y}_t$.

Finally, we point out that it is difficult with our hybrid approach to establish the theoretical properties of the resulting estimator. However, this drawback is largely compensated for by significant computational advantages compared to a more formal inferential framework.

3.3 Dynamic model averaging and Dynamic model selection for the TVP-MAI-SV

Time-varying parameter models are well suited to estimate the gradual evolution of coefficients. However, they may not work well for sudden changes, and allowing for switches between different models can accommodate more abrupt breaks. Indeed, model switching is a potentially useful addition for TVP-MAI-SV. Equation (5) can be generalized to accommodate a set $\tilde{\mathcal{M}} = \{\mathcal{M}_1, \mathcal{M}_2, \ldots, \mathcal{M}_K\}$ of possible models based on a different combination of: number of indexes (q), value for the decay factors (κ), number of lags (p), etc. Taking into account the $\tilde{\mathcal{M}}$ possible models, equation (5) can be rewritten as follows:

$$
\begin{aligned}
\mathbf{y}_t &= \mathbf{Z}_t^{(k)} \boldsymbol{\beta}_t^{(k)} + \boldsymbol{\varepsilon}_t^{(k)}, \qquad & \boldsymbol{\varepsilon}_t^{(k)} &\sim \mathcal{N}\left(0, \mathbf{H}_t^{(k)}\right), \\
\boldsymbol{\beta}_t^{(k)} &= \boldsymbol{\beta}_{t-1}^{(k)} + \boldsymbol{\eta}_t^{(k)}, \qquad & \boldsymbol{\eta}_t^{(k)} &\sim \mathcal{N}\left(0, \mathbf{Q}_t^{(k)}\right),
\end{aligned}
\tag{7}
$$

where the index k denotes a specification of model (5) based on a selection of q, κ, p and other parameters to be defined later.

Equation (7) shows that there are, potentially, many models to estimate at each time point t. When faced with multiple models, it is common to use model selection or model averaging techniques that have to be dynamic in our framework. More specifically, in a model selection exercise, we allow the selected model to change over time, performing DMS. Instead, in a model averaging exercise, we allow the weights used in averaging the models to change over time, leading to DMA. In this paper, we do both using the same approach of Raftery et al. (2010), see Appendix A.

In DMS and DMA the main objective is to calculate $\pi_{t|t-1,k}$, which is the probability that the model $\mathcal{M}_k$ should be used for the forecast at time t, given the information up to time $t-1$. Once $\pi_{t|t-1,k}$ is obtained, it can be used to perform model averaging or model selection.

DMS arises if, at each time point, the model with the highest value for $\pi_{t|t-1,k}$ is used to forecast. Note that $\pi_{t|t-1,k}$ will vary over time, and hence the selected model may change over time. DMA arises if the model averaging is performed in period t using $\pi_{t|t-1,k}$ $\forall k$ as weights. Raftery et al. (2010) developed a fast recursive algorithm to compute $\pi_{t|t-1,k}$ given an initial condition ($\pi_{0|0,k}$) and a forgetting factor (α):

$$\pi_{t|t-1,k} = \frac{\pi^{\alpha}_{t-1|t-1,k}}{\sum_{j=k}^{K} \pi^{\alpha}_{t-1|t-1,k}},$$

and a model updating equation:

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} f_k(\mathbf{y}_t|\mathbf{y}_{1:t-1})}{\sum_{k=1}^{K} \pi_{t|t-1,k} f_k(\mathbf{y}_t|\mathbf{y}_{1:t-1})},$$

where $f_k(\mathbf{y}_t|\mathbf{y}_{1:t-1})$ is the predictive likelihood of model k (i.e. the predictive density for model k evaluated at $\mathbf{y}_t$) that is produced by the KF and has a standard formula, see Frúhwirth-Schnatter (2006). The forgetting factor $0 < \alpha \leq 1$ adjusts the frequency of switches between models over time. Low values of α correspond to a rapid switch, and high values give the opposite, when $\alpha = 1$ we get the traditional Bayesian Model Averaging (BMA). Finally, the initial condition is set to equal probability $\pi_{0|0,k} = 1/K$, $\forall k$.

4 Forecasting Results

We investigate the performance of TVP-MAI-SV in forecasting US macroeconomic variables. Subsection 4.1 presents the dataset considered in the study, while Subsection 4.2 discusses the forecast exercise.

4.1 Data description

The dataset used in the forecast exercise is made up of 25 major quarterly US macroeconomic variables sourced from the Fred Database that run from 1959:Q1 to 2023:Q2. All variables, transformed as suggested in McCracken and Ng (2020), are listed in table 1.

4.2 Forecasting exercise

This section provides the out-of-sample performance of the TVP-MAI-SV against a set of competitors. We focus on three main variables: CPI, GDP, and FFR. The forecasting exercise is performed using an expanding window with an initial estimation sample that runs from 1960:Q1 to 1971:Q4. The model is then recursively estimated in a forecast window that starts from 1972:Q1 to 2023:Q2 for a total of 183 quarterly vintages. The forecast window covers the two oil shocks of 1973 and 1979, the 2001 Dot-com bubble, the 2008 Great Recession, and the 2020 Covid-19 pandemic.

TVP-MAI-SV is featured by several parameters that are dynamically selected

Table 1: Data Description. The Table reports: the Mnemonic Code (Code), the Variable name (Variables), and the transformation used (Transformation) accordingly to McCracken and Ng (2020).

Code	Variables	Transformation
PAYEMS	Employees on nonfarm payroll	Log-First-Difference
CES3000000008	Average hourly earnings	Log-First-Difference
DSPIC96	Personal Income	Log-Second-Difference
PCECC96	Real Consumption	Log-First-Difference
INDPRO	Industrial Production Index	Log-First-Difference
MCUMFN	Capacity Utilization	Level
UNRATE	Unemployment rate	First Difference
HOUST	Housing Starts	Log-First-Difference
CPIAUCSL	CPI all items	Log-Second-Difference
WPSFD49207	Produce Price Index (finished goods)	Log-First-Difference
PCECTPI	Price deflator for personal cons. exp	Log-Second-Difference
PPIACO	PPI ex food and energy	Log-Second-Difference
FEDFUNDS	Federal funds effective	First Difference
M1SL	M1 money stock	Log-First-Difference
M2SL	M2 money stock	Log-Second-Difference
TOTRESNS	Total reserves of depository inst.	Log-Second-Difference
NOBORRES	Nonborrowed reserves of depository inst.	Log-Second-Difference
S&P500	S&P common stock price index	Log-First-Difference
GS10	Int. Rate on Tr. Bills, 10 Year Const. Mat.	First Difference
EER	Effective Exchange Rate	Log-First-Difference
GDPC1	Gross Domestic Product	Log-First-Difference
BORROW	Total Borrowings from the Federal Reserve	Log-Second-Difference
OILPRICEx	Oil Price	Log-First-Difference
Y033RC1Q027SBEAx	Real Gross Private Domestic Investment	Log-First-Difference
OPHPBS	Labor Productivity per Hour - All Employees	Log-First-Difference

using DMS or DMA: the number of indexes (q), the values of the forgetting factors (λ and α) see Appendix A for details, the values of the decay factors (κ) and the number of lags (p). For the selection process, we consider a set of $q = \{1, 2, 3, 4\}$ indexes and a range of values for the forgetting factor $\lambda \in \{0.97, 0.98, 0.99, 1\}$ that covers from rapid coefficient change to no change. Following the recent literature, we dynamically selected α in the grid $\alpha = \{0.50, 0.70, 0.80, 0.95, 0.99, 1\}$ as suggested in Beckmann et al. (2020). For the decay factors, we use the grid of values $\kappa \in \{0.96, 0.97, 0.98, 0.99, 1\}$ for EWMA and $\kappa_1, \kappa_2 \in \{0.96, 0.97, 0.98, 0.99, 1\}$ for DCC-EWMA. Finally, the lag length (p) is set to 4.

Imposing simple restrictions, the TVP-MAI-SV encompasses some multivariate models:

- The original MAI of Reinsel (1983), estimated as in Cubadda et al. (2017) when $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1}$ and $\mathbf{H}_t$ is time invariant ($\mathbf{Q}_t = 0$, $\kappa = 1$ and $\mathbf{H}$ sets to the OLS estimate). This model is our benchmark;

- The MAI-SV similar to Carriero et al. (2018) when $\boldsymbol{\beta}_t = \boldsymbol{\beta}_{t-1}$ is time invariant ($\mathbf{Q}_t = 0$) and $\mathbf{H}_t$ evolves over time;

- The TVP-MAI model without time-varying volatility when $\boldsymbol{\beta}_t$ is time-varying ($\mathbf{Q}_t \neq 0$) but $\mathbf{H}_t$ is time invariant and set to the OLS estimates (sample variance);

We also consider results from several other models:

- TVP-VAR with four lags, where the optimal Minnesota shrinkage coefficient (γ) is set to 0.005, see Koop and Korobilis (2013);

- VAR of order 1 estimated with OLS;

- DFM with factors dynamically selected to explain 90% of the total variability;

- Random Walk process (RW);

- TVP-VAR-SV as in Koop and Korobilis (2013) with different specifications;

- TVP-FAVAR-SV as in Koop and Korobilis (2014) with different specifications;

Table 2: The table reports all the models considered in the forecasting exercise including the benchmark ($\mathcal{M}_{11}$). The first column is the model label (Label). The second column provides a description of each model (Full Description).

Label	*Full Description*
$\mathcal{M}_1$	TVP-MAI-SV with DCC-EWMA. The number of indexes and the optimal value of λ, κ_1 and κ_2 are selected using DMA as described in Koop and Korobilis (2013).
$\mathcal{M}_2$	TVP-MAI-SV with DCC-EWMA. Number of indexes and the optimal value of the λ, κ_1 and κ_2 are selected using DMS as described in Koop and Korobilis (2013).
$\mathcal{M}_3$	TVP-MAI-SV with EWMA. The number of indexes and the optimal value of λ and κ are selected using DMA as described in Koop and Korobilis (2013).
$\mathcal{M}_4$	TVP-MAI-SV with EWMA. The number of indexes and the optimal value of λ and κ are selected using DMS as described in Koop and Korobilis (2013).
$\mathcal{M}_5$	MAI-SV, with fix $\beta_t(\lambda = 1)$ and DCC-EWMA. The number of indexes and the optimal value of κ_1 and κ_2 are selected using DMA as described in Koop and Korobilis (2013).
$\mathcal{M}_6$	MAI-SV, with fix $\beta_t(\lambda = 1)$ and DCC-EWMA. The number of indexes and the optimal value of κ_1 and κ_2 are selected using DMS as outlined in Koop and Korobilis (2013).
$\mathcal{M}_7$	MAI-SV, with fix $\beta_t(\lambda = 1)$ and EWMA. The number of indexes and the optimal value of κ are selected using DMA as described in Koop and Korobilis (2013).
$\mathcal{M}_8$	MAI-SV, with fix $\beta_t(\lambda = 1)$ and EWMA. The number of indexes and the optimal value of κ are selected using DMS as outlined in Koop and Korobilis (2013).
$\mathcal{M}_9$	TVP-MAI, with fix H ($\kappa = 1$). The number of indexes and the optimal value of λ are selected using DMA as described in Koop and Korobilis (2013).
$\mathcal{M}_{10}$	TVP-MAI, with fix H ($\kappa = 1$). The number of indexes and the optimal value of λ are selected using DMS as outlined in Koop and Korobilis (2013).
$\mathcal{M}_{11}$	MAI. The number of indexes is selected using DMA as described in Koop and Korobilis (2013). **Benchmark model**.
$\mathcal{M}_{12}$	MAI. The number of indexes is selected using DMS as outlined in Koop and Korobilis (2013).
$\mathcal{M}_{13}$	Random Walk.
$\mathcal{M}_{14}$	Vector Autoregressive(1) estimated using the OLS.
$\mathcal{M}_{15}$	Dynamic Factor Model
$\mathcal{M}_{16}$	TVP-VAR-SV with 4 lags and time-varying volatility. The optimal value of the shrinkage parameter is selected. using DMS as described in Koop and Korobilis (2013). In this model $\lambda = 0.99$, $\kappa = 0.98$ and $\alpha = 0.99$.
$\mathcal{M}_{17}$	TVP-VAR-SV with 4 lags and time-varying volatility. The optimal value of the shrinkage parameter is selected using DMS as described in Koop and Korobilis (2013). In this model λ is dynamically selected, $\kappa = 0.98$ and $\alpha = 0.99$.
$\mathcal{M}_{18}$	FAVAR with 4 lags and 4 factors as in Koop and Korobilis (2014).
$\mathcal{M}_{19}$	TVP-FAVAR with 4 lags and 4 factors as in Koop and Korobilis (2014). In this model $\lambda_1 = 1$, $\lambda_2 = 1$, $\kappa_1 = 0.99$ and $\kappa_2 = 0.96$.
$\mathcal{M}_{20}$	TVP-FAVAR-SV with 4 lags and 4 factors as in Koop and Korobilis (2014). In this model $\lambda_1 = 0.99$, $\lambda_2 = 0.99$, $\kappa_1 = 0.96$ and $\kappa_2 = 0.96$.
$\mathcal{M}_{21}$	AR+PC, autoregressive model with four lags plus a factor estimated using principal components.

- AR+PC as autoregressive model with four lags plus a factor estimated using principal components;

The full set of competitive models considered is reported in Table 2.

We evaluate the accuracy of our model in terms of point and density forecasts following Chan et al. (2020a). Let $\hat{y}_t^{(k)}$ denotes the forecast made by model k for the variables of interest and y_t^* their realizations, then the root mean squared forecast error (RMSFE) and the mean absolute forecast error (MAFE) are:

$$
\begin{aligned}
\mathrm{RMSFE}_{j,h}^{(k)} &= \sqrt{\frac{\sum_{t=t_0}^{T-h}\left(y_{j,t+h}^{*} - \hat{y}_{j,t+h}^{(k)}\right)^2}{T-h-t_0+1}}, \\
\mathrm{MAFE}_{j,h}^{(k)} &= \frac{\sum_{t=t_0}^{T-h}\left|y_{j,t+h}^{*} - \hat{y}_{j,t+h}^{(k)}\right|}{T-h-t_0+1}.
\end{aligned}
\tag{8}
$$

where k is the model, $h = \{1, \ldots, H\}$ are the forecast steps ahead and $j = \{1, \ldots, J\}$ are the target variables. To evaluate density forecasts, we use the average log-predictive likelihood (ALPL) as described in Korobilis (2021) and Chan et al. (2020a) as the broadest measure of density accuracy, see also Geweke (2005).[3]

Tables 3 to 5 report for GDP, CPI, and FFR, the RMSFE and MAFE, relative to the benchmark model, for $h = \{1, \ldots, 8\}$ steps ahead. Numbers smaller (larger) than 1 indicate forecasts that are more (less) accurate than the benchmark. The tables also report the ALPL as a spread from the ALPL of the benchmark model. Positive (negative) values signify better (worst) performance relative to the benchmark.

With three different variables, eight different forecast horizons, and three different forecast metrics, virtually every model performs well in some cases, but several

[3]Notice that the Diebold-Mariano test (Diebold, 2015) is not suitable to test the accuracy of the Bayesian model.

Table 3: ***Results for GDP, 25 series, Covid case.***
Point and density forecast results for GDP. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicate better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The model description is reported in Table 2. Values in the table are capped at 3.

Root Mean Squared Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	1.043	1.054	1.051	1.055	1.051	1.058	1.052	1.054	1.275	1.289	1.002	1.373	1.557	0.958	1.190	1.191	1.037	1.099	1.147	1.424
2	1.027	1.037	1.020	1.020	1.030	1.037	1.022	1.026	1.374	1.423	1.003	1.385	1.328	1.146	1.016	1.020	1.177	1.048	1.066	3.000
3	0.993	0.996	0.990	0.993	0.996	0.997	0.993	0.994	2.016	2.169	1.002	1.350	1.159	1.019	0.980	0.972	1.100	1.008	1.030	1.095
4	0.999	1.000	0.994	0.995	1.001	1.001	0.994	0.994	3.000	3.000	1.001	1.384	1.056	1.016	0.988	0.981	1.069	1.003	1.022	0.998
5	1.000	1.003	0.999	1.000	1.005	1.003	1.001	1.001	3.000	3.000	0.998	1.365	1.014	1.027	0.990	0.991	1.025	1.002	1.005	1.005
6	1.003	1.002	1.003	1.003	1.004	1.003	1.002	1.002	3.000	3.000	1.003	1.427	1.032	1.026	0.989	0.989	1.025	1.002	1.010	1.007
7	0.998	0.997	0.998	0.997	0.997	0.997	0.997	0.997	3.000	3.000	1.004	1.403	1.062	1.008	0.989	0.989	1.011	1.000	0.999	1.001
8	0.999	0.999	0.998	0.998	0.999	0.999	0.999	0.999	3.000	3.000	1.001	1.404	1.070	0.994	0.992	0.992	1.002	1.002	0.999	1.003

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.975	1.001	1.005	1.014	0.990	1.002	1.011	1.011	1.089	1.121	1.003	1.230	1.142	0.995	1.366	1.314	1.164	0.990	1.005	1.147
2	1.004	1.017	0.998	1.000	1.010	1.013	0.997	1.005	1.144	1.141	1.009	1.284	1.189	1.131	0.982	0.989	1.184	0.975	0.991	2.689
3	0.966	0.981	0.969	0.979	0.976	0.982	0.974	0.978	1.269	1.299	1.004	1.374	1.100	0.983	0.950	0.936	1.148	0.977	1.006	1.036
4	0.980	0.984	0.970	0.975	0.988	0.989	0.974	0.974	1.515	1.572	1.002	1.451	1.074	1.009	0.973	0.961	1.117	1.012	1.043	0.992
5	0.999	1.004	0.995	0.995	1.003	1.001	0.996	0.997	2.047	2.163	1.006	1.439	1.057	1.039	0.996	0.989	1.082	1.001	1.015	1.009
6	0.995	0.995	0.996	0.995	0.996	0.995	0.995	0.994	3.000	3.000	1.006	1.468	1.085	1.015	0.984	0.984	1.067	0.997	1.014	1.004
7	0.988	0.988	0.987	0.987	0.987	0.988	0.987	0.987	3.000	3.000	1.007	1.509	1.071	1.003	1.000	0.992	1.033	0.985	0.985	0.991
8	0.990	0.988	0.990	0.989	0.989	0.989	0.990	0.990	3.000	3.000	1.003	1.553	1.092	0.986	1.016	1.010	1.010	0.998	0.995	0.996

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	0.225	0.239	0.231	0.233	0.220	0.236	0.225	0.228	-0.000	-0.000	0.000	-0.051	-0.007	-0.023	0.286	0.247
2	0.179	0.196	0.209	0.215	0.175	0.192	0.209	0.214	-0.003	-0.002	0.000	-0.166	-0.117	-0.001	0.288	0.279
3	0.155	0.170	0.196	0.205	0.153	0.168	0.203	0.208	-0.001	0.000	0.001	-0.159	-0.110	0.011	0.249	0.254
4	0.156	0.171	0.195	0.203	0.156	0.170	0.204	0.208	-0.002	-0.001	0.001	-0.153	-0.105	0.017	0.246	0.237
5	0.165	0.181	0.205	0.213	0.166	0.180	0.213	0.218	0.002	0.003	0.001	-0.142	-0.094	0.029	0.257	0.288
6	0.166	0.181	0.206	0.215	0.166	0.180	0.215	0.220	0.002	0.003	0.001	-0.140	-0.092	0.031	0.250	0.250
7	0.166	0.181	0.207	0.216	0.166	0.180	0.216	0.221	0.002	0.004	0.001	-0.139	-0.091	0.032	0.260	0.251
8	0.167	0.181	0.208	0.217	0.166	0.180	0.217	0.222	0.003	0.004	0.001	-0.138	-0.090	0.033	0.276	0.253

Table 4: **Results for CPI, 25 series, Covid case.**
Point and density forecast results for CPI Inflation. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicate better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The description of the model is reported in Table 2. Values in the table are capped at 3.

Root Mean Squared Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.968	0.982	1.013	1.015	0.978	0.979	1.017	1.018	1.539	1.590	1.002	1.559	1.202	1.413	0.979	0.978	0.954	0.968	0.965	0.997
2	0.994	0.993	0.995	0.992	0.990	0.991	0.992	0.992	2.365	2.526	1.007	1.547	1.265	1.179	0.989	0.990	0.993	0.967	0.976	3.000
3	0.971	0.972	0.974	0.973	0.968	0.968	0.973	0.974	3.000	3.000	1.006	1.232	1.168	1.110	0.969	0.971	0.979	0.954	0.958	1.271
4	0.970	0.973	0.976	0.976	0.970	0.970	0.976	0.976	3.000	3.000	1.002	1.475	1.074	1.316	0.973	0.976	0.981	0.977	0.983	0.979
5	0.987	0.987	0.988	0.988	0.986	0.987	0.987	0.987	3.000	3.000	1.003	1.375	1.027	1.130	0.985	0.985	0.992	0.992	0.993	0.997
6	1.003	1.002	1.004	1.005	1.003	1.002	1.005	1.005	3.000	3.000	1.001	1.342	1.010	1.007	1.000	1.000	1.012	1.007	1.010	1.006
7	0.991	0.991	0.992	0.993	0.991	0.991	0.992	0.992	3.000	3.000	1.001	1.402	0.990	1.029	0.986	0.986	0.997	0.989	0.990	0.995
8	0.994	0.996	0.994	0.994	0.994	0.994	0.994	0.994	3.000	3.000	0.999	1.465	0.997	1.044	0.985	0.985	0.998	0.988	0.990	0.998

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.942	0.953	0.973	0.973	0.953	0.958	0.977	0.979	1.130	1.145	1.008	1.539	1.036	1.108	0.955	0.958	0.906	0.922	0.919	0.910
2	0.968	0.968	0.975	0.972	0.964	0.965	0.971	0.972	1.262	1.280	1.013	1.559	1.095	1.063	0.965	0.970	0.970	0.951	0.964	3.000
3	0.971	0.971	0.972	0.969	0.966	0.967	0.970	0.972	1.535	1.582	1.013	1.255	1.097	1.046	0.965	0.970	0.989	0.963	0.971	1.112
4	0.956	0.961	0.964	0.963	0.957	0.957	0.964	0.964	2.044	2.146	1.006	1.510	1.033	1.068	0.969	0.974	0.988	0.968	0.978	0.980
5	0.981	0.982	0.980	0.980	0.980	0.980	0.980	0.980	3.000	3.000	1.001	1.518	1.042	1.044	0.985	0.985	1.000	0.985	0.990	1.001
6	0.999	0.996	1.002	1.002	1.000	0.999	1.002	1.002	3.000	3.000	1.003	1.402	1.018	0.999	1.000	1.000	1.016	1.007	1.007	1.002
7	0.989	0.988	0.992	0.992	0.989	0.989	0.991	0.991	3.000	3.000	1.004	1.417	0.999	1.017	0.985	0.984	0.998	0.991	0.995	0.986
8	0.997	0.998	0.998	0.998	0.996	0.996	0.998	0.998	3.000	3.000	0.998	1.607	1.004	1.030	0.984	0.984	0.998	0.984	0.986	0.996

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	-0.044	-0.046	-0.090	-0.093	-0.052	-0.048	-0.095	-0.099	-0.000	-0.000	0.000	0.107	0.130	-0.009	-0.003	0.015
2	-0.005	-0.005	-0.047	-0.049	-0.011	-0.007	-0.051	-0.054	-0.002	-0.004	-0.001	0.165	0.181	0.050	0.055	0.073
3	0.003	0.003	-0.038	-0.041	-0.002	0.002	-0.042	-0.045	-0.009	-0.010	0.000	0.094	0.089	0.059	0.065	0.083
4	0.012	0.012	-0.028	-0.031	0.008	0.012	-0.032	-0.035	-0.010	-0.010	0.000	0.085	0.100	0.070	0.075	0.093
5	0.035	0.036	-0.005	-0.007	0.032	0.036	-0.008	-0.011	-0.014	-0.013	-0.000	0.111	0.125	0.095	0.101	0.119
6	0.037	0.038	-0.003	-0.005	0.034	0.039	-0.006	-0.008	-0.015	-0.014	0.000	0.101	0.128	0.098	0.121	0.121
7	0.039	0.040	-0.001	-0.003	0.037	0.041	-0.004	-0.006	-0.015	-0.014	0.000	0.115	0.130	0.100	0.106	0.124
8	0.040	0.041	-0.000	-0.002	0.038	0.042	-0.002	-0.005	-0.016	-0.015	-0.000	0.117	0.131	0.101	0.107	0.125

Table 5: ***Results for FFR, 25 series, Covid case.***

Point and density forecast results for Interest Rate. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicate better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The description of the model is reported in Table 2. Values in the table are capped at 3.

Root Mean Squared Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.957	0.970	1.005	1.016	0.971	0.969	1.020	1.025	1.071	1.074	1.007	1.265	1.281	1.361	1.027	1.023	1.319	1.086	1.080	1.266
2	0.944	0.941	0.932	0.927	0.937	0.943	0.928	0.930	1.206	1.219	1.012	1.315	1.454	1.019	0.894	0.891	1.110	0.999	1.007	0.976
3	0.934	0.934	0.928	0.930	0.931	0.932	0.929	0.933	1.884	2.005	1.010	1.240	1.376	0.984	0.926	0.925	1.082	0.968	0.986	1.075
4	0.912	0.912	0.912	0.911	0.912	0.910	0.912	0.911	3.000	3.000	1.001	1.261	1.297	1.070	0.909	0.908	1.047	0.954	0.991	1.000
5	0.981	0.984	0.981	0.982	0.982	0.981	0.982	0.984	3.000	3.000	0.999	1.230	1.266	1.087	0.972	0.973	1.065	1.013	1.030	1.056
6	0.973	0.975	0.974	0.974	0.976	0.975	0.974	0.974	3.000	3.000	0.996	1.337	1.145	1.001	0.953	0.953	1.032	0.993	1.008	1.002
7	0.965	0.967	0.964	0.964	0.965	0.965	0.964	0.964	3.000	3.000	1.008	1.532	1.079	0.977	0.938	0.938	0.958	0.941	0.942	0.973
8	0.965	0.964	0.965	0.965	0.965	0.965	0.965	0.965	3.000	3.000	1.000	1.394	0.986	1.013	0.944	0.945	0.970	0.969	0.977	0.986

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.853	0.862	0.887	0.888	0.853	0.850	0.888	0.891	1.022	1.044	1.025	0.884	1.189	1.140	0.879	0.895	1.194	0.805	0.824	0.911
2	0.870	0.863	0.870	0.864	0.864	0.865	0.866	0.867	1.057	1.065	1.016	1.093	1.298	0.988	0.810	0.808	1.183	0.963	0.978	0.874
3	0.920	0.922	0.890	0.887	0.908	0.910	0.885	0.891	1.209	1.244	1.011	1.176	1.219	0.987	0.865	0.864	1.182	0.964	0.991	0.902
4	0.881	0.880	0.888	0.884	0.871	0.868	0.884	0.884	1.474	1.545	0.997	1.210	1.191	1.034	0.873	0.873	1.110	0.949	0.987	0.889
5	0.983	0.988	0.991	0.990	0.976	0.974	0.991	0.991	2.084	2.200	0.993	1.395	1.235	1.106	0.960	0.961	1.119	1.034	1.063	1.034
6	0.962	0.964	0.968	0.967	0.959	0.958	0.966	0.965	3.000	3.000	1.009	1.470	1.104	1.030	0.919	0.919	1.066	0.999	1.031	0.981
7	0.962	0.967	0.967	0.966	0.961	0.960	0.966	0.966	3.000	3.000	1.006	1.614	1.093	1.003	0.912	0.912	0.997	0.958	0.970	0.951
8	0.976	0.980	0.980	0.979	0.976	0.976	0.979	0.979	3.000	3.000	1.000	1.558	1.012	1.046	0.939	0.940	0.994	0.988	1.006	0.989

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	0.319	0.371	0.328	0.351	0.341	0.380	0.354	0.367	-0.004	-0.004	0.002	0.118	0.124	-0.008	0.301	0.342
2	0.242	0.299	0.276	0.308	0.278	0.318	0.323	0.335	0.004	0.005	-0.001	0.141	0.147	-0.012	0.281	0.293
3	0.204	0.256	0.245	0.275	0.242	0.278	0.292	0.302	0.001	-0.002	-0.001	0.132	0.136	-0.010	0.233	0.216
4	0.203	0.258	0.248	0.278	0.244	0.281	0.297	0.308	-0.000	-0.004	0.001	0.133	0.138	0.022	0.283	0.273
5	0.189	0.245	0.237	0.265	0.231	0.269	0.286	0.297	-0.002	-0.004	0.002	0.124	0.127	0.032	0.258	0.255
6	0.193	0.251	0.245	0.276	0.236	0.274	0.295	0.305	-0.002	-0.006	0.000	0.133	0.136	0.045	0.255	0.255
7	0.192	0.252	0.245	0.278	0.236	0.275	0.295	0.306	0.000	-0.000	0.001	0.132	0.135	0.054	0.272	0.247
8	0.192	0.252	0.244	0.278	0.236	0.275	0.295	0.307	0.003	0.002	0.002	0.124	0.127	0.058	0.273	0.252

observations can be made. First, TVP-FAVAR-SV ($\mathcal{M}_{18}$, $\mathcal{M}_{19}$ and $\mathcal{M}_{20}$), MAI-SV ($\mathcal{M}_5$, $\mathcal{M}_6$, $\mathcal{M}_7$ and $\mathcal{M}_8$) and TVP-MAI-SV ($\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$ and $\mathcal{M}_4$) generally outperform competitors. This result confirms that time-varying volatility is necessary to improve forecast performance in a dataset subject to large shocks such as the Covid-19 pandemic. Models that feature data reduction (e.g. TVP-MAI or DFM) without time-varying volatility often degenerate or forecast poorly, which is in line with the findings of Lenza and Primiceri (2022), Primiceri and Tambalotti (2020) and Carriero et al. (2022a).

Looking at both RMSFE and MAFE as well as the ALPL, TVP-MAI-SV is one of the best models for all the variables, outperforming its counterparts (MAI, TVP-MAI and MAI-SV) for both short and long horizons. Moreover, TVP-MAI-SV with the volatility estimated using DCC-EWMA always outperforms the EWMA counterpart. This indicates that using two decay factors, one for the variances and one for the correlations, could be helpful in forecasting.

Another point concerns the usefulness of adding time-varying parameters in the MAI-SV. The tables show that the two models have comparable RMSFE and MAFE, but TVP-MAI-SV always has a better ALPL.

In general, the results show that TVP-MAI-SV guarantees safe forecasts compared to other competitors, such as TVP-VAR-SV (see Koop and Korobilis, 2013) and more similar models such as TVP-FAVAR-SV, as described in Koop and Korobilis (2014). In Appendix B we provide two robustness checks: the first considers a forecasting period that excludes Covid-19, the second a smaller dataset of 7 variables. While in the first case, our model still outperforms the competitors, in the second it performs well, but with a lower gain.

5 Conclusions

Many economic variables are characterized by changing means and volatilities, to model these features, TVP-VAR-SV are commonly used. Although reliable with a small or medium dataset, when the number of variables increases, this approach becomes computationally unfeasible and provides imprecise estimation due to overparameterization. In this paper, we introduce TVP-MAI-SV, which can easily handle large data sets that feature a time-varying mean and volatility. Furthermore, we present a novel estimation methodology that significantly reduces the computational burden. Interestingly, our approach allows for real-time selection of the number of indexes and other data features using Dynamic Model Selection and Dynamic Model Averaging without further computational cost.

In the empirical application, we use 25 US quarterly time series to forecast three key macroeconomic variables, namely, the real gross domestic product, the consumer price index, and the effective federal funds rate. To assess the reliability of our model, we consider both point and density forecasts for the time window 1972:Q1-2023:Q2. Our results suggest that, using both point and density forecasts, the TVP-MAI-SV performs well compared to a set of multivariate and univariate competitors.

Extending this methodology to reduce the dimensions in both the conditional mean and the conditional variance in a single unifying idea, in the spirit of Chan et al. (2020b), is a promising avenue for future research.

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Appendix A

Methodology

From a technical point of view, handling a large number of models is not only computationally cumbersome, but also memory intensive. Raftery et al. (2010) and Koop and Korobilis (2012) recently proposed the forgetting factor methodology, which allows online estimation of time-varying parameters plus Dynamic Model Averaging

(DMA) and Dynamic Model Selection (DMS).

Following the discussion in Subsection 3.3, let us consider $\tilde{\mathcal{M}} = \{\mathcal{M}_1, \ldots, \mathcal{M}_K\}$ possible models at each time point t. The number of models is a combination of the number of indexes, values of λ (see below), and values of κ. Taking into account all possible combinations, the number of models will increase exponentially as the number of indexes, values of λ and κ increase. The state space model takes the following form:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{Z}_t^{(k)} \boldsymbol{\beta}_t^{(k)} + \boldsymbol{\varepsilon}_t^{(k)}, & \boldsymbol{\varepsilon}_t^{(k)} &\sim \mathcal{N}\left(\mathbf{0}, \mathbf{H}_t^{(k)}\right), \\ \boldsymbol{\beta}_t^{(k)} &= \boldsymbol{\beta}_{t-1}^{(k)} + \boldsymbol{\eta}_t^{(k)}, & \boldsymbol{\eta}_t^{(k)} &\sim \mathcal{N}\left(\mathbf{0}, \mathbf{Q}_t^{(k)}\right), \end{aligned} \tag{9}$$

see Subsection 3.1 for details.

Concurrent estimation of these models can be computationally cumbersome and even infeasible with the maximum likelihood or MCMC methods. To overcome this problem, Raftery et al. (2010) introduces an approximate KF that avoids calculating $\mathbf{Q}_t$ using a hyperparameter λ. Koop and Korobilis (2012) applied this methodology to forecast inflation and also added the estimation of a time-varying variance ($\mathbf{H}_t$) using an EWMA that requires a decay factor κ.

The main step in the KF recursions, for a given model k, is:

$$\boldsymbol{\beta}_{t-1}^{(k)} | \mathrm{Y}_{t-1} \sim \mathcal{N}\left(\hat{\boldsymbol{\beta}}_{t-1|t-1}^{(k)}, \Sigma_{t-1|t-1}^{(k)}\right), \tag{10}$$

where $\mathrm{Y}_{t-1} = (\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{t-1})$, $\hat{\boldsymbol{\beta}}_{t-1|t-1}^{(k)} = \mathrm{E}\left(\boldsymbol{\beta}_{t-1}^{(k)} | \mathrm{Y}_{t-1}\right)$ and $\Sigma_{t-1|t-1}^{(k)} = \mathrm{Var}\left(\boldsymbol{\beta}_{t-1}^{(k)} | \mathrm{Y}_{t-1}\right)$.

At each time point t, the algorithm iterates between the prediction equation, the updating equation, and the predictive density:

$$\boldsymbol{\beta}_t^{(k)}|\mathrm{Y}_{t-1} \sim \mathcal{N}\left(\hat{\boldsymbol{\beta}}_{t|t-1}^{(k)}, \Sigma_{t|t-1}^{(k)}\right), \tag{11}$$

$$\boldsymbol{\beta}_t^{(k)}|\mathrm{Y}_t \sim \mathcal{N}\left(\hat{\boldsymbol{\beta}}_{t|t}^{(k)}, \Sigma_{t|t}^{(k)}\right), \tag{12}$$

$$\mathbf{y}_t|\mathrm{Y}_{t-1} \sim \mathcal{N}\left(\mathbf{Z}_t^{(k)}\hat{\boldsymbol{\beta}}_{t|t-1}^{(k)}, \mathbf{H}_t^{(k)} + \mathbf{Z}_t^{(k)}\Sigma_{t|t-1}^{(k)}\mathbf{Z}_t^{(k)\prime}\right). \tag{13}$$

The quantity $\Sigma_{t|t-1}^{(k)}$ depends on the error variances: $\Sigma_{t|t-1}^{(k)} = \Sigma_{t-1|t-1}^{(k)} + \mathbf{Q}_t^{(k)}$. Raftery et al. (2010) proposed an approximation given by:

$$\Sigma_{t|t-1}^{(k)} = \frac{1}{\lambda}\Sigma_{t-1|t-1}^{(k)}. \tag{14}$$

Consequently, $\mathbf{Q}_t^{(k)} = \left(\frac{1}{\lambda} - 1\right)\Sigma_{t-1|t-1}^{(k)}$ with $\lambda \in (0, 1]$. The tuning parameter λ plays a crucial role in the adjustment of the effective memory of the algorithm, leading to a weighted estimate in which the data at i time points in the past have weight λ^i. For example, in the case of quarterly macroeconomic data, $\lambda = 0.99$ implies that observations five years ago received approximately 80% as much weight as the last period of observation, which leads to a fairly stable model where the coefficients change gradually. When $\lambda = 1$, we have the constant parameter case.

It is well known that both macroeconomic and financial time series are characterized by heteroskedastic effects; therefore, Koop and Korobilis (2012) assume that

$\mathbf{H}_t^{(k)}$ follows an EWMA such that:

$$\hat{\mathbf{H}}_t^{(k)} = \kappa \hat{\mathbf{H}}_t^{(k)} + (1-\kappa)\, \hat{\boldsymbol{\varepsilon}}_t^{(k)} \hat{\boldsymbol{\varepsilon}}_t^{(k)\prime}$$

where $\hat{\boldsymbol{\varepsilon}}_t^{(k)} = \mathbf{y}_t - \mathbf{Z}_t^{(k)} \hat{\boldsymbol{\beta}}_{t|t-1}^{(k)}$ is an output of the KF. The EWMA requires a value for the decay factor κ.

To carry out the model selection dynamically, we use the following posterior probabilities:

$$\begin{aligned} p\left(\boldsymbol{\beta}_t, \mathcal{M}_t | \mathrm{Y}_t\right) &= \sum_{k=1}^{K} p\left(\boldsymbol{\beta}_t^{(k)} | \mathcal{M}_t = k, \mathrm{Y}_t\right) Pr\left(\mathcal{M}_t = k | \mathrm{Y}_t\right) = \\ &= \sum_{k=1}^{K} p\left(\boldsymbol{\beta}_t^{(k)} | \mathcal{M}_t = k, \mathrm{Y}_t\right) \pi_{t|t,k}. \end{aligned} \tag{15}$$

where $\pi_{t|t,k} = Pr\left(\mathcal{M}_t = k | \mathrm{Y}_t\right)$ is estimated recursively using the prediction equation (16) and updating equation (17):

$$\pi_{t|t-1,k} = \sum_{l=1}^{K} \pi_{t-1|t-1,l}\, p_{kl}, \tag{16}$$

$$\pi_{t|t,k} = \frac{\pi_{t|t-1,k} f_k\left(\mathbf{y}_t | \mathrm{Y}_{t-1}\right)}{\sum_{l=1}^{K} \pi_{t|t-1,l} f_l\left(\mathbf{y}_t | \mathrm{Y}_{t-1}\right)}. \tag{17}$$

where $f_k\left(\mathbf{y}_t | \mathrm{Y}_{t-1}\right)$ is the predictive density. We have to underline that the model prediction equation (16) requires estimating the $K \times K$ elements of p_{kl}, this is replaced by an approximation:

$$\pi_{t|t-1,k} = \frac{\pi_{t-1|t-1,k}^{\alpha}}{\sum_{l=1}^{K} \pi_{t-1|t-1,l}^{\alpha}}. \tag{18}$$

To interpret α, let us take:

$$\pi_{t|t-1,k} \propto \left[\pi_{t-1|t-2,k} p_k \left(\mathbf{y}_{t-1} | \mathrm{Y}_{t-2}\right)\right]^{\alpha} = \prod_{i=1}^{t-1} \left[f_k \left(\mathbf{y}_{t-i} | \mathrm{Y}_{t-i-1}\right)\right]^{\alpha^i}. \tag{19}$$

where $f_k \left(\mathbf{y}_{t-i} | \mathrm{Y}_{t-i-1}\right)$ is the predictive density for the model k evaluated at $\mathbf{y}_{t-i}$ with $i = 1, \ldots, t-1$.

The forgetting factor $\alpha \in (0, 1]$ gives a measure of the decay rate of model performance, the forecast performance recorded in i periods in the past has significance equal to α^i. Note that when $\alpha = 0$ all models are equally probable for every t, the weights of the models remain unchanged from the prior $\pi_{0|0,k} = 1/K$. Finally, Koop and Korobilis (2012) refer to the special case $\alpha = 1$ as Bayesian Model Averaging (BMA) which is very popular in macroeconomics and finance, see Koop and Potter (2004).

From the recursive iteration, a prediction for every model k is obtained:

$$\mathbf{y}_t | \mathcal{M}_t = k, \mathrm{Y}_{t-1} \sim \mathcal{N}\left(\mathbf{Z}_t^{(k)} \hat{\boldsymbol{\beta}}_{t|t-1}^{(k)}, \mathbf{H}_t^{(k)} + \mathbf{Z}_t^{(k)} \Sigma_{t|t-1}^{(k)} \mathbf{Z}_t^{(k)\prime}\right). \tag{20}$$

DMA comes from a weighted average of all the models' weights that are the conditional probabilities $P\left(\mathcal{M}_t = k | \mathrm{Y}_{t-1}\right) = \pi_{t|t-1,k}$ computed using the information up to time $t-1$ for $k = 1, 2, \ldots, K$:

$$\mathbf{y}_{DMAt} = \mathrm{E}\left(\mathbf{y}_t | \mathrm{Y}_{t-1}\right) = \sum_{k=1}^{K} \pi_{t|t-1,k} \mathbf{Z}_t^{(k)} \hat{\boldsymbol{\beta}}_{t|t-1}^{(k)}. \tag{21}$$

DMS selects and uses, at time t the model with the highest predictive power to make predictions about the dependent variable. The definition of a prior for $\pi_{0|0,k}$ and $\boldsymbol{\beta}_0^{(k)}$ is essential to implement DMA, DMS and BMA. A non-informative prior is chosen for both the states and the weights. In particular, $\pi_{0|0,k} = 1/K$ and $\boldsymbol{\beta}_0^{(k)} \sim \mathcal{N}\left(\mathbf{0}_{Nqp\times 1}, 4\mathrm{I}_{Nqp}\right) \forall k$. This means that at first, all models are equally likely.

DCC-EWMA

We describe here the DCC-EWMA estimator for the variance-covariance matrix reported in Section 3.2. This approach follows from Johansson et al. (2023) and has the great advantage of setting two forgetting factors, one for variances (κ_1) and one for correlation (κ_2). This gives more flexibility in the estimation and improves the forecast.

The typical DCC model is given by:

$$\mathrm{E}\left(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t'\right) = \mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t, \tag{22}$$

where: $\mathbf{D}_t^2 = \mathrm{diag}[\mathrm{E}(\boldsymbol{\varepsilon}_t \odot \boldsymbol{\varepsilon}_t)]$, $\odot$ denotes element-wise multiplication, and $\mathbf{R}_t$ is the correlation matrix of $\boldsymbol{\varepsilon}_t$. The DCC-EWMA can be estimated as follows:

a) Estimate the $\mathbf{D}_t^2$ using a univariate EWMA with κ_1:

$$\hat{\mathbf{D}}_t^2 = \kappa_1 \hat{\mathbf{D}}_{t-1}^2 + (1-\kappa_1)\text{diag}\left(\hat{\boldsymbol{\varepsilon}}_t \odot \hat{\boldsymbol{\varepsilon}}_t\right).$$

where diag$(\cdot)$ indicates a diagonal matrix in which the diagonal elements are the vector in the argument.

b) Estimate the correlation matrix as follows:

- Standardize the $\hat{\boldsymbol{\varepsilon}}_t$, $\tilde{\boldsymbol{\varepsilon}}_t = \hat{\mathbf{D}}_t^{-1}\hat{\boldsymbol{\varepsilon}}_t$;

- Calculate the EWMA version of the correlation:

$$\tilde{\mathbf{R}}_t = \kappa_2 \tilde{\mathbf{R}}_{t-1} + (1-\kappa_2)\tilde{\boldsymbol{\varepsilon}}_t \tilde{\boldsymbol{\varepsilon}}_t';$$

- Create the $\tilde{\mathbf{S}}_t^2 = \text{diag}(\tilde{r}_{11,t}, \ldots, \tilde{r}_{NN,t})$, where $\tilde{r}_{ij,t}$ is a generic element of the $\tilde{\mathbf{R}}_t$.

- Calculate the $\hat{\mathbf{R}}_t = \tilde{\mathbf{S}}_t^{-1}\tilde{\mathbf{R}}_t\tilde{\mathbf{S}}_t^{-1}$ that has the ones in the main diagonal.

c) Finally the estimation of $\mathbf{H}_t$ is done using the following formula:

$$\hat{\mathbf{H}}_t = \hat{\mathbf{D}}_t \hat{\mathbf{R}}_t \hat{\mathbf{D}}_t.$$

Appendix B: Robustness checks

To check the robustness of our model, we provide two exercises. In the first exercise, we consider a different forecast window that excludes the Covid-19 period: 1972:Q1 to 2019:Q4. The results in tables 6 to 8 show that, as already pointed out in the main text, models without time-varying volatility have worse performance. In particular, MAI with time-varying volatility works well in both the short and long horizons and shows good results in terms of RMSFE, MAFE, and ALPL.

In the second exercise, we used a smaller dataset of seven variables. Tables 9 to 14 suggest that our model still performs well, but with a lower gain.

Table 6: ***Results for GDP, NoCovid Case, 25 Series.***

Point and density forecast results for GDP. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicates better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The model description is reported in Table 2. Values in the table are capped at 3.

Root Mean Square Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.962	0.987	0.981	0.992	0.969	0.987	0.993	0.990	1.063	1.104	1.003	1.134	1.112	1.063	1.393	1.369	1.310	0.956	0.959	1.044
2	1.050	1.072	1.044	1.045	1.053	1.071	1.043	1.051	1.526	1.614	1.009	1.239	1.143	1.177	1.009	1.021	1.251	0.943	0.946	4.009
3	0.984	0.989	0.980	0.988	0.991	0.994	0.984	0.988	2.671	2.894	1.005	1.238	1.122	0.953	0.958	0.955	1.172	0.996	1.013	1.016
4	1.003	1.007	0.990	0.992	1.010	1.010	0.992	0.992	3.000	3.000	1.000	1.313	1.115	1.021	0.986	0.980	1.124	1.010	1.019	0.995
5	1.010	1.013	1.006	1.006	1.016	1.011	1.007	1.008	3.000	3.000	0.994	1.368	1.031	1.028	1.004	1.002	1.066	1.017	1.019	1.015
6	1.002	1.002	0.999	0.999	1.003	1.003	0.997	0.996	3.000	3.000	1.006	1.350	1.056	0.992	0.978	0.978	1.053	1.000	1.000	1.003
7	0.997	0.997	0.997	0.998	0.997	0.998	0.998	0.998	3.000	3.000	1.009	1.406	1.074	1.026	0.983	0.982	1.021	0.996	0.998	1.006
8	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	3.000	3.000	1.003	1.480	1.135	0.983	0.990	0.989	1.012	0.998	0.999	1.011

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.948	0.975	0.970	0.981	0.959	0.973	0.979	0.977	1.009	1.049	1.000	1.154	1.036	0.999	1.433	1.380	1.246	0.932	0.937	1.030
2	1.016	1.031	1.007	1.010	1.021	1.024	1.008	1.017	1.080	1.085	1.011	1.197	1.090	1.109	1.014	1.019	1.202	0.922	0.932	2.633
3	0.953	0.972	0.955	0.967	0.966	0.973	0.962	0.967	1.224	1.252	1.005	1.272	1.035	0.924	0.951	0.938	1.139	0.951	0.968	0.959
4	0.973	0.980	0.957	0.962	0.985	0.986	0.961	0.961	1.567	1.623	1.000	1.352	1.075	0.988	0.980	0.968	1.118	0.995	1.014	0.973
5	1.011	1.018	1.003	1.003	1.015	1.013	1.004	1.004	2.299	2.423	1.007	1.380	1.071	1.018	1.025	1.017	1.111	1.015	1.023	1.013
6	0.992	0.993	0.988	0.987	0.994	0.993	0.986	0.986	3.000	3.000	1.006	1.374	1.083	0.982	0.982	0.982	1.079	0.991	0.996	0.993
7	0.985	0.987	0.984	0.984	0.986	0.987	0.984	0.984	3.000	3.000	1.008	1.448	1.066	1.011	0.994	0.985	1.039	0.980	0.981	0.991
8	0.988	0.987	0.988	0.988	0.988	0.988	0.989	0.989	3.000	3.000	1.004	1.515	1.097	0.976	1.013	1.006	1.016	0.992	0.993	0.997

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	0.232	0.245	0.235	0.237	0.227	0.243	0.229	0.232	-0.000	-0.000	0.000	-0.058	-0.007	-0.024	0.286	0.289
2	0.190	0.206	0.216	0.222	0.186	0.203	0.216	0.220	0.002	0.003	0.000	-0.169	-0.119	-0.002	0.257	0.280
3	0.168	0.182	0.205	0.214	0.166	0.180	0.211	0.215	0.004	0.006	0.001	-0.162	-0.111	0.010	0.251	0.241
4	0.169	0.184	0.204	0.212	0.169	0.183	0.210	0.215	0.004	0.005	0.001	-0.156	-0.106	0.016	0.256	0.249
5	0.179	0.194	0.214	0.222	0.179	0.193	0.221	0.225	0.009	0.010	0.002	-0.145	-0.094	0.028	0.307	0.300
6	0.180	0.194	0.216	0.224	0.179	0.193	0.222	0.227	0.009	0.010	0.002	-0.143	-0.093	0.030	0.302	0.302
7	0.180	0.194	0.217	0.225	0.179	0.193	0.223	0.228	0.010	0.011	0.002	-0.141	-0.091	0.031	0.298	0.276
8	0.181	0.194	0.218	0.226	0.179	0.193	0.224	0.229	0.010	0.011	0.002	-0.140	-0.090	0.033	0.276	0.244

Table 7: ***Results for CPI, NoCovid Case, 25 Series.***

Point and density forecast results for CPI Inflation. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicates better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The description of the model is reported in Table 2. Values in the table are capped at 3.

Root Mean Square Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.950	0.965	0.960	0.960	0.957	0.959	0.962	0.964	1.578	1.642	1.002	1.543	1.214	1.110	0.981	0.984	0.985	0.940	0.947	0.904
2	1.015	1.016	1.010	1.007	1.013	1.014	1.007	1.007	2.544	2.728	1.000	1.610	1.250	1.076	1.020	1.021	1.009	0.987	0.994	3.000
3	0.975	0.976	0.977	0.975	0.973	0.974	0.976	0.977	3.000	3.000	1.008	1.259	1.170	0.986	0.984	0.985	0.993	0.964	0.971	1.113
4	0.965	0.968	0.970	0.969	0.966	0.965	0.969	0.969	3.000	3.000	1.001	1.451	1.075	0.991	0.974	0.977	0.980	0.962	0.964	0.986
5	0.990	0.991	0.991	0.991	0.990	0.990	0.991	0.991	3.000	3.000	1.001	1.405	1.037	0.991	0.997	0.997	0.991	0.997	1.000	0.999
6	1.004	1.004	1.006	1.006	1.004	1.004	1.006	1.006	3.000	3.000	1.001	1.322	1.016	1.005	1.010	1.010	1.018	1.010	1.013	1.007
7	0.989	0.989	0.990	0.990	0.989	0.989	0.990	0.990	3.000	3.000	1.001	1.409	0.987	0.991	0.990	0.990	0.996	0.987	0.987	0.993
8	0.990	0.992	0.990	0.990	0.990	0.990	0.990	0.990	3.000	3.000	0.999	1.455	0.994	0.987	0.987	0.987	0.995	0.984	0.987	0.995

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.930	0.942	0.936	0.934	0.939	0.944	0.939	0.941	1.133	1.152	1.008	1.530	1.047	1.020	0.963	0.973	0.938	0.921	0.928	0.872
2	0.987	0.988	0.986	0.983	0.985	0.986	0.982	0.983	1.292	1.320	1.010	1.628	1.079	1.023	1.001	1.005	0.980	0.967	0.976	3.000
3	0.975	0.976	0.975	0.972	0.973	0.974	0.974	0.975	1.591	1.643	1.016	1.275	1.098	1.001	0.988	0.993	1.009	0.980	0.989	1.079
4	0.956	0.961	0.963	0.962	0.957	0.958	0.963	0.963	2.152	2.266	1.005	1.486	1.036	0.987	0.979	0.984	0.989	0.955	0.961	0.997
5	0.992	0.993	0.991	0.992	0.991	0.991	0.992	0.991	3.000	3.000	0.999	1.565	1.058	0.989	1.010	1.009	1.002	0.997	1.005	1.009
6	1.001	0.998	1.003	1.003	1.002	1.001	1.003	1.003	3.000	3.000	1.003	1.373	1.027	0.998	1.015	1.015	1.024	1.010	1.010	1.001
7	0.984	0.984	0.987	0.987	0.985	0.985	0.986	0.986	3.000	3.000	1.003	1.402	0.994	0.987	0.990	0.989	0.997	0.988	0.991	0.983
8	0.993	0.994	0.993	0.994	0.992	0.992	0.993	0.993	3.000	3.000	0.999	1.617	1.002	0.990	0.984	0.984	0.994	0.981	0.983	0.990

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	-0.035	-0.037	-0.082	-0.085	-0.042	-0.039	-0.087	-0.092	-0.000	-0.000	0.000	0.024	0.029	-0.009	0.004	0.022
2	0.006	0.006	-0.035	-0.038	0.001	0.005	-0.039	-0.043	-0.003	-0.005	-0.001	0.084	0.083	0.052	0.065	0.083
3	0.013	0.013	-0.028	-0.031	0.009	0.013	-0.032	-0.036	-0.007	-0.008	0.000	0.093	0.090	0.060	0.073	0.091
4	0.023	0.022	-0.018	-0.021	0.019	0.023	-0.022	-0.025	-0.008	-0.008	0.000	0.035	0.040	0.071	0.084	0.102
5	0.047	0.048	0.007	0.005	0.045	0.049	0.004	0.001	-0.012	-0.012	0.000	0.101	0.104	0.098	0.092	0.082
6	0.049	0.050	0.009	0.007	0.047	0.051	0.007	0.003	-0.013	-0.013	0.000	0.103	0.090	0.071	0.132	0.132
7	0.051	0.052	0.011	0.009	0.050	0.054	0.009	0.006	-0.013	-0.013	0.000	0.116	0.133	0.103	0.117	0.134
8	0.052	0.053	0.012	0.010	0.051	0.055	0.010	0.007	-0.014	-0.013	0.000	0.137	0.134	0.104	0.118	0.135

Table 8: ***Results for FFR. NoCovid Case. 25 Series.***
Point and density forecast results for Interest Rate. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are reported relative to the benchmark specification (M_{11}) for which the values are equal to 1. RMSFE-MAFE lower (higher) than 1 indicates better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The description of the model is reported in Table 2. Values in the table are capped at 3.

Root Mean Square Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.931	0.944	0.970	0.983	0.944	0.942	0.990	0.994	1.065	1.067	1.004	1.273	1.173	1.137	1.031	1.027	1.125	1.039	1.033	1.203
2	0.942	0.938	0.928	0.922	0.932	0.937	0.923	0.925	1.220	1.231	1.011	1.341	1.240	1.001	0.908	0.906	1.038	0.963	0.973	0.992
3	0.927	0.927	0.923	0.926	0.924	0.924	0.925	0.929	1.887	2.008	1.011	1.239	1.222	0.961	0.925	0.924	1.034	0.932	0.936	1.077
4	0.917	0.916	0.916	0.915	0.916	0.914	0.916	0.916	3.000	3.000	1.001	1.266	1.244	0.944	0.913	0.912	1.000	0.946	0.969	1.005
5	0.981	0.984	0.982	0.982	0.982	0.981	0.983	0.984	3.000	3.000	0.999	1.226	1.256	1.003	0.979	0.980	1.032	1.006	1.012	1.058
6	0.972	0.973	0.973	0.973	0.973	0.973	0.973	0.973	3.000	3.000	0.996	1.340	1.150	0.986	0.967	0.967	1.024	0.990	1.002	1.002
7	0.965	0.966	0.965	0.965	0.964	0.964	0.965	0.965	3.000	3.000	1.009	1.549	1.082	0.963	0.956	0.956	0.956	0.941	0.940	0.973
8	0.962	0.962	0.962	0.962	0.962	0.962	0.963	0.963	3.000	3.000	1.000	1.398	0.985	0.973	0.958	0.959	0.970	0.968	0.976	0.984

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.819	0.829	0.843	0.847	0.819	0.817	0.849	0.851	1.023	1.044	1.022	0.901	1.112	1.042	0.887	0.903	1.107	0.765	0.791	0.870
2	0.863	0.854	0.859	0.852	0.852	0.851	0.855	0.856	1.085	1.089	1.012	1.138	1.219	0.977	0.833	0.832	1.171	0.933	0.956	0.900
3	0.902	0.904	0.877	0.875	0.891	0.891	0.873	0.878	1.206	1.239	1.012	1.170	1.137	0.961	0.863	0.863	1.140	0.918	0.938	0.905
4	0.896	0.893	0.899	0.896	0.886	0.882	0.896	0.896	1.497	1.566	0.994	1.212	1.163	0.969	0.886	0.886	1.087	0.937	0.963	0.904
5	0.984	0.988	0.990	0.989	0.976	0.973	0.990	0.990	2.124	2.240	0.991	1.376	1.219	1.038	0.974	0.975	1.087	1.012	1.028	1.038
6	0.961	0.961	0.966	0.965	0.956	0.954	0.964	0.963	3.000	3.000	1.009	1.465	1.109	1.002	0.949	0.949	1.048	0.983	1.011	0.980
7	0.966	0.970	0.970	0.969	0.963	0.962	0.969	0.968	3.000	3.000	1.006	1.633	1.096	0.989	0.956	0.955	0.991	0.961	0.971	0.952
8	0.973	0.977	0.976	0.975	0.971	0.972	0.975	0.975	3.000	3.000	1.000	1.558	1.014	1.012	0.972	0.973	0.997	0.988	1.005	0.985

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	0.319	0.366	0.329	0.352	0.339	0.375	0.348	0.360	-0.002	-0.002	0.003	0.076	0.131	0.001	0.327	0.338
2	0.235	0.286	0.270	0.302	0.270	0.306	0.310	0.322	-0.001	0.001	0.001	0.092	0.148	-0.016	0.291	0.283
3	0.201	0.249	0.242	0.274	0.235	0.269	0.282	0.292	0.004	0.002	-0.001	0.089	0.145	-0.004	0.293	0.302
4	0.194	0.242	0.239	0.271	0.231	0.263	0.281	0.291	-0.001	-0.004	0.002	0.086	0.141	0.024	0.314	0.311
5	0.182	0.232	0.229	0.262	0.220	0.254	0.271	0.282	0.002	0.000	0.003	0.076	0.131	0.038	0.265	0.285
6	0.187	0.238	0.237	0.270	0.226	0.259	0.279	0.290	0.002	-0.003	0.000	0.080	0.136	0.051	0.303	0.303
7	0.184	0.238	0.234	0.268	0.224	0.258	0.277	0.287	0.001	-0.001	0.002	0.074	0.132	0.056	0.303	0.317
8	0.185	0.237	0.234	0.267	0.224	0.258	0.276	0.287	0.003	0.001	0.002	0.072	0.126	0.059	0.329	0.319

Table 9: **Results for GDP, Covid Case, 7 Series.**

Point and density forecast results for GDP. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicate better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The model description is reported in Table 2. Values in the table are capped at 3.

Root Mean Squared Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.993	1.120	1.092	1.140	0.976	1.046	1.064	1.135	1.200	1.208	1.007	1.377	1.100	1.011	1.008	1.008	1.147	1.032	1.012	1.012
2	1.008	1.070	1.038	1.039	0.995	1.028	1.016	1.040	1.199	1.212	0.998	1.396	1.015	1.040	1.015	1.009	1.058	1.022	1.014	1.014
3	1.001	1.026	1.017	1.014	0.987	0.989	0.999	1.003	1.300	1.305	1.001	1.367	0.990	0.995	0.979	0.979	1.030	1.010	1.016	1.016
4	1.009	1.024	1.020	1.013	0.998	1.002	1.007	1.003	1.499	1.495	1.003	1.393	0.999	1.007	0.969	0.974	1.000	1.004	1.014	1.014
5	0.998	1.008	1.003	0.998	0.998	1.000	1.000	0.997	1.672	1.646	1.005	1.363	0.995	1.002	0.989	0.985	0.997	0.997	0.996	0.996
6	1.003	1.013	1.006	1.004	1.001	1.004	1.001	1.001	2.159	2.058	1.002	1.422	0.995	0.998	0.991	0.991	0.994	0.999	0.999	0.999
7	1.002	1.017	1.002	1.004	0.997	0.999	0.998	0.999	2.584	2.219	1.001	1.405	0.997	1.005	0.996	0.997	1.000	0.999	0.998	0.998
8	0.998	1.008	0.996	0.999	0.999	0.998	0.999	0.998	3.000	2.810	1.001	1.404	0.995	0.998	0.997	0.997	0.996	0.998	0.995	0.995

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.983	1.038	1.018	1.045	0.976	0.976	1.046	1.046	1.071	1.076	1.010	1.308	1.118	1.015	1.121	1.124	1.212	1.011	1.006	1.006
2	1.022	1.082	1.055	1.057	0.995	0.995	1.046	1.046	1.095	1.117	1.013	1.370	1.027	1.068	1.016	1.013	1.063	1.000	1.000	1.000
3	0.992	1.025	1.002	0.995	0.970	0.970	0.987	0.987	1.115	1.125	1.010	1.429	0.972	0.981	0.949	0.948	1.031	1.000	1.007	1.007
4	0.995	1.006	1.009	0.996	0.982	0.982	0.989	0.989	1.157	1.151	1.005	1.463	0.977	0.996	0.939	0.938	0.990	1.001	1.027	1.027
5	1.007	1.028	1.016	1.002	0.998	0.998	1.005	1.005	1.191	1.187	1.009	1.439	0.989	1.007	0.983	0.982	0.987	0.992	0.994	0.994
6	1.007	1.022	1.010	1.003	1.000	1.000	0.997	0.997	1.264	1.250	1.006	1.469	0.991	1.000	0.978	0.978	0.987	0.992	0.997	0.997
7	1.011	1.040	1.008	1.013	0.997	0.997	0.999	0.999	1.311	1.250	1.003	1.530	0.997	1.019	1.004	1.002	1.001	0.993	0.990	0.990
8	0.994	1.014	0.988	0.996	0.995	0.995	0.995	0.995	1.539	1.362	1.002	1.559	0.989	0.996	1.011	1.010	0.989	0.995	0.992	0.992

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	0.198	0.278	0.193	0.269	0.191	0.268	0.196	0.280	-0.000	-0.000	0.000	0.343	0.339	-0.023	0.247	0.265
2	0.142	0.217	0.146	0.213	0.148	0.216	0.155	0.219	0.005	0.009	-0.003	0.315	0.313	0.003	0.213	0.291
3	0.133	0.211	0.143	0.209	0.144	0.216	0.159	0.223	0.007	0.014	-0.004	0.299	0.295	0.019	0.268	0.280
4	0.125	0.204	0.135	0.200	0.141	0.213	0.156	0.217	0.005	0.013	-0.004	0.297	0.293	0.022	0.271	0.299
5	0.123	0.204	0.136	0.200	0.139	0.213	0.158	0.220	0.007	0.016	-0.005	0.302	0.297	0.029	0.312	0.314
6	0.121	0.201	0.135	0.199	0.138	0.213	0.158	0.220	0.007	0.016	-0.005	0.302	0.298	0.031	0.310	0.310
7	0.120	0.200	0.135	0.198	0.138	0.213	0.159	0.221	0.007	0.016	-0.005	0.303	0.298	0.031	0.311	0.319
8	0.119	0.200	0.134	0.198	0.137	0.212	0.159	0.221	0.006	0.015	-0.005	0.303	0.299	0.032	0.301	0.313

Table 10: **Results for CPI, Covid Case, 7 Series.**
Point and density forecast results for CPI Inflation. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicate better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The description of the model is reported in Table 2. Values in the table are capped at 3.

Root Mean Squared Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.971	0.989	1.006	1.065	0.977	1.014	1.016	1.041	1.014	1.026	1.013	1.594	0.984	1.092	1.034	1.033	0.945	0.967	0.986	0.986
2	1.006	1.036	1.014	1.007	1.000	1.038	0.992	1.009	1.031	1.031	1.007	1.577	1.057	1.026	1.032	1.031	0.984	0.991	0.991	0.991
3	1.003	1.009	1.004	1.008	0.994	1.021	1.006	1.012	1.058	1.097	0.996	1.275	1.001	1.027	1.042	1.034	0.998	0.991	1.003	1.003
4	1.012	0.999	1.015	1.024	1.016	1.030	1.027	1.028	0.995	1.012	1.010	1.510	0.990	1.026	1.016	1.013	1.000	1.000	1.009	1.009
5	1.008	1.012	1.007	1.011	1.002	1.008	1.000	1.005	1.026	1.045	1.001	1.395	1.001	1.024	1.003	1.001	1.016	1.006	1.010	1.010
6	1.001	0.997	1.005	1.006	1.003	1.005	1.005	1.009	1.032	1.033	0.998	1.342	1.003	1.014	1.003	1.003	1.006	1.003	1.004	1.004
7	1.002	0.999	1.001	1.001	0.997	1.000	0.999	1.000	1.030	1.046	1.000	1.412	0.999	1.006	0.993	0.994	0.996	0.998	1.000	1.000
8	1.004	1.004	1.001	1.001	1.001	1.004	0.998	0.999	1.046	1.018	1.000	1.474	0.999	1.004	0.990	0.990	0.998	0.996	0.998	0.998

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	1.017	1.037	1.030	1.061	1.014	1.014	1.051	1.051	1.031	1.046	1.023	1.669	1.043	1.103	1.102	1.103	0.977	0.986	1.003	1.003
2	1.016	1.028	1.012	1.011	1.009	1.009	1.015	1.015	1.021	1.004	1.005	1.626	1.078	1.022	1.027	1.027	0.983	0.997	1.002	1.002
3	0.997	1.001	0.997	1.004	0.994	0.994	1.013	1.013	1.042	1.069	0.995	1.294	1.011	1.040	1.039	1.037	1.016	0.997	1.014	1.014
4	1.026	1.009	1.025	1.032	1.025	1.025	1.029	1.029	0.992	1.015	1.009	1.578	0.996	1.042	1.032	1.030	1.015	1.012	1.027	1.027
5	1.010	1.017	1.007	1.021	1.003	1.003	1.010	1.010	1.033	1.047	1.002	1.545	0.998	1.031	1.006	1.006	1.019	1.006	1.016	1.016
6	1.007	1.002	1.007	1.011	1.008	1.008	1.011	1.011	1.044	1.045	1.000	1.405	1.004	1.030	1.007	1.007	1.007	1.004	1.005	1.005
7	1.002	0.994	1.000	1.001	0.996	0.996	0.995	0.995	1.033	1.045	1.001	1.423	0.995	1.009	0.988	0.989	0.988	0.998	1.005	1.005
8	1.009	1.012	1.008	1.007	1.005	1.005	1.006	1.006	1.046	1.027	0.999	1.620	1.004	1.015	0.991	0.991	0.996	0.997	0.999	0.999

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	-0,030	-0,017	-0,041	-0,039	-0,025	-0,014	-0,041	-0,041	-0,000	-0,000	-0,000	0,177	0,178	-0,009	-0,017	-0,003
2	-0,059	-0,041	-0,055	-0,055	-0,044	-0,031	-0,050	-0,054	0,002	0,008	-0,003	0,158	0,157	0,021	0,013	0,026
3	-0,056	-0,034	-0,049	-0,047	-0,038	-0,023	-0,042	-0,044	0,002	0,009	-0,005	0,167	0,166	0,035	0,027	0,041
4	-0,054	-0,029	-0,042	-0,039	-0,036	-0,019	-0,035	-0,036	0,002	0,012	-0,006	0,173	0,172	0,045	0,037	0,051
5	-0,060	-0,033	-0,045	-0,043	-0,040	-0,021	-0,037	-0,038	0,004	0,016	-0,007	0,182	0,181	0,056	0,048	0,062
6	-0,061	-0,034	-0,045	-0,044	-0,040	-0,021	-0,036	-0,038	0,003	0,016	-0,007	0,184	0,182	0,058	0,064	0,064
7	-0,061	-0,034	-0,045	-0,044	-0,040	-0,021	-0,036	-0,037	0,003	0,016	-0,007	0,185	0,183	0,059	0,051	0,065
8	-0,062	-0,034	-0,045	-0,044	-0,040	-0,021	-0,035	-0,037	0,002	0,016	-0,007	0,186	0,184	0,060	0,052	0,066

Table 11: **Results for FFR, Covid Case, 7 Series.**

Point and density forecast results for Interest Rate. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicate better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The description of the model is reported in Table 2. Values in the table are capped at 3 .

Root Mean Square Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.911	0.984	0.939	0.941	0.948	1.038	0.943	0.961	0.989	1.029	1.011	1.174	1.019	0.952	0.987	0.987	1.028	1.010	0.997	0.997
2	1.003	1.051	1.013	1.029	0.986	1.001	1.002	1.021	1.019	1.069	1.015	1.421	1.092	1.013	0.966	0.968	1.078	1.047	1.029	1.029
3	0.998	1.027	1.018	1.032	0.979	1.009	0.994	1.027	1.075	1.083	1.001	1.328	0.998	0.996	0.995	0.993	1.071	1.007	1.011	1.011
4	1.034	1.037	1.026	1.015	1.009	1.017	0.999	1.000	1.167	1.237	1.002	1.350	0.997	0.991	0.956	0.954	1.049	1.008	1.029	1.029
5	1.009	1.026	1.012	1.013	1.003	1.011	1.003	0.999	1.151	1.181	1.002	1.257	1.014	1.015	1.003	1.003	1.030	1.018	1.016	1.016
6	1.010	1.005	1.010	1.009	1.006	0.991	1.005	1.006	1.268	1.263	1.006	1.373	1.001	1.037	0.993	0.993	1.016	1.013	1.018	1.018
7	1.007	1.023	1.010	1.014	1.004	1.011	1.007	1.005	1.376	1.307	1.001	1.597	1.005	1.026	0.990	0.989	0.989	0.985	0.984	0.984
8	1.005	1.010	1.001	1.002	1.001	0.998	0.998	0.999	1.765	1.523	1.007	1.437	0.995	1.009	0.991	0.990	1.004	1.001	1.007	1.007

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.866	0.892	0.878	0.878	0.887	0.887	0.899	0.899	0.940	1.000	1.052	0.905	0.978	0.872	0.959	0.956	0.961	0.789	0.798	0.798
2	0.988	1.020	0.988	1.019	0.967	0.967	0.999	0.999	0.988	1.011	1.025	1.234	1.115	0.996	0.967	0.960	1.182	1.059	1.054	1.054
3	1.011	1.062	1.013	1.051	0.981	0.981	1.020	1.020	1.031	1.042	1.030	1.275	0.987	0.950	0.993	0.985	1.163	1.040	1.053	1.053
4	1.031	1.052	1.035	1.049	1.004	1.004	1.014	1.014	1.088	1.116	1.010	1.324	0.998	0.989	1.009	0.999	1.128	1.028	1.058	1.058
5	1.018	1.044	1.030	1.036	1.005	1.005	1.009	1.009	1.105	1.128	1.006	1.418	1.011	1.009	1.064	1.056	1.052	1.040	1.038	1.038
6	1.022	1.033	1.026	1.031	1.003	1.003	1.006	1.006	1.132	1.135	1.006	1.521	0.997	1.011	1.029	1.029	1.032	1.035	1.045	1.045
7	1.017	1.041	1.017	1.031	1.003	1.003	1.003	1.003	1.123	1.106	1.000	1.680	1.002	1.023	1.037	1.030	1.003	0.994	0.997	0.997
8	1.015	1.025	1.015	1.014	1.001	1.001	0.998	0.998	1.201	1.154	1.005	1.584	0.994	1.016	1.048	1.042	0.996	1.004	1.012	1.012

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	0.254	0.369	0.261	0.366	0.241	0.349	0.254	0.353	0.007	0.004	-0.009	0.129	0.130	0.029	0.339	0.328
2	0.187	0.274	0.198	0.276	0.196	0.294	0.214	0.305	0.003	0.010	-0.000	0.118	0.118	-0.012	0.264	0.212
3	0.171	0.254	0.182	0.254	0.177	0.266	0.200	0.285	0.006	0.011	-0.004	0.112	0.110	0.003	0.293	0.286
4	0.154	0.234	0.167	0.236	0.171	0.266	0.193	0.279	-0.001	0.006	-0.002	0.111	0.111	0.021	0.304	0.311
5	0.149	0.225	0.160	0.229	0.169	0.257	0.189	0.272	-0.002	0.005	-0.003	0.099	0.098	0.042	0.313	0.323
6	0.150	0.228	0.162	0.230	0.170	0.269	0.191	0.275	0.001	0.007	-0.004	0.103	0.102	0.045	0.333	0.333
7	0.150	0.233	0.161	0.233	0.170	0.266	0.191	0.274	0.006	0.016	-0.004	0.103	0.102	0.049	0.342	0.331
8	0.151	0.235	0.164	0.236	0.168	0.263	0.191	0.279	0.006	0.015	-0.004	0.097	0.096	0.049	0.323	0.321

Table 12: ***Results for GDP, 7 series, NoCovid case.***

Point and density forecast results for GDP. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicates better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The model description is reported in Table 2. Values in the table are capped at 3.

Root Mean Squared Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.935	0.948	0.977	0.986	0.939	0.942	0.979	0.989	1.007	1.009	1.013	1.223	1.046	1.015	1.165	1.163	1.312	0.999	1.005	1.005
2	0.988	1.033	1.014	1.032	0.984	1.022	1.004	1.028	0.989	1.008	1.003	1.285	0.982	1.089	1.032	1.028	1.134	0.958	0.961	0.961
3	0.981	0.991	0.995	0.997	0.966	0.962	0.977	0.970	1.005	1.021	1.012	1.261	0.958	0.979	0.962	0.959	1.044	1.006	1.017	1.017
4	0.996	1.008	0.998	0.989	0.986	0.985	0.991	0.965	1.022	1.030	1.008	1.313	0.985	1.001	0.967	0.967	1.003	1.000	1.011	1.011
5	0.992	0.998	1.001	0.994	0.989	0.993	0.995	0.990	1.018	1.025	1.011	1.348	0.988	1.003	0.988	0.988	0.989	0.992	0.991	0.991
6	1.001	1.024	1.003	1.000	0.995	0.999	0.996	0.995	1.012	1.027	1.004	1.344	0.988	0.988	0.975	0.975	0.986	0.997	0.995	0.995
7	1.005	1.029	1.006	1.012	0.998	1.002	0.998	0.999	0.995	0.995	1.003	1.409	0.993	1.013	0.991	0.991	0.993	0.996	0.995	0.995
8	0.993	1.002	0.993	0.998	0.998	0.998	0.998	0.997	0.997	1.008	1.003	1.475	0.989	0.995	0.996	0.996	0.989	0.991	0.987	0.987

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.984	1.011	1.005	1.021	0.980	0.980	1.024	1.024	1.015	1.015	1.013	1.273	1.086	1.021	1.168	1.166	1.222	1.014	1.026	1.026
2	1.018	1.061	1.040	1.043	0.995	0.995	1.029	1.029	0.993	1.007	1.017	1.282	1.010	1.080	1.029	1.030	1.106	0.967	0.979	0.979
3	0.978	0.993	0.980	0.974	0.962	0.962	0.963	0.963	1.009	1.016	1.018	1.342	0.951	0.978	0.966	0.963	1.028	0.994	0.998	0.998
4	0.988	0.993	0.994	0.979	0.978	0.978	0.968	0.968	1.009	1.001	1.007	1.382	0.975	0.994	0.957	0.955	1.000	0.996	1.019	1.019
5	0.997	1.009	1.004	0.992	0.986	0.986	0.995	0.995	1.010	1.008	1.012	1.359	0.982	1.006	0.993	0.992	0.972	0.985	0.982	0.982
6	1.002	1.023	1.003	0.995	0.992	0.992	0.989	0.989	1.022	1.028	1.008	1.379	0.987	0.992	0.980	0.980	0.984	0.989	0.990	0.990
7	1.013	1.039	1.012	1.018	0.999	0.999	0.999	0.999	0.994	0.987	1.004	1.471	0.994	1.020	1.005	1.004	0.997	0.990	0.987	0.987
8	0.994	1.007	0.990	0.999	0.997	0.997	0.998	0.998	0.999	1.010	1.004	1.525	0.988	0.997	1.019	1.019	0.988	0.993	0.991	0.991

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	0.202	0.279	0.198	0.272	0.194	0.272	0.201	0.283	-0.000	0.000	0.000	0.342	0.338	-0.024	0.296	0.275
2	0.145	0.218	0.150	0.216	0.152	0.219	0.159	0.221	0.009	0.013	-0.003	0.314	0.312	0.003	0.322	0.301
3	0.137	0.214	0.148	0.213	0.149	0.221	0.165	0.227	0.012	0.019	-0.004	0.298	0.294	0.018	0.238	0.217
4	0.129	0.206	0.141	0.204	0.145	0.218	0.161	0.221	0.011	0.018	-0.004	0.296	0.292	0.021	0.291	0.280
5	0.127	0.206	0.142	0.204	0.144	0.219	0.164	0.224	0.014	0.022	-0.005	0.301	0.297	0.029	0.278	0.228
6	0.125	0.203	0.141	0.203	0.143	0.218	0.164	0.225	0.014	0.023	-0.005	0.302	0.298	0.030	0.253	0.253
7	0.124	0.203	0.141	0.202	0.143	0.218	0.165	0.225	0.014	0.023	-0.005	0.302	0.298	0.031	0.251	0.230
8	0.124	0.202	0.141	0.202	0.142	0.218	0.165	0.225	0.014	0.023	-0.005	0.303	0.298	0.032	0.251	0.230

Table 13: **Results for CPI, 7 series, NoCovid case**

Point and density forecast results for CPI Inflation. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicates better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The description of the model is reported in Table 2. Values in the table are capped at 3.

Root Mean Squared Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	1	1.031	0.992	1.002	0.974	1.022	0.972	0.973	1.017	1.017	1.012	1.588	0.987	1.105	1.046	1.049	0.9766	0.9654	0.9755	0.9755
2	1	1.023	1.009	1.007	1.007	1.044	0.994	1.006	1.035	1.027	1.001	1.615	1.058	1.026	1.050	1.047	0.9725	0.9942	0.9906	0.9906
3	1	1.016	1.010	1.013	1.003	1.025	1.005	1.017	1.057	1.098	0.995	1.304	1.001	1.027	1.061	1.053	1.0083	0.9999	1.0166	1.0166
4	1	1.006	1.008	1.000	1.004	1.014	1.005	1.004	1.006	1.025	1.012	1.496	0.991	1.038	1.033	1.030	1.0129	0.9931	0.9963	0.9963
5	1	1.010	1.008	1.014	1.002	1.007	1.002	1.006	1.029	1.053	1.000	1.423	1.003	1.027	1.013	1.012	1.0153	1.0079	1.0109	1.0109
6	1	0.998	1.004	1.004	1.004	1.004	1.005	1.008	1.020	1.023	0.997	1.322	1.006	1.019	1.014	1.014	1.009	1.0051	1.007	1.007
7	1	0.997	1.001	1.001	0.997	0.999	0.998	0.999	1.034	1.051	1.001	1.423	1.000	1.007	1.000	1.000	0.9957	0.9975	0.9998	0.9998
8	1	1.005	1.002	1.001	1.002	1.004	0.999	0.999	1.012	1.013	1.000	1.469	1.000	1.006	0.996	0.996	0.9999	0.996	0.9984	0.9984

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	1	1.063	1.023	1.035	1.012	1.012	1.021	1.021	1.023	1.035	1.021	1.655	1.040	1.111	1.117	1.127	1.001	0.9939	1.0041	1.0041
2	1	1.029	1.010	1.010	1.018	1.018	1.014	1.014	1.023	0.996	1.002	1.678	1.085	1.025	1.053	1.051	0.970	1.001	1.0001	1.0001
3	1	1.004	1.003	1.005	1.002	1.002	1.016	1.016	1.042	1.067	0.992	1.310	1.014	1.040	1.064	1.062	1.030	1.0106	1.0335	1.0335
4	1	1.014	1.017	1.015	1.014	1.014	1.010	1.010	1.009	1.034	1.011	1.557	0.994	1.054	1.050	1.048	1.026	1.0031	1.0146	1.0146
5	1	1.008	1.007	1.019	1.001	1.001	1.007	1.007	1.033	1.050	1.002	1.578	0.998	1.030	1.020	1.019	1.011	1.005	1.0114	1.0114
6	1	1.005	1.009	1.012	1.010	1.010	1.013	1.013	1.029	1.030	1.000	1.379	1.009	1.039	1.025	1.025	1.011	1.0079	1.0112	1.0112
7	1	0.994	0.998	0.998	0.996	0.996	0.992	0.992	1.036	1.043	1.002	1.413	0.995	1.009	0.996	0.996	0.989	0.9967	1.005	1.005
8	1	1.012	1.007	1.006	1.005	1.005	1.004	1.004	1.019	1.016	0.999	1.636	1.004	1.018	0.995	0.995	0.997	0.9961	0.9977	0.9977

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	-0.022	-0.010	-0.033	-0.031	-0.018	-0.008	-0.033	-0.034	-0.000	-0.000	-0.000	0.176	0.177	-0.009	-0.011	0.004
2	-0.053	-0.035	-0.049	-0.049	-0.038	-0.025	-0.044	-0.047	0.004	0.010	-0.004	0.159	0.157	0.021	0.019	0.035
3	-0.050	-0.027	-0.042	-0.040	-0.032	-0.017	-0.034	-0.037	0.005	0.012	-0.005	0.167	0.166	0.036	0.033	0.049
4	-0.047	-0.021	-0.035	-0.033	-0.029	-0.011	-0.027	-0.029	0.005	0.016	-0.006	0.174	0.172	0.046	0.044	0.059
5	-0.053	-0.026	-0.038	-0.036	-0.033	-0.014	-0.029	-0.030	0.007	0.020	-0.007	0.184	0.182	0.058	0.056	0.071
6	-0.054	-0.027	-0.038	-0.037	-0.032	-0.014	-0.028	-0.030	0.007	0.020	-0.008	0.185	0.184	0.060	0.073	0.073
7	-0.055	-0.027	-0.038	-0.037	-0.032	-0.014	-0.028	-0.030	0.007	0.021	-0.008	0.186	0.185	0.061	0.059	0.074
8	-0.055	-0.027	-0.038	-0.037	-0.032	-0.013	-0.028	-0.029	0.006	0.020	-0.008	0.187	0.186	0.062	0.060	0.075

Table 14: **Results for FFR, 7 series, NoCovid case**

Point and density forecast results for Interest Rate. Root Mean Squared Forecast Error (RMSFE) upper panel. Median Absolute Forecast Error (MAFE). middle panel. Average Log Predictive Likelihood (ALPL) bottom panel. The results are relative to the benchmark specification ($\mathcal{M}_{11}$) whose values, all equal to 1, are not reported in the table. RMSFE-MAFE lower (higher) than 1 indicates better (worse) performance than the benchmark. ALPL higher (lower) than 1 indicates better (worse) performance. The description of the model is reported in Table 2. Values in the table are capped at 3.

Root Mean Squared Forecast Error (RMSFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.908	0.933	0.910	0.911	0.938	1.001	0.926	0.935	0.971	1.011	1.011	1.192	0.957	0.961	0.990	0.990	1.013	0.997	0.992	0.992
2	1.001	1.024	1.001	1.006	0.987	0.987	0.991	0.997	1.008	1.059	1.015	1.436	1.041	1.016	0.971	0.973	1.068	1.029	1.022	1.022
3	0.996	1.015	1.004	1.016	0.981	1.000	0.988	1.010	1.009	1.018	1.001	1.335	0.993	1.001	1.000	0.998	1.070	0.997	0.996	0.996
4	1.032	1.028	1.016	1.006	1.007	1.007	0.993	0.990	1.097	1.173	1.002	1.348	0.995	0.992	0.960	0.959	1.044	1.002	1.017	1.017
5	1.008	1.027	1.009	1.010	1.002	1.004	1.001	0.996	1.027	1.066	1.002	1.255	1.015	1.015	1.010	1.009	1.029	1.015	1.013	1.013
6	1.010	1.006	1.010	1.009	1.007	0.988	1.005	1.006	1.037	1.052	1.006	1.380	1.001	1.040	1.008	1.008	1.017	1.014	1.020	1.020
7	1.006	1.024	1.009	1.014	1.003	1.008	1.006	1.004	0.973	0.969	1.001	1.615	1.004	1.027	1.006	1.005	0.988	0.985	0.982	0.982
8	1.005	1.011	1.001	1.002	1.000	0.994	0.997	0.999	1.013	1.036	1.007	1.446	0.995	1.011	1.006	1.006	1.005	1.002	1.007	1.007

Mean Absolute Forecast Error (MAFE)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{13}$	$\mathcal{M}_{14}$	$\mathcal{M}_{15}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$	$\mathcal{M}_{21}$
1	0.862	0.864	0.855	0.856	0.880	0.880	0.882	0.882	0.924	0.980	1.052	0.925	0.947	0.874	0.960	0.957	0.935	0.767	0.788	0.788
2	0.988	0.999	0.982	1.005	0.969	0.969	0.984	0.984	0.980	0.999	1.029	1.265	1.087	0.999	0.986	0.978	1.166	1.034	1.049	1.049
3	1.015	1.056	1.006	1.044	0.985	0.985	1.010	1.010	1.000	1.012	1.033	1.289	0.988	0.963	1.010	1.002	1.165	1.026	1.038	1.038
4	1.026	1.040	1.021	1.038	0.998	0.998	0.998	0.998	1.041	1.069	1.010	1.311	0.997	0.992	1.023	1.015	1.119	1.016	1.036	1.036
5	1.013	1.044	1.020	1.027	0.997	0.997	0.997	0.997	1.037	1.063	1.006	1.399	1.009	1.009	1.080	1.072	1.046	1.027	1.025	1.025
6	1.025	1.039	1.026	1.033	1.002	1.002	1.005	1.005	1.038	1.046	1.006	1.525	1.001	1.017	1.069	1.069	1.034	1.033	1.048	1.048
7	1.016	1.043	1.014	1.030	0.998	0.998	0.999	0.999	0.988	0.984	1.000	1.693	1.000	1.024	1.078	1.072	1.000	0.991	0.992	0.992
8	1.015	1.031	1.014	1.014	0.998	0.998	0.997	0.997	1.009	1.008	1.005	1.590	0.993	1.018	1.082	1.076	0.998	1.003	1.012	1.012

Average Log Predictive Likelihood (ALPL)

H	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$	$\mathcal{M}_5$	$\mathcal{M}_6$	$\mathcal{M}_7$	$\mathcal{M}_8$	$\mathcal{M}_9$	$\mathcal{M}_{10}$	$\mathcal{M}_{12}$	$\mathcal{M}_{16}$	$\mathcal{M}_{17}$	$\mathcal{M}_{18}$	$\mathcal{M}_{19}$	$\mathcal{M}_{20}$
1	0.231	0.351	0.237	0.343	0.219	0.333	0.230	0.330	0.009	0.006	-0.010	0.133	0.134	0.037	0.322	0.314
2	0.162	0.260	0.171	0.256	0.175	0.286	0.190	0.281	0.003	0.011	-0.001	0.121	0.120	-0.011	0.221	0.237
3	0.148	0.240	0.157	0.235	0.162	0.256	0.179	0.262	0.008	0.014	-0.005	0.116	0.114	0.002	0.237	0.245
4	0.133	0.221	0.146	0.218	0.156	0.258	0.175	0.259	0.004	0.012	-0.003	0.118	0.118	0.024	0.327	0.323
5	0.129	0.211	0.141	0.210	0.155	0.248	0.172	0.252	0.004	0.010	-0.003	0.102	0.101	0.044	0.323	0.322
6	0.131	0.213	0.146	0.213	0.156	0.258	0.175	0.254	0.006	0.012	-0.005	0.101	0.100	0.045	0.327	0.327
7	0.135	0.219	0.149	0.219	0.159	0.258	0.178	0.258	0.012	0.023	-0.004	0.100	0.099	0.050	0.334	0.333
8	0.134	0.217	0.149	0.218	0.158	0.255	0.177	0.259	0.011	0.020	-0.004	0.094	0.093	0.050	0.335	0.332

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