

Optimal Banking Regulation with Endogenous Liquidity Provision

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very, very preliminary, please do not circulate

Abstract

In a money-search model where deposits are used as means-of-payments, banks have expertise to obtain higher returns from assets with a cost and an economy of scale but are subject to limited commitment and moral hazard. They can pledge a proportion of asset holdings to issue deposits. Optimal regulation trades off efficiency in asset-management and liquidity service banks provide. An optimal charter system restricts banking licence to create profits for banks to sustain a leverage ratio above the laissez-faire level to improve liquidity. A moral hazard problem for banks is also considered where banks may choose to gamble with the assets to obtain a stochastic higher private returns but with lower overall expected returns and we characterize the optimal capital requirement. As moral hazard becomes more serious, optimal regulation allows banks to be larger and have higher profits to compensate for stricter capital requirement due to moral hazard. However, we also show that when such capital requirement becomes too restrictive, it is in fact optimal to allow banks to gamble.

Keywords: Capital requirement, banking, moral hazard, deposit insurance

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1 Introduction

The serious interruption of the real economy from the Global Financial Crisis of 2008 has given rise to a renowned interest in understanding the role of financial intermediaries and how to regulate them. Two particular issues have surfaced both in the mass media and in the policy debate: first, bankers seem to make unjustified profits;¹ second, the banking sector seem to be too concentrated in few big banks. These issues surfaced to the public domain partly because the banking sector has been under two government protections: the deposit insurance that allows them to raise more deposits, and government bail-outs to many banking failures. These privileges seem even more unreasonable as it has been difficult to persecute any potential fraud in the sector.²

We contribute to this debate by studying optimal banking regulations and their interactions with bank sizes and profits. Our framework has two main features that are suitable for this study related to the balance sheet of a bank. On the asset side, banks have an environment where banks play a useful role as financial intermediaries to improve social welfare. In particular, aspect that may result in externality which requires regulation is the liquidity role banks' liabilities serve;³ in most advanced economies, the majority of money supply consists of bank deposits. This role, which is mainly concerned with bank liabilities, motivates various regulations that promote stability, as bank failures would affect not only banks' shareholders but also the welfare of the general public who rely on banks' liquidity services.

This liquidity service is provided by an asset transformation process: while banks issue

¹In a comment about the Dodd-Frank reform, New Yorker article (“Banking’s New Normal,” 2016 issue) has argued that “Bankers still make absurd amounts of money.”

²For a popular view on difficulty in such persecutions, see New Yorker article, “Why Corrupt Bankers Avoid Jail,” 2017 issue.

³See, for example, the Controversies section in *Economic Journal*, issue 106, May 1996, where all articles mentioned that banking is special because they produce “money,” or assets that can be used as means-of-payments.

deposits on the liability side, they also hold various assets to back those deposits. This process, as the financial crisis reveals, involves many credit market frictions from the banks, such as uncertainty to bank returns and limited commitment. The feature that banks supply liquidity and they are subject to frictions has important macroeconomic consequences, and thus, implications on policy and regulations.

In this paper, we propose a model of financial intermediaries with endogenous liquidity provision. We do this by introducing *banks* into a standard monetary model à la Lagos and Wright (2005) to maintain tractability. On the asset side, banks are the only agents with the necessary expertise to manage/monitor loans (modeled as one-period Lucas trees) to receive dividends. There is economy of scale in the sector by way of a fixed cost of operation, which determines the efficient level of asset holdings and hence the efficient size of banks. On the liability side, banks may issue deposits to finance their asset holdings, and, under the usual frictions that render means-of-payments essential (lack of commitment and monitoring) from the depositor side, this can generate a higher profits to banks by doing so. We consider two main frictions in the banking sector. First, banks cannot fully commit to honor their future obligations; instead, they could only credibly pledge a fraction of their assets that can be seized by the court upon bankruptcy. This friction constraints the amount of liquidity banks can provide and may prevent the first-best level of consumption for the depositors to be achieved. Second, the banks' efforts in managing the assets may not be observable and this moral hazard issue may hinder the liquidity role of the banks.

We first consider the limited commitment of the banks but no moral hazard. When banks can only make static contracts, the amount of deposits a bank can issue is constrained by limited pledgeability of assets through market discipline; no one would deposit in a bank unless it can credibly repay. Under free entry of banks, bank sizes are determined by a zero-profit condition that balances the variable cost of asset management and the fixed cost of entry, which coincide with the efficient level of asset holdings as far as the

asset-management is concerned for the economy of scale. However, unless the pledgeability constraint is slack, depositors cannot achieve first-best level of consumption due to lack of liquidity and asset pricing exhibits liquidity premium. This pledgeability constraint also implies a capital requirement imposed by the market: the bank will not repay any deposit beyond what is pledgeable because of limited commitment and hence the difference has to be financed by bank capital.

Against this free market arrangement, we show that a charter system with a banking regulator can improve social welfare. Under the charter system, the regulator can shut down a bank when it does not honor its obligation and hence allows for a dynamic incentive to relax the pledgeability constraint. For this dynamic incentive to be effective, however, it is necessary to limit the number of charters relative to the efficient number under free entry and to allow banks to earn economic profits. This scheme makes it incentive feasible for banks to issue unsecured deposits beyond the pledgeable assets they own, and hence can increase the leverage ratio of banks. The optimal policy then trades off two inefficiencies: on the one hand, a smaller number of charters increases bank profits and hence helps increase liquidity, which improves depositors' welfare; on the other hand, a smaller number of charters increases the overall cost of banking operations as each one gets inefficiently large. Our main result demonstrates that, whenever liquidity is tight under static bank contracts, it is optimal to limit the number of charters relative to the number under efficient asset management, and to relax the pledgeability constraint through a lower overall leverage ratio requirement than its *laissez faire* level.

Our model can not only generate insights about overall bank size and profits, it also allows us to consider how optimal regulation affects distribution of bank sizes and profits. In our model, bank sizes are endogenously determined by either free entry (in the absence of charter), or by the number of charters. We extend our model by allowing for heterogeneous management costs. In the absence of regulation, more efficient banks end up being larger

in terms of asset holding, and it is efficient to do so. When liquidity is tight, we show that an optimal charter system would in fact make large banks even larger by allowing more generous unsecured borrowing. The intuition is simple: when the number of charters is limited, large banks make higher profits and hence it is more efficient to incentivise them to repay unsecured deposits. As a result, we obtain a positive correlation between bank size and leverage ratio under the optimal policy arrangement.

We then introduce the moral hazard issue with asset-management by banks, in which banks may gamble on their assets to obtain a private gain with certain probability, although doing so would lower the overall return on average and hence socially suboptimal. Precisely because of the two-sided nature of bank contracts, banks may have incentives to gamble, as that may increase their profits because of the private gain while the depositors have to suffer (most of) the consequences. We show that, under static contract and free-market arrangement, market discipline would impose an additional proportional capital requirement to a bank's asset holding to ensure efforts. This, however, can be harmful to liquidity provision as it lowers the level of deposits banks can offer. In particular, as moral hazard becomes more serious, the liquidity service becomes poorer.

Against this background environment we study how an optimal charter system can handle this moral hazard issue. We first fully characterize the optimal proportional capital requirement necessary for the moral hazard issue and the accompanied overall leverage ratio, for any given number of licences. Then we can employ our earlier methodology to search for the optimal number of banking licences. Thus, the model delivers a clear distinction between two capital regulations: the first is a capital requirement proportional to a bank's asset holdings to deal with moral hazard, and the second is an overall leverage ratio that would depend, among other things, on the market power and hence profitability of the bank. There is a nontrivial interaction between these two regulations as well. We show that, under some conditions, as moral hazard becomes worse, it is in fact optimal to allow for higher profits

and to make banks larger. This is due to the welfare impact on banks' liquidity provision: as moral hazard gets worse, a higher proportional capital requirement is needed and that hurts liquidity provision; to compensate for that loss, it is optimal to increase unsecured borrowing from banks by making them bigger and hence more profitable.

Finally, while traditionally it is assumed that gambling would be suboptimal in banking, it is not obvious whether gambling would be better or worse for liquidity provision in our framework. We first show that, without further capital requirements, an equilibrium where all banks gamble exist if the moral hazard issue is binding. As mentioned earlier, to induce prudent behavior it is necessary to introduce further capital requirement. Comparing against the regime in which all banks gamble, the regulator faces a nontrivial trade-off, which does not exist in earlier literature without endogenous liquidity needs, to introduce the capital requirement that induces prudent behavior: on the one hand, it discourages gambling and hence increase overall return; on the other hand, it directly decreases liquidity provision. We give a full characterization on how the trade-off resolves. It turns out that the regime without the capital requirement and hence with gambling banks yields a higher welfare whenever the necessary capital requirement is sufficiently stringent.

Related Literature

Our pledgeability constraint is similar to that in Gertler and Kiyotaki (2010). There are two key differences, however. First, we explicit model the deposits as means-of-payments and hence focus on banks' liquidity role for depositors. Second, we focus on banks' limited commitment and moral hazard problem and the optimal regulations to deal with them. Our approach to model bank assets as Lucas trees effectively assumes that all agency issue between the banks and the end borrowers is captured by the management/monitoring cost, an approach shared by some recent papers such as Begenau and Landvoigt (2017).

This paper is not the first one to point out that future bank profits play an essential role

in banking regulations. On the empirical side, Keeley (1990) provides some evidence that charter value restricts banks' risk-taking behavior. On the theory side, Hellmann, Murdock, and Stiglitz (2000), in a model where banks have market powers and face moral hazard, show that it is optimal to use a combination of capital requirement and deposit-rate ceilings to create sufficient franchise value for banks to ameliorate the moral-hazard problem. Future bank profits are the main incentive device for prudent behavior, using deposit-rate ceiling to maintain profits. In contrast, profits are maintained by restricting entry and deposit-rate ceilings would be sub-optimal in our model. Two main modeling ingredients explain the difference: first, while deposit demand is exogenously given there, in our model it is driven by endogenous liquidity needs; second, bank entry decision and sizes are endogenously determined in our model.

Our paper is also related to the literature on liquidity provision by banks. Using a means-of-payment-in-advance model with currency and deposits, Chari and Phelan (2014) show that if there is insufficient deflation, fractionally backed banks which offer interest-bearing deposits may be good, but such banks are subject to socially costly runs. Williamson (2016), shows that, when banks face limited commitment, and when short-maturity government debt has a greater degree of pledgeability than long-maturity government debt, quantitative easing can improve liquidity. These papers, however, do not address optimal financial regulations. Gorton and Winton (2017) also features a trade-off of raising capital requirement because bank debt is used for transactions purposes, while more bank capital can reduce the chance of bank failure; however, they assume exogenously given banks' charter value. Phelan (2016), in a model where deposits serve the liquidity function, shows that leverage also increases asset price volatility and so limiting leverage decreases the likelihood that the financial sector is undercapitalized. However, the model assumes that deposits exogenously generate utility to depositors, and hence it is then not clear how regulations may affect banks' function in providing means-of-payment, and through which, the economic activity.

2 The Environment

The environment is borrowed from Rocheteau and Wright (2005). Time is discrete and has an infinite horizon, $t \in \mathbb{N}_0$. The economy is populated by three sets of agents; each set has a continuum of infinitely-lived agents with measure one. The first set consists of *buyers*, denoted by \mathbb{B} , and the second consists of *sellers*, denoted by \mathbb{S} . The third set consists of potential *banks*. Each date has two stages: the first has pairwise meetings of buyers and sellers in a decentralized market (called the DM), and the second has centralized meetings (called the CM) where all agents meet. In each DM, the probability that a buyer has a successful meeting with a seller is σ . There is a single perishable good produced in each stage, with the CM good taken as the numéraire. Agents' labels as buyers and sellers depend on their roles in the DM where only sellers are able to produce and only buyers wish to consume. While all agents can produce and consume in the CM, potential banks do not consume nor produce in the DM.

Buyers' preferences are represented by the following utility function

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t [u(q_t) + x_t - h_t],$$

where $\beta \equiv (1 + r)^{-1} \in (0, 1)$ is the discount factor, q_t is DM consumption, x_t is CM consumption, and h_t is the supply of hours in the CM. Sellers' preferences are given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t [-c(q_t) + x_t - h_t],$$

where $c(q)$ is the seller's disutility of producing q in the DM. The first-stage utility functions, $u(q)$ and $-c(q)$, are increasing and concave, with $u(0) = v(0) = 0$. The surplus function, $u(q) - c(q)$, is strictly concave, with $q^* = \arg \max [u(q) - c(q)]$. Moreover, $u'(0) = c'(\infty) = \infty$ and $c'(0) = u'(\infty) = 0$. All agents have access to a linear technology to produce the CM

output from their own labor, $x = h$.

There is only one real assets, loans to entrepreneurs. There is a competitive market for loans to financing entrepreneurs' projects, which, for simplicity, are assumed to materialize within a single period, each unit has a gross return τ (in terms of CM goods) at pays off in the next CM. The average supply (per buyer) of the projects is \bar{A} at each period. To obtain the return from the entrepreneurs, however, it requires a potential bank to perform costly monitoring/management. As in Gertler and Kiyotaki (2010), one can also think of the bank's claim on these projects as equity.⁴ The firms are subject to an agency problem such that costly monitoring is required, as in the delegated monitoring model of financial intermediaries proposed by Williamson (1986) or Diamond (1984).⁵ In contrast to those papers, however, our main focus is on the role of banks in providing liquid assets as a means of payment. Specifically, each bank can issue deposit certificates in an open market. We assume that these certificates are perfectly divisible, perfectly durable, and cannot be counterfeited. Such liabilities are payable to the bearer. Thus, buyers may use such certificates to finance their consumptions in the DM. There is a public record of banks' liabilities and asset holdings, but there is no record keeping of buyers' or sellers' deposit holdings and their transaction records. A historical resemble of this deposit claim is banknotes, and a modern counterpart is stored-value cards issued by banks.⁶

There are two frictions associated with this financial intermediation. The first friction is the cost associated with managing/monitoring the loans. Only active banks can hold assets and issue deposits; to become active, a bank has to pay a fixed cost of γ each period. There is also a marginal cost of asset-management: for a banker to hold a units of loans, he needs to

⁴They also consider capital accumulation for firms and an interbank loan market, which are absent here.

⁵Of course, in those models one needs to introduce asymmetric information between borrowers and lenders to give the financial intermediaries a role, while here the return of the Lucas trees is certain. One can interpret the return of Lucas trees here as the *diversified* return in those models where each bank represents a large number of depositors.

⁶The point here is that the record keeping technology should not be too good to completely destroy the anonymity of agents in the DM market; otherwise trade can be conducted by using credit.

pay $\psi(a)$ (as a labor cost) to monitor/manage the entrepreneurs. We assume that $\psi(0) = 0$, $\psi(a)$ is strictly increasing and strictly convex, and $\psi(\bar{A}) = \infty$.

Second, banks have limited liability and cannot commit to their future actions. However, we assume that if a bank files for bankruptcy, the court could seize ρ proportion of his claims on the entrepreneurs. Thus, by holding a units of bonds, a bank can credibly pledge ρ fraction of the returns from the projects he invested in but can take the rest away, a friction similar to that in Kiyotai and Moore (1997). In contrast, banks can fully pledge the value of all their tree holdings. Banks maximize their life-time profits with discount factor β .

Finally, we define social welfare in our economy. It is convenient to define

$$\Pi(A) = \psi'(A)A - \psi(A). \quad (1)$$

We assume that

$$\Pi(\bar{A}) < \gamma < \Pi \left[(\psi')^{-1} \left(\frac{\tau}{1+r} \right) \right]. \quad (2)$$

An allocation consists of both DM trade per successful meeting, denoted by q , and the number of active banks, denoted by m (and hence the amount of loan holding for each bank is \bar{A}/m).⁷ Given an allocation (q, m) , the total welfare is given by

$$\mathcal{W}(q, m) = \alpha[u(q) - c(q)] - [m\psi(\bar{A}/m) + m\gamma]. \quad (3)$$

The first-best allocation, defined as the allocation (q, m) that maximizes $\mathcal{W}(q, m)$ without any constraint, is denoted by (q^*, m^*) and it satisfies

$$\begin{aligned} u'(q^*) - c'(q^*) &= 0, \\ \Pi(\bar{A}/m^*) &= \gamma. \end{aligned}$$

⁷Note that, as typically in the Lagos-Wright models, CM production and consumption does not affect welfare due to linearity.

Assumption (2) then ensures that $m^* < 1$.

3 Bank contracts

In this section we consider equilibrium bank contracts with the depositors. We begin with the case where there is free entry without any regulations, and highlight the potential inefficiency under this free-market arrangement. Then, we introduce the charter system with a regulator whose goal is to maximize the social welfare, and characterize the optimal interventions that respect voluntary participation and incentive compatibility due to limited commitment of all agents and anonymity of buyers.

We first describe the time line and the general characteristics of the banking contracts.

The course of events. In the CM, the course of events is as follows:

1. first, banks settle deposit obligations with depositors;
2. then, banks buy loans in competitive market at price ϕ (in terms of CM good);
3. finally, banks may issue deposit contract, promising a gross return R (in exchange for CM good).

We use d to denote the total amount of deposits that the bank promises to give out in the next CM (and hence it will receive d/R in the current CM). Note that there are two different markets in the CM—a spot market for deposits, and a spot market for assets. Because only banks can manage Lucas trees to receive dividends, with no loss of generality we assume that buyers and sellers do not participate in the asset market.⁸ We also assume that only

⁸We implicitly assume that there is no friction within the two spot markets in the sense that all agents (especially buyers and bankers) can make promises to deliver the CM goods within the same-date CM stage when making the portfolio decisions, and hence, as usual in Lagos and Wright (2005) frameworks, the timing of the trades within CM does not matter and we can work with the *net* consumption in the CM for various agents.

buyers enter the deposit contracts in the CM but not sellers.

In the DM, upon a successful meeting with a seller, the buyer makes a take-it-or-leave-it offer, (q, z) , where q is the DM consumption and z is the amount of deposit (in terms of the coming CM goods) transfer. This is feasible because, as mentioned earlier, there is record-keeping technology under which the accounts of the buyer can be transferred to the seller.

3.1 Static bank contracts

Here we consider the case where the free entry of banks implies a zero-profit condition, which in turn implies that banks cannot credibly promise any amount beyond what could be seized by the court.

As a benchmark, we first begin with the situation where banks cannot issue deposits at all. In this case, the price of the Lucas trees can be easily pinned down by a no arbitrage condition (i.e., banks' profit-maximizing condition) and the number of banks pinned down by free entry. Note that assumption (2) also ensures that there is sufficient entry to the banking sector. Indeed, as will be clear below, $\Pi(A) - \gamma$ will be the profit for a bank with A units of trees. Free entry then requires banks to hold $A = \Pi^{-1}(\gamma)$ and hence only a measure $m^* = \bar{A}/\Pi^{-1}(\gamma)$ of banks will enter, where m^* is also the first-best number of active banks. Thus, (2) ensures that a unit measure of banks is sufficient to provide free entry. We may define the fundamental value of trees as

$$\phi^* = \frac{\tau}{1+r} - \psi' \left(\frac{\bar{A}}{m^*} \right), \quad (4)$$

which will be the price for the asset if the banks cannot issue any deposits.

For expositional purposes, we define a variable,

$$\iota \equiv \frac{1+r}{R} - 1.$$

Given R (and hence ι), φ , and ϕ , and for a given loan holding, a , a given tree holding, b , and a given deposit issuance, d , (in terms of next CM promised value), a bank's profit is given by

$$\begin{aligned} \pi(a, d; \phi, R) &= \frac{d}{R} - \phi a - \gamma - \psi(a) + \beta\{\tau a - d\} \\ &= \beta\{\iota d + [\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma]\}, \end{aligned} \quad (5)$$

and is subject to the pledgeability constraint,

$$d \leq \rho \tau a. \quad (6)$$

As mentioned, under static contracts, banks can only pledge what could be seized by the court, namely, ρ fraction of the dividends of their assets; (6) captures this constraint.

Let $A(\phi, \iota)$ be the optimal asset holding that maximizes (5) subject to (6). Note that whenever $\iota > 0$, the constraint (6) is binding and $A(\phi, \iota)$ is determined by the following FOC:

$$-(1+r)\phi + (1+\iota\rho)\tau = (1+r)\psi'(a). \quad (7)$$

When the pledgeability constraint is binding, the bank needs to own capital, $\phi a - \frac{d}{R}$, to finance some of its asset holdings.

Now we turn to depositors' behavior. Given R , a depositor's problem is given by

$$\max_{d \geq 0} -\frac{d}{R} + \beta \{ \sigma [u(q(d)) - c(q(d))] + d \}, \quad (8)$$

where $c(q(d)) = d$ if $d < c(q^*)$ and $q(d) = q^*$ otherwise.

Note that d is the *promised value* of the deposit in the coming CM. The FOC to (8) is

$$\iota = \frac{\sigma [u'(q(d)) - c'(q(d))]}{c'(q(d))}. \quad (9)$$

Let $D(\iota)$, the deposit demand per depositor, be the solution to (9). Note that for any $\iota > 0$, $D(\iota)$ is uniquely determined; when $\iota = 0$, $D(\iota)$ is not pinned down but $D(\iota) \geq c(q^*)$. Without loss of generality we may take $D(0)$ as its minimum. Then, $D(\iota)$ is continuous and strictly decreasing in ι .

Equilibrium then requires market clearing conditions for deposits and assets:

$$D(\iota) = \rho \tau \bar{A}; \quad (10)$$

$$mA(\iota, \phi) = \bar{A}. \quad (11)$$

Finally, free-entry implies that all active banks have to have zero profits.

Lemma 3.1. *There is a unique equilibrium allocation, (m, ϕ, ι, q, d) , in which $m = m^*$, and (ϕ, ι, q, d) is characterized as follows.*

(a) *Suppose that*

$$\rho \tau \bar{A} \geq c(q^*). \quad (12)$$

Then, $\phi = \phi^$, $q = q^*$, and $\iota = 0$.*

(b) Suppose that (12) does not hold. Then,

$$\phi = \frac{(\iota\rho + 1)\tau}{1 + r} - \psi' \left(\frac{\bar{A}}{m^*} \right), \quad (13)$$

with $q = c^{-1}(D(\iota)) < q^*$ and with $\iota > 0$ as the unique solution to

$$D(\iota) = \rho\tau\bar{A}. \quad (14)$$

Moreover, in this case, the constraint (6) is binding in equilibrium.

According to Lemma 3.1, the equilibrium interest rate on deposits is determined by (14). When the real pledgeable value of the loans, $\rho\tau\bar{A}$, is small, ι can be large and possibly larger than r and hence the deposit contract has a negative net return. However, in a system with fiat money alone, a monetary equilibrium with zero gross return exists and would dominate such an equilibrium with banking. In the next section we show that by introducing a banking authority that imposes a reserve requirement would make the banking system essential in the sense that it will dominate fiat money in terms of social welfare.

3.2 Reserve requirements

Here we introduce the charter system with a banking authority or regulator. The regulator, or the central bank, set a reserve requirement. The bank reserve, as in the current system, should be regarded as a nominal liability of the central bank. As such, the bank can fully pledge its holding of the reserves. One can think of reserves as an instrument to settle interbank payments. For now we assume that the reserve money is of constant supply, and bears no interest, and hence, has a zero gross return. The reserve thus may be regarded as outside money for the banking system.

Consider a bank that holds a units of loan and z units of reserve (measured in terms of

coming CM good), and issue d units of deposits. Given the return on deposit, R , and price for loans, ϕ , such a bank's profit is given by

$$\begin{aligned}\pi(a, d, z; \phi, R) &= \frac{d}{R} - \phi a - z - \gamma - \psi(a) + \beta\{\tau a + z - d\} \\ &= \beta\{\iota d + [\tau - (1+r)\phi]a - rz - (1+r)[\psi(a) + \gamma]\},\end{aligned}\tag{15}$$

and is subject to the pledgeability constraint and reserve requirement,

$$d \leq \rho\tau a + z,\tag{16}$$

$$z \geq \eta d,\tag{17}$$

where z is the reserve holding. The constraint (17) is the reserve requirement, and, since z enters (15) as a negative term, (17) is binding. Thus, the profit to a bank is given by

$$\pi(a, d, z; \phi, R) = \beta\{(\iota - \eta r)d + [\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma]\}.$$

Note that $d > 0$ only if

$$\iota - \eta r \geq 0.\tag{18}$$

Assuming that this holds, (16) is binding, and hence

$$\pi(a, d, z; \phi, R) = \beta\left\{\frac{\iota - \eta r}{1 - \eta}\rho\tau a + [\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma]\right\}.$$

This then gives rise to the following FOC:

$$\frac{\iota - \eta r}{1 - \eta}\rho\tau + [\tau - (1+r)\phi] = (1+r)[\psi'(a) + \gamma].\tag{19}$$

We use $A(\phi, \iota)$ to denote the solution.

A depositor's problem is still given by (8), and deposit demand, $D(\iota)$, remains the same. For given η and m , the market-clearing conditions are given by

$$D(\iota) = \frac{\rho\tau}{1-\eta}\bar{A}; \quad (20)$$

$$mA(\phi, \iota) = \bar{A}. \quad (21)$$

Moreover, we only consider m 's that satisfy

$$\tau \geq \psi' \left(\frac{\bar{A}}{m} \right) (1+r). \quad (22)$$

By (2) and convexity of ψ , there exists a unique $\bar{m} < m^*$ such that (22) is satisfied for all $m \in [\bar{m}, m^*]$. It can be verified that it is never optimal to have $m < \bar{m}$.

Lemma 3.2. *Let $m \in [\bar{m}, m^*]$ and let η be given. Assuming that all banks issue deposits, there is a unique allocation (ϕ, ι, q, d) that satisfies the market-clearing conditions that can be characterized as follows:*

$$\phi = \frac{\left(1 + \rho \frac{\iota - r\eta}{1-\eta}\right) \tau - \psi' \left(\frac{\bar{A}}{m} \right) (1+r)}{1+r}, \quad (23)$$

and $q = c^{-1}(D(\iota))$ with $\iota \equiv \iota(\eta)$ as the unique ι such that $\iota \geq 0$ and

$$D(\iota) \leq \frac{\rho\tau}{1-\eta}\bar{A}, \quad (24)$$

with equality whenever $\iota > 0$.

Now, to ensure that the allocation that satisfies market clearing is indeed an equilibrium, we need to check (18). To do so, note that for any η , (24) determines ι uniquely, denoted by

$\iota(\eta)$. Note that $\iota(\eta)$ is strictly decreasing in η . It is an equilibrium if and only if

$$\eta r \leq \iota(\eta). \quad (25)$$

To maximize social welfare, (3), the optimal reserve requirement would maximize the total liquidity, i.e., the right-side of (24), subject to (25). The following theorem characterizes such optimal reserve requirement.

Theorem 3.1. *Assume that $\rho > 0$. Then, in any equilibrium with free entry, we have $m = m^*$. If (12) holds, then the optimal reserve requirement is to set $\eta = 0$. Otherwise, the optimal reserve requirement is to set*

$$\eta = \frac{\iota}{r}. \quad (26)$$

with $\iota \in (0, r)$ determined by

$$D(\iota) = \frac{r\tau\rho\bar{A}}{r - \iota}. \quad (27)$$

Compared with (14) in Lemma 3.1, aggregate liquidity shown in (27) is larger (because $r > \iota$) due to higher pledgeability of banks' asset. The reserve requirement improves banks' pledgeability because reserve money is more pledgeable than loans. According to Theorem 3.1, the equilibrium return on bank contracts is always more attractive than a system with fiat money, regardless of asset supply. This is in contrast with Lemma 3.1 where such return can be negative. The reserve requirement in fact performs asset transformation for the economy with banking—by choosing the optimal level of reserve requirements, the central bank can ensure that banks will compete to guarantee a positive gross return. This is a form of asset-transformation; the banking sector combines the reserves and loans to entrepreneurs into a liquid asset that pays a positive return.

However, despite the optimal use of bank reserves, the total amount of liquidity the banking sector can provide is still bounded by pledgeable asset supply, as formalized in (27),

while the banking sector is still fully efficient in terms of asset monitoring. Indeed, it is easy to show that equilibrium ι is strictly decreasing in $\rho\tau\bar{A}$ and hence equilibrium amount of deposit holding per depositor is strictly increasing in $\rho\tau\bar{A}$. In the next section we introduce a charter system that can relax further the pledgeability constraint.

3.3 Charter system

In many modern banking systems banks are highly regulated and entry is restricted through permission of a banking authority. One example is the charter system in the US. Here we show that such a system, by restricting entry and by potentially depriving banking privilege as a way to discipline bank behavior, can relax the pledgeability constraint and improve the efficiency of banking system in terms of liquidity provision. Intuitively, such a system can relax the constraint because it provides a dynamic incentive for banks to repay their deposit liability. On the one hand, restriction on entry implies that all operational banks would earn economic profits due to less intensive competition. On the other hand, any bank that does not repay would be excluded from banking forever. The relaxation of the pledgeability constraint then allows for more liquidity the banking system can provide, but at the expense of less efficient monitoring of their assets due to larger than efficient amount of asset holding for each bank.

We begin with a more relaxed pledgeability constraint and reserve requirement:

$$d \leq \rho\tau a + z + \kappa, \tag{28}$$

$$z \geq \max\{0, \eta(d - \kappa')\}. \tag{29}$$

Compare to (16), (28) allows each bank to issue κ units of additional deposits relative to its pledgeable assets. Compared to (17), (29) allows the first κ' units of deposits to be exempted from reserve requirement. As we shall see later, incentive compatibility conditions

are needed to sustain these relaxed constraints. Obviously there are other ways to relax these constraints; in particular, one may increase ρ to allow for higher issuance of deposits for any given level of asset holding. In the Appendix we show that the forms of allowing the issuance of unsecured deposits and of the reserve requirement given here are in fact optimal.

Note that bank profit is still given by (15); thus, it is always optimal to choose $d \geq \kappa'$, and to have (29) binding. Thus, the profit is given by

$$\pi(a, d, z; \phi, R) = \beta \left\{ \frac{\iota - \eta r}{1 - \eta} \rho \tau a + [\tau - (1 + r)\phi]a - (1 + r)[\psi(a) + \gamma] + \frac{\iota - \eta r}{1 - \eta} (\kappa - \eta \kappa') + r \eta \kappa' \right\}.$$

Now, given κ and κ' , the market-clearing conditions now become:

$$D(\iota) = \frac{\rho \tau}{1 - \eta} \bar{A} + m \frac{1}{1 - \eta} (\kappa - \eta \kappa'); \quad (30)$$

$$mA(\phi, \iota) = \bar{A}. \quad (31)$$

We have the following lemma.

Lemma 3.3. *Let $m \in [\bar{m}, m^*]$ and let η, κ be given. Assuming that all banks issue deposits, there is a unique allocation (ϕ, ι, q, d) that satisfies the market-clearing conditions that can be characterized as follows: ϕ is still given by (23), and $q = c^{-1}(D(\iota))$ with $\iota = \iota(m, \eta_1, \kappa) \in [0, r)$ as the unique ι such that $\iota \geq 0$ and*

$$D(\iota) \leq \frac{\rho \tau}{1 - \eta} \bar{A} + m \frac{1}{1 - \eta} (\kappa - \eta \kappa'), \quad (32)$$

with equality whenever $\iota > 0$. Moreover, the profit for each bank is given by

$$\Pi \left(\frac{\bar{A}}{m} \right) - \gamma + \frac{1}{1 + r} \left[\frac{\iota - \eta r}{1 - \eta} (\kappa - \eta \kappa') + r \eta \kappa' \right].$$

Now, we need to verify two incentive compatibility constraints. First, to convince banks

to issue deposits beyond κ' , it is necessary that (18) holds. Second, banks have to be willing to repay the unsecured component, κ . Since the court can only seize ρ proportion of a bank's asset and the reserves, the bank has temptation not to repay the κ component of his liability in (28). To deter this temptation, the regulator can remove the bank charter and stop the bank from future business if the bank fails to honor his deposit obligations. Thus, if a bank defaults, he loses the pledged assets, $\rho\tau\bar{A}/m$, and the reserves, z , as well as the charter to run the business, beginning from the period when he fails to repay. As a result, a bank is willing to repay deposits if and only if

$$-\kappa - \rho\tau\bar{A}/m - z + \sum_{t=0}^{\infty} \beta^t \left\{ \Pi \left(\frac{\bar{A}}{m} \right) - \gamma + \frac{1}{1+r} \left[\frac{\iota - \eta r}{1 - \eta} (\kappa - \eta\kappa') + r\eta\kappa' \right] \right\} \geq -\rho\tau\bar{A}/m - z.$$

This constraint can be simplified as

$$-\frac{r - \iota}{1 - \eta} (\kappa - \eta\kappa') + (1 + r) \left[\Pi \left(\frac{\bar{A}}{m} \right) - \gamma \right] \geq 0. \quad (33)$$

We have the following lemma.

Lemma 3.4. *Let $m \in [\bar{m}, m^*]$ and $\iota < r$ be given. The optimal (η, κ, κ') that solves*

$$\mathcal{S}(m, \iota) = \max_{\eta, \kappa, \kappa'} \frac{\rho\tau}{1 - \eta} \bar{A} + m \frac{1}{1 - \eta} (\kappa - \eta\kappa'), \quad (34)$$

subject to (18) and (33) is given by (26) and

$$\kappa = \kappa' = \frac{(1 + r) \left[\Pi \left(\frac{\bar{A}}{m} \right) - \gamma \right]}{r - \iota}. \quad (35)$$

According to Lemma 3.4, it is optimal to set $\kappa' = \kappa$, and to set both equal to the level according to (35). As a result, we may refer to a policy as a pair (m, η, κ) , with the understanding that we set $\kappa' = \kappa$. Moreover, Lemma 3.4 shows that to solve for optimal

policy, we may use the following equilibrium condition:

$$D(\iota) \leq \mathcal{S}(m, \iota) = \frac{r\rho\tau}{r-\iota} \bar{A} + m \frac{(1+r) \left[\Pi \left(\frac{\bar{A}}{m} \right) - \gamma \right]}{r-\iota}, \quad (36)$$

and hence the planner's problem is now to maximize (3) subject to (36).

The following theorem characterizes the optimal charter system.

Theorem 3.2. *Assume (A0). There exists an optimal policy (m, η, κ) that maximizes welfare subject to implementability.*

(a) *If (12) holds, then $(m, \eta, \kappa) = (m^*, 0, 0)$ is an optimal policy.*

(b) *Suppose that (12) does not hold. Then, any optimal policy has $m < m^*$, $\eta > 0$, and $\kappa > 0$.*

Theorem 3.2 shows that, when designing an optimal charter system, the regulator has to balance efficiency in asset management and efficiency in liquidity provision. When there is abundant pledgable assets so that (12) holds, full efficiency can be achieved on both aspects, according to Theorem 3.2 (a). Otherwise, according to Theorem 3.2 (b), the constrained efficient arrangement has to sacrifice full efficiency on both aspects. Restricting the number of charters reduces competition and increases banks' profits; this is suboptimal regarding efficiency in asset-management. However, higher profits make it easier for banks' incentive constraint, (33), to hold and allow for a positive κ without having the banks defaulting on their debts. Thus, this financial stability in our framework is possible because of positive profits banks enjoy, and it is useful to enhance social welfare because banks provides liquidity services as their liabilities are used as means-of-payments.

Since the optimal κ regulates the amount of deposits a bank can issue through the pledgeability constraint (28), one can interpret the policy parameter κ as an *overall* leverage ratio requirement. Again, consider the balance sheet of each bank in equilibrium, where the

value of the asset is given by $\tau \frac{\bar{A}}{m}$. Then the leverage ratio, defined as the ratio between liability to asset, would be

$$\mathcal{L} = \frac{\rho \tau \frac{\bar{A}}{m} + z + \kappa}{\tau \frac{\bar{A}}{m} + z}.$$

Theorem 3.2 (2) then implies that this leverage ratio would be higher than under the free entry when liquidity is tight.

In contrast to the common capital requirements that depend only on the asset characteristics a bank holds, optimal κ also depends on other bank characteristics such as ρ (proportion of asset that can be secured for repayment) and ψ (marginal cost of asset-management). Our framework then implies a holistic approach to capital requirement that would take both idiosyncratic feature of a specific bank as well as the global environment into account when designing the optimal capital regulations.

We emphasize that the charter system can implement the optimal equilibrium uniquely by allowing the issuance of unsecured debt and terminating banks that fail to repay deposits. In principle, the regulator can simply restrict the number of banking licences and the market will demand an appropriate pledgeability constraint accordingly. However, in that case there would be multiple equilibrium: any κ below the highest level consistent with the incentive compatibility constraint (33) can be an equilibrium, including $\kappa = 0$.

4 Heterogenous bank sizes and profits

Our basic framework can be readily extended to discuss many issues, including heterogeneity of bank sizes, profits, and leverage ratios. Here we consider heterogenous banks in terms of their efficiency in asset management. Specifically, for each $n \in \{1, \dots, N\}$, the economy has measure μ_n of type- n banks with $\sum_{n=1}^N \mu_n = 1$, and the cost function for a bank of type- n is $\lambda_n \psi(a) + \gamma$. The parameter λ_n is then a measurement of how efficient type- n banks are

in terms of asset management. We assume that $\lambda_n \in [1, \bar{\lambda}]$ is strictly increasing in n , and hence type-1 banks are the most efficient ones while type- n are the least efficient ones.

4.1 Efficient asset management

First we consider efficient asset management in this environment. Without deposit issuance, efficient asset management requires the measures of type- n active banks, denoted by m_n , to solve

$$\begin{aligned} & \min_{m_n \in [0, \mu_n], A_n \geq 0, n=1, \dots, N} \sum_{n=1}^N [m_n \gamma + \lambda_n \psi(A_n)] \\ \text{s.t. } & \sum_{n=1}^N m_n A_n = \bar{A}. \end{aligned} \quad (37)$$

Parallel to (2), to ensure that there is sufficient entry we assume that

$$\sum_{n=1}^N \mu_n \Pi^{-1}(\gamma/\lambda_n) > \bar{A}. \quad (38)$$

To characterize the solution to (37), first for each $\mathbf{m} = (m_1, \dots, m_N)$ with $m_1 > 0$, we define $\{A_n(\mathbf{m})\}_{n=1}^N$ as the solution to

$$\sum_{n=1}^N m_n A_n = \bar{A}, \quad \lambda_1 \psi'(A_1) = \lambda_n \psi'(A_n) \text{ if } m_n > 0, \quad A_n = 0 \text{ otherwise.} \quad (39)$$

We have the following claim.

Claim 4.1. *Assume (38). The solution to (37) is unique, denoted by \mathbf{m}^* , is characterized*

by $\bar{n} \in \{1, \dots, N\}$ and $0 < m_{\bar{n}}^* \leq \mu_{\bar{n}}$ such that

$$\mathbf{m}^* = (\mu_1, \dots, \mu_{\bar{n}-1}, m_{\bar{n}}^*, 0, \dots, 0), \quad (40)$$

$$\lambda_n \Pi(A_n(\mathbf{m}^*)) \geq \gamma, \text{ for all } n = 1, \dots, \bar{n}, \quad (41)$$

$$\lambda_n \Pi(A_n(\mathbf{m}^*)) = \gamma \text{ if } m_{\bar{n}} < \mu_{\bar{n}}, \quad (42)$$

$$\lambda_n \Pi(A_n(\mathbf{m}^*)) < \gamma, \text{ for all } n = \bar{n} + 1, \dots, N. \quad (43)$$

4.2 Reserve requirement

Now we consider only reserve requirements with free entry. Given R (and hence ι) and ϕ , and for a given asset holding, a , reserve holding, z , and deposits giving out, d , (in terms of next CM promised value), the profit of a type- n bank is given by

$$\pi_n(a, z, d; \phi, R) = \beta \{ \iota d - [(1+r)\phi - \tau]a - rz - (1+r)[\lambda_n \psi(a) + \gamma] \},$$

and is subject to the pledgeability constraint and reserve requirement,

$$d \leq \rho \tau a + z, \quad (44)$$

$$z \geq \eta d. \quad (45)$$

Note that the reserve requirement is not size-dependent because η is the same across banks (though the amounts of reserves held are different across banks because deposits issued are heterogenous). As will be clear later, making the reserve required ratio size-dependent will not improve welfare, because it does not affect aggregate liquidity. As before, to ensure that banks issue deposits, we need (18) to hold. Assuming (18), this gives rise to a well-defined

asset demand $A_n(\phi, \iota, \eta)$ determined by the following FOC:

$$-(1+r)\phi + \left[1 + \frac{\rho(\iota - r\eta)}{1-\eta}\right] \tau = (1+r)\lambda_n \psi'(a). \quad (46)$$

That is,

$$A_n(\phi, \iota, \eta) = (\psi')^{-1} \left(\frac{-(1+r)\phi + \left[1 + \frac{\rho(\iota - r\eta)}{1-\eta}\right] \tau}{(1+r)\lambda_n} \right). \quad (47)$$

Let $\phi_{\mathbf{m}}^*$ be the unique solution to

$$\sum_{n=1}^N m_n A_n(\phi, 0, 0) = \bar{A}.$$

As before, we may call $\phi_{\mathbf{m}^*}^*$ the fundamental value of the asset, the price for the trees if banks were not allowed to issue deposits; in that situation the measures of active banks would be given by \mathbf{m}^* .

Let m_n be the measure of active type- n banks, $n = 1, \dots, N$. Then, equilibrium objects include asset price ϕ , returns to deposits ι , and the measure of active type- n banks, m_n for each $n = 1, \dots, N$ ($m_n = 0$ means that no type- n bank is active). The market-clearing conditions and free entry condition are given by (note that $D(\iota)$ is still given by (9))

$$D(\iota) = \frac{\rho\tau}{1-\eta} \bar{A}; \quad (48)$$

$$\sum_{n=1}^N m_n A_n(\phi, \iota, \eta) = \bar{A}; \quad (49)$$

$$\lambda_n \Pi[A_n(\phi, \iota, \eta)] \geq \gamma \text{ if } m_n > 0, \quad \lambda_n \Pi[A_n(\phi, \iota, \eta)] \leq \gamma \text{ if } m_n < \mu_n. \quad (50)$$

We have the following lemma.

Lemma 4.1. *Assume (38) and let η be given. There exists a unique allocation that satisfies market clearing. The equilibrium measures of active banks are given by \mathbf{m}^* and equilibrium*

asset holding is given by $A_n(\mathbf{m}^*)$ according to (39) for type- n banks.

Lemma 4.1 shows that even with heterogenous banks, the result that efficiency of asset-management is achieved without charters still holds. However, here we obtain an endogenous distribution of bank balance sheets. Specifically, (39) implies that $A_n(\mathbf{m}^*) > A_{n+1}(\mathbf{m}^*)$ for all $n = 1, \dots, \bar{n} - 1$, and hence, under free entry, more efficient banks are also larger in terms of asset holdings. Moreover, the FOC also implies that the profit for bank of type- n is given by $\lambda_n \Pi[A_n(\phi, \iota)] - \gamma$, and hence Claim 4.1 implies that even under the efficient arrangement for asset management, some banks may make positive profits. Strict convexity also implies that $\lambda_n \Pi(A_n(\mathbf{m}^*)) > \lambda_{n+1} \Pi(A_{n+1}(\mathbf{m}^*))$ for all $n = 1, \dots, \bar{n} - 1$, and hence, more efficient banks also make higher profits. In what follows, we assume that the solution satisfies $m_{\bar{n}}^* < \mu_{\bar{n}}$.

Theorem 4.1. *Assume that $\rho > 0$. Then, in any equilibrium with free entry, we have $\mathbf{m} = \mathbf{m}^*$. If (12) holds, then the optimal reserve requirement is to set $\eta = 0$. Otherwise, the optimal reserve requirement is to set (26) with $\iota \in (0, r)$ determined by*

$$D(\iota) = \frac{r\tau\rho\bar{A}}{r - \iota}. \quad (51)$$

Remark 4.1. It is straightforward to see that allowing for varying η across bank types does not increase welfare.

4.3 Heterogenous bank leverages

Here we consider the charter system. We assume that bank efficiency, λ_n , is observable. Given this assumption, the policy parameters also include a measure of banks for each type, $\mathbf{m} = (m_1, \dots, m_N)$. The pledgeability and reserve requirements for bank of type n are as

follows:

$$d \leq \rho\tau a + z + \kappa_n, \quad (52)$$

$$z \geq \max\{0, \eta(d - \kappa'_n)\}. \quad (53)$$

As before, we assume that $\kappa'_n \geq \kappa_n$ for each $n = 1, \dots, N$. Note that the demand for assets from banks of type- n is still given by (47) (since κ_n and κ'_n do not affect the FOC). For given \mathbf{m} and $\{\kappa_n\}$, the market-clearing conditions are given by (note that $D(\iota)$ is still given by (9))

$$D(\iota) = \frac{\rho\tau}{1-\eta}\bar{A} + \sum_{n=1}^N m_n \frac{\kappa_n - \eta\kappa'_n}{1-\eta}; \quad (54)$$

$$\int_{n=1}^N m_n A_n(\phi, \iota, \eta) = \bar{A}. \quad (55)$$

We have the following lemma.

Lemma 4.2. *Let $\mathbf{m} \leq \mathbf{m}^*$ with $m_1 > 0$ and $\{\kappa_n\}$ be given. There is a unique allocation (ϕ, ι, q, d) that satisfies the market-clearing conditions, and can be characterized as follows:*

$$A_n = A_n(\mathbf{m}),$$

$$\phi = \frac{\left[1 + \frac{\rho(\iota - r\eta)}{1-\eta}\right] \tau - (1+r)\lambda_1 \psi'(A_1)}{1+r}, \quad (56)$$

$$D(\iota) \leq \frac{\rho\tau}{1-\eta}\bar{A} + \sum_{n=1}^N m_n \frac{\kappa_n - \eta\kappa'_n}{1-\eta}, \text{ with equality if } \iota > 0. \quad (57)$$

Moreover, the profit for bank of type n is given by

$$\lambda_n \Pi(A_n(\mathbf{m})) - \gamma + \frac{1}{1+r} \left[\frac{\iota - \eta r}{1-\eta} (\kappa_n - \eta\kappa'_n) + r\eta\kappa'_n \right]. \quad (58)$$

The assumption that $m_1 > 0$ is with no loss of generality; if, instead, $m_1 = 0$ but $m_n > 0$ for some other n , then we can simply replace 1 by n in (56) and (57). Note also that since we are only concerned with market clearing and not entry, banks may make negative profits (because of the fixed cost γ). However, a full equilibrium analysis also requires incentive compatibility for repayment of κ , which would require nonnegative profits. As before, banks fail to repay depositors will be closed and hence lose their future profits. Thus, given a policy, \mathbf{m} and $\{(\kappa_n, \kappa'_n)\}$, a bank of type- n is willing to repay deposits if and only if

$$-\kappa_n + \sum_{t=0}^{\infty} \beta^t \left\{ \lambda_n \Pi(A_n(\mathbf{m})) - \gamma + \frac{1}{1+r} \left[\frac{\iota - \eta r}{1 - \eta} (\kappa_n - \eta \kappa'_n) + r \eta \kappa'_n \right] \right\} \geq 0.$$

This constraint can be simplified as

$$-\frac{r - \iota}{1 - \eta} (\kappa_n - \eta \kappa'_n) + (1 + r) [\lambda_n \Pi(A_n(\mathbf{m})) - \gamma] \geq 0. \quad (59)$$

The regulator then chooses policy parameters to maximize the social welfare. For a given policy \mathbf{m} and $\{\kappa_n\}$ and the DM trade q , the regulator maximizes the welfare given by

$$\sigma[u(q) - c(q)] - \sum_{n=1}^N m_n [\lambda_n \psi(A_n(\mathbf{m})) - \gamma], \quad (60)$$

subject to equilibrium implementation $c(q) = D(\iota(\mathbf{m}, \{\kappa_n\}))$ and incentive compatibility condition (59). The following lemma characterize optimal $\{\kappa_n\}$ for a given \mathbf{m} .

Lemma 4.3. *Let \mathbf{m} be given such that*

$$\lambda_n \Pi(A_n(\mathbf{m})) \geq \gamma \text{ for all } n \text{ with } m_n > 0.$$

For any given ι , it is optimal to set η according to (26), and to set $\kappa'_n = \kappa_n$.

(a) Let $\hat{\kappa}_n(\mathbf{m}) = \frac{1+r}{r}[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]$ for each $n = 1, \dots, N$. If

$$c(q^*) \leq \rho \frac{\tau}{1+r} \bar{A} + \sum_{n=1}^N m_n \hat{\kappa}_n(\mathbf{m}), \quad (61)$$

then $\iota = 0$ and $q = q^*$ in equilibrium under $\eta = 0$.

(b) Suppose that (61) does not hold. Then, there exists an optimal $\{\kappa_n\}$ under \mathbf{m} , denoted by $\{\bar{\kappa}_n(\mathbf{m})\}$, such that the constraint (59) is binding for all n with $m_n > 0$.

Now we are ready to characterize optimal policy.

Theorem 4.2. *Suppose that $\rho > \tilde{\rho}$. There exists an optimal policy \mathbf{m} and $\{\kappa_n\}$; in any optimal policy, we have that $\mathbf{m} \leq \mathbf{m}^*$, and that $m_n = \mu_n$ or $m_n = 0$ except for at most one n .*

(a) *Suppose that (61) holds for $\mathbf{m} = \mathbf{m}^*$, then $(\mathbf{m}^*, \{\hat{\kappa}_n(\mathbf{m}^*)\})$ is an optimal policy.*

(b) *Suppose that (61) does not hold for $\mathbf{m} = \mathbf{m}^*$.*

(b.1) *Any optimal policy $(\mathbf{m}, \{\bar{\kappa}_n(\mathbf{m})\})$ have $m_{\bar{n}} < m_{\bar{n}}^*$.*

(b.2) *Suppose that $\psi(A) = A^x$ for some $x > 1$. Then, for any optimal policy $(\mathbf{m}, \{\bar{\kappa}_n(\mathbf{m})\})$,*

$$\mathcal{L}_n = \frac{\rho \tau A_n(\mathbf{m}) + z_n + \bar{\kappa}_n(\mathbf{m})}{\tau A_n(\mathbf{m}) + z_n}$$

is strictly decreasing in n .

Theorem 4.2 (b.1) shows that unless the first-best is implementable, restriction in banking licence is optimal. This generalizes Theorem 3.2. Moreover, (b.2) shows that under the optimal arrangement, not only the regulator would allow higher unsecured deposit issuance for larger banks, the ratio between total debt and total asset also increases with the bank size; that is, it is optimal to allow for higher leverage ratio requirements for larger banks. The underlying intuition is this. When liquidity is tight, it is desirable to allow for more unsecured

deposit issuance to banks with higher profits (or with potential for higher profits). Because larger banks are more efficient in terms of asset management, they also have potential to obtain higher profits.

5 Moral hazard

We have shown that when designing an optimal policy in the presence of limited commitment of banks, the regulator needs to trade off efficiency and stability. Here we introduce another friction that is more akin the conventional moral hazard issue discussed in the literature. Our main focus is to what extent the competitive market can correct this issue and how this issue would interact with the optimal overall leverage ratio requirements we obtained in the last section.

Suppose that the return to a bank's loan holdings are subject to moral hazard and the bank may *gamble* on the assets. There are two benefits when banks commit gambling behavior. First it lowers the cost of managing assets. By gambling the cost of managing a units of assets is $\psi(a) - ea + \gamma$. Second, while by gambling the return is stochastic and lower on average, when it succeeds, banks receive private returns. Specifically, with probability $q \in (0, 1)$ the return from gambling will be $\tau_h > \tau$, and with probability $1 - q$ it will be $\tau_\ell < \tau$. When the return τ_h realizes, the difference $\tau_h - \tau$ is not observable and hence is the private gain to the bank. The decision to gamble is not observable, but when return τ_ℓ occurs, it is observable to all. We assume that

$$(1 + r)e < \tau - [q\tau_h + (1 - q)\tau_\ell]. \quad (62)$$

Condition (62) ensures that no gambling, or *prudent behavior*, is socially beneficial.

5.1 Static contracts

First we begin with the case where there is only reserve requirement and hence only static contracts are feasible. We shall impose free entry later, but for now assume that the number of active banks is given by a fixed m . Even in the absence of regulation, depositors can potentially discipline the bank not to gamble by not depositing in a bank without sufficient capital in place. In this subsection, we focus on equilibrium in which no banks gamble; we shall return to equilibria with gambling toward the end of the section.

To induce prudent behavior, the constraint (6) may no longer be appropriate as it may not induce efforts. Indeed, this is the case when ρ is in a certain range, as will be shown later. The market can discipline banks by demanding additional capital requirement. A more general pledgeability constraint thus is:

$$d \leq \rho\tau_\ell a + \omega(\tau - \tau_\ell)a + z, \tag{63}$$

for some $\omega \in [0, \rho]$. We still keep the reserve requirement as in (17). Note that when $\omega = \rho$, (63) coincides with (6). The parameter ω also has a simple interpretation: $1 - \omega$ stands for the share of the additional return that goes to the bank by being prudent. Thus, ω basically represents a capital requirement in which lower ω means that banks have to place more capital to finance their asset holdings. As we shall see later, when $\omega = 0$, all banks are willing to be prudent.

Here we give a remark about what we mean by equilibrium that induces prudent behavior under moral hazard, or a *prudent equilibrium*. The market would discipline the banks by not depositing in banks that do not satisfy the appropriate pledgeability constraints. Thus, in equilibrium the pledgeability constraint (63) must satisfy two conditions: first, it has to ensure that the banks are willing to be prudent; second, no banks can credibly issue more deposits than what the constraint requires. Equivalently, equilibrium requires the *highest*

ω under which no bank has incentive to gamble. It has to be the highest as for otherwise banks can credibly deviate to issue more deposits than the constraint (63) requires.⁹

To do this, we modify our previous analysis and obtain market clearing conditions. Recall that we assume a fixed number of active banks, m . By being prudent, the bank profit is obtained by substituting (63) at equality into (5) (as before, whenever $\iota \geq 0$, it is without loss of generality to assume that (63) is binding):

$$\begin{aligned} \pi(a, z, d; \phi, R) &= \frac{d}{R} - z - \phi a - [\psi(a) + \gamma] + \beta\{\tau a + z - d\} \\ &= \beta \left\{ \frac{\iota - r\eta}{1 - \eta} [\rho\tau_\ell + \omega(\tau - \tau_\ell)] a + [\tau - (1 + r)\phi] a - (1 + r)[\psi(a) + \gamma] \right\}. \end{aligned} \quad (64)$$

The FOC for (64) is thus

$$\frac{\iota - r\eta}{1 - \eta} [\rho\tau_\ell + \omega(\tau - \tau_\ell)] + [\tau - (1 + r)\phi] = (1 + r)\psi'(a).$$

Thus, the equilibrium price for trees is pinned down by market-clearing, $a = \bar{A}/m$:

$$\phi = \frac{\frac{\iota - r\eta}{1 - \eta} [\rho\tau_\ell + \omega(\tau - \tau_\ell)] + \tau - \psi'\left(\frac{\bar{A}}{m}\right) (1 + r)}{1 + r}; \quad (65)$$

Note that, as before, the bank profit is then given by $\Pi(\bar{A}/m) - \gamma$. Given ϕ , the equilibrium ι is then the unique solution to (with equality whenever $\iota > 0$)

$$D(\iota) \leq \frac{\rho\tau_\ell + \omega(\tau - \tau_\ell)}{1 - \eta} \bar{A}. \quad (66)$$

Finally, we also need to consider the profit to a bank if it gambles, taking ϕ as given. Though a gambling bank may issue deposits constrained by (63) (since the gambling decision is not

⁹Obviously, here we assume monotonicity in terms of incentivising banks to be prudent in terms of ω , a fact that will be confirmed in our equilibrium analysis.

observable), he only pays $d_\ell = \rho\tau_\ell a + z$ to depositors once the return turns out to be τ_ℓ ; hence, the current bank profit is given by (again, when $\iota \geq 0$, we may assume that (63) is binding)

$$\begin{aligned}\pi^s(a, z, d; \phi, R) &= \frac{d}{R} - z - \phi a - [\psi(a) - ea + \gamma] + \beta\{[q\tau_h + (1-q)\tau_\ell]a + z - qd - (1-q)d_\ell\} \\ &= \beta \left\{ \begin{array}{l} \frac{\iota - r\eta}{1-\eta}[\rho\tau_\ell + \omega(\tau - \tau_\ell)]a + [\tau - (1+r)\phi]a - (1+r)[\psi(a) + \gamma] \\ + [q\tau_h + (1-q)\tau_\ell - \tau + (1-q)\omega(\tau - \tau_\ell)]a + (1+r)ea \end{array} \right\}. \quad (67)\end{aligned}$$

The FOC implies that the asset holding for a gambling bank is given by A^s that solves

$$\frac{\iota - r\eta}{1-\eta}[\rho\tau_\ell + \omega(\tau - \tau_\ell)] + [\tau - (1+r)\phi] + [q\tau_h + (1-q)\tau_\ell - \tau + (1-q)\omega(\tau - \tau_\ell)] = (1+r)[\psi'(A^s) - e].$$

Hence, the bank profit under shirking is given by $\Pi(A^s) - \gamma$. Thus, to ensure that banks have no incentive to shirk, we the following condition:

$$\Pi(\bar{A}/m) - \Pi(A^s) \geq 0. \quad (68)$$

To summarize, equilibrium conditions then consist (65), (66), and (68). We have the following lemma.

Lemma 5.1. *Consider the static contract. In equilibrium $m = m^*$. The highest ω under which no bank gambles in equilibrium is given by $\min\{\omega_1, \rho\}$ with*

$$\omega_1 \equiv 1 - \frac{(1+r)e + q(\tau_h - \tau)}{(1-q)(\tau - \tau_\ell)}. \quad (69)$$

It is also straightforward to verify from the proof of Lemma 5.1 that banks have no incentive to gamble if and only if $\omega \leq \min\{\omega_1, \rho\}$ in equilibrium, and hence the highest ω is also one consistent with the incentive to issue deposits. Moreover, since equilibrium ι is

decreasing in ω , the highest ω is also optimal from the depositors' perspective.

Lemma 5.1 shows that the market can discipline banks to be prudent by demanding additional capital requirement parameterized by ω_1 . When ρ is relatively small, i.e., when $\rho \leq \omega_1$, the highest ω consistent with efforts is ρ and the presence of moral hazard does not affect the equilibrium allocation. In contrast, when ρ is relatively large and hence $\omega_1 < \rho$, the presence of moral hazard does limit the ability of the banks to provide liquidity. Finally, under fee entry, $m = m^*$; that is, with an appropriate capital requirement to counter the moral hazard problem, the equilibrium measure of banks is still m^* .

5.2 Charter system with moral hazard

Now we turn to the charter system with moral hazard. Relative to the literature, the novelty here is to study the two capital regulations together, one parameterized by ω and the other by κ . For simplicity we assume that for the given κ the reserve requirement is given by (29) with $\kappa' = \kappa$. In the Appendix we show that this is in fact optimal. Under the charter system with moral hazard, the general pledgeability constraint is given by:

$$d \leq \rho\tau_\ell a + \omega(\tau - \tau_\ell)a + z + \kappa. \quad (70)$$

Again, here we consider only equilibria that induce prudent behavior. Thus, the policy parameter now becomes $(m, \eta, \kappa, \omega)$.

First we do the equilibrium analysis for a given policy parameter. By being prudent, the bank profit is given by (assuming that $\eta \leq \iota/r$ so that all constraints are binding and banks

are willing to hold deposits):

$$\begin{aligned}\pi(a, z, d; \phi, R) &= \frac{d}{R} - z - \phi a - [\psi(a) + \gamma] + \beta\{\tau a + z - d\} \\ &= \beta \left\{ \frac{\iota - r\eta}{1 - \eta} [\rho\tau_\ell + \omega(\tau - \tau_\ell)]a + \iota\kappa + [\tau - (1 + r)\phi]a - (1 + r)[\psi(a) + \gamma] \right\}.\end{aligned}\quad (71)$$

The profit to each bank in equilibrium is then $\Pi(\bar{A}/m) - \gamma + \beta\iota\kappa$. Given the policy parameter $(m, \eta, \kappa, \omega)$, the equilibrium ι that satisfies market clearing is then the unique solution to (with equality whenever $\iota > 0$)

$$D(\iota) \leq \frac{\rho\tau_\ell + \omega(\tau - \tau_\ell)}{1 - \eta} \bar{A} + m\kappa. \quad (72)$$

Again, note that the only difference between (66) and (72) is the term $m\kappa$.

Now we turn the incentive compatibility of banks to be prudent and to repay κ . We assume that banks with return τ_ℓ will have their charters terminated. That is, losing the charter value is used as a threat by the regulator to prevent gambling behavior, and it is easy to see that this is the optimal punishment. Thus, a gambling bank with asset holding a only pays d_ℓ given by

$$d_\ell = \rho\tau_\ell a + z \quad (73)$$

to depositors under return τ_ℓ . Thus, the profit to a gambling bank is given by

$$\begin{aligned}\pi^s(a, z, d; \phi, R) & \\ &= \frac{d}{R} - z - \phi a - [\psi(a) - ea + \gamma] + \beta\{[q\tau_h + (1 - q)\tau_\ell]a + z - qd - (1 - q)d_\ell\} \\ &= \beta \left\{ \begin{aligned} &\frac{\iota - r\eta}{1 - \eta} [\rho\tau_\ell + \omega(\tau - \tau_\ell)]a + \iota\kappa + [\tau - (1 + r)\phi]a - (1 + r)[\psi(a) + \gamma] \\ &+ (1 - q)\kappa + [q\tau_h + (1 - q)\tau_\ell - \tau + \omega(1 - q)(\tau - \tau_\ell)]a + (1 + r)ea \end{aligned} \right\}.\end{aligned}\quad (74)$$

Again, note that the only difference between (74) and (67) is the term $\beta(\iota + 1 - q)\kappa$ and

hence has no bearings on FOC; so the optimal asset holding is still given by A^s and the profit is $\Pi(A^s) - \gamma + \beta(1 - q + \iota)\kappa$.

To ensure that banks follow equilibrium behavior, we have two incentive compatibility conditions, one for repaying κ , the other for being prudent. Since we assume that in equilibrium all banks are prudent, the first one is the same as before, (33) (with $\kappa' = \kappa$ though); note that, however, equilibrium ι is affected by ω through (72). Now, since a shirking bank will be closed only if the return τ_ℓ realizes, the second condition is new and is given by

$$\begin{aligned} & [\Pi(A^s) - \gamma] + \beta\iota\kappa + q\frac{\beta}{1-\beta} \left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r} \right] + (1-q)\beta\kappa \\ \leq & [\Pi(\bar{A}/m) - \gamma] + \beta\iota\kappa + \frac{\beta}{1-\beta} \left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r} \right]. \end{aligned}$$

The above condition is obtained by checking by the one-shot deviation. After some algebra, this is equivalent to

$$- [\Pi(A^s) - \Pi(\bar{A}/m) + \beta(1-q)\kappa] + \frac{\beta(1-q)}{1-\beta} \left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1+r} \right] \geq 0. \quad (75)$$

Theorem 5.1. *Let $m < m^*$ be given. Suppose that $\rho \in (0, 1)$. The optimal capital requirement is such that $\omega = \min\{\rho, \omega_1\}$ given by (69), and κ is the highest κ that satisfies (33) with ι determined by (72) and with $\omega = \min\{\rho, \omega_1\}$.*

Compared against Lemma 5.1, Theorem 5.1 shows that under the charter system the optimal ω is the same as that under market-discipline, and that it is optimal to use the dynamic incentive to increase κ and κ only. Note that, however, optimal κ is indeed affected by moral hazard, since the choice of ω does affect the amount of liquidity banks can provide through asset prices and returns on deposits. Moreover, since Theorem 5.1 holds for any given m , it follows that we can extend Theorem 3.2 to the case with moral hazard. In particular, Theorem 5.1 states that the highest κ exists for which (33) holds with ι determined by (72)

and with $\omega = \omega_1$. One can then solve for the optimal m and, as in Theorem 3.2, we will have $m < m^*$ and $\kappa > 0$ unless the first-best is implementable under $m = m^*$ and $\kappa = 0$, as well as $\omega = \omega_1$.

But how would the optimal m , and hence optimal κ and profits vary with the moral hazard issue? The following theorem gives a partial characterization when the parameters are close to the region where the first-best is implementable.

Theorem 5.2. *Suppose that $\psi(A) = \lambda A^x/x$, $x > 1$, and $c(q) = q$, and that*

$$\iota + \frac{D'(\iota)}{D(\iota)} \text{ is strictly increasing.} \tag{76}$$

If the first-best is not implementable under market discipline, then, as τ_h increases, optimal m decreases, and optimal profit increases.

Examples of utility functions that satisfy (76) include the functional form $u(q) = \theta q^\alpha/\alpha$ for any $\alpha \in (0, 1)$. As τ_h increases and hence the moral hazard issue becomes more serious, ω_1 decreases as well by Theorem 5.1. This directly decreases the amount of liquidity banks can provide. However, Theorem 5.2 shows that the optimal response to such change is to decrease m , which allow banks to acquire higher profits, and, therefore, permits the regulator to set the highest incentive feasible κ . This implies a nontrivial interaction between the conventional capital requirement designed to counter the moral hazard issue and the overall leverage ratio requirement in our charter system, with the aim to balance stability and liquidity. Crucially, this result follows from our explicit treatment of liquidity provision from banks. The threat to removing charters is used to control for banks' incentive to be prudent, as well as to provide sufficient liquidity that is otherwise tightened up by preventing banks from gambling.

5.3 Is gambling always bad?

Up to now we have focused on equilibrium with regulations that induce banks to be prudent, and have shown that the best equilibrium with prudent banks requires the capital requirement be given by $\omega = \omega_1$ even under the charter system. However, this can be very costly in terms of liquidity provision; in particular, when τ_h is high so that ω_1 is close to zero, it seems inefficient to insist on prudent behavior. To investigate whether it is indeed optimal for banks not to gamble, it is then necessary to understand whether an equilibrium with banks gambling exists in the first place. For the analysis below, we first begin with a given m .

In contrast to the previous section, when the regulator expects the banks to gamble, the pledgeability constraint has to take this into account. In particular, since the returns are now stochastic, the repayment should also depend on the bank return. Specifically, when a bank with asset holding a has return τ_h , he can repay up to

$$d_h = \rho\tau_h a + z + \kappa,$$

where κ is the unsecured lending; note that, as assumed earlier, the court cannot seize the difference $\tau_h - \tau$. When the return is τ_ℓ , the bank can only repay

$$d_\ell = \rho\tau_\ell a + z + \kappa.$$

Note that this differs from (73) by κ as gambling is expected and the regulator still requires the repayment κ to stay in business. Thus, the pledgeability constraint is bounded by the *expected* amount the bank can repay, and hence is given by

$$d \leq \rho[q\tau + (1 - q)\tau_\ell]a + z + \kappa, \tag{77}$$

and, as in the previous section, we set the reserve requirement as (29) with $\kappa' = \kappa$.

Note that here there is no extra capital requirement than what the court could enforce under the assumption that all banks gamble. Thus, the profit for a gambling bank is given by

$$\begin{aligned}\pi^s(a, d; \phi, R) &= \frac{d}{R} - z - \phi a - [\psi(a) - ea + \gamma] + \beta\{(q\tau_h + (1-q)\tau_\ell)a + z - qd_h - (1-q)d_\ell\} \\ &= \beta \left\{ \begin{aligned} &\frac{\iota-r\eta}{1-\eta}\rho[q\tau + (1-q)\tau_\ell]a + [q\tau_h + (1-q)\tau_\ell - (1+r)\phi]a \\ &+ \iota\kappa - (1+r)[\psi(a) - ea + \gamma] \end{aligned} \right\}. \quad (78)\end{aligned}$$

Note that the demand for deposits is not affected by the stochastic returns since both buyers and sellers are only concerned with the *expected* return in the CM. Thus, the market clearing condition for deposits,

$$D(\iota) \leq \rho \frac{[q\tau + (1-q)\tau_\ell]}{1-\eta} \bar{A} + m\kappa, \quad (79)$$

with equality whenever $\iota > 0$.

Finally, banks need incentives to gamble. Indeed, if $q = 0$, then no bank is willing to gamble by (62), as all the additional return apart from τ_ℓ is to the banks and (62) ensures that such gain is higher than the saved management cost. In general, we need to consider the profit for a bank not to gamble under pledgeability constraint (77), which is given by

$$\begin{aligned}\pi^p(a, d; \phi, R) &= \frac{d}{R} - z - \phi a - [\psi(a) + \gamma] + \beta\{\tau a + z - d_h\} \\ &= \beta \left\{ \begin{aligned} &\frac{\iota-r\eta}{1-\eta}\rho[q\tau + (1-q)\tau_\ell]a + [q\tau_h + (1-q)\tau_\ell - (1+r)\phi]a + \iota\kappa \\ &+ [\tau - q\tau_h - (1-q)\tau_\ell]a - \rho(1-q)(\tau - \tau_\ell)a - (1+r)[\psi(a) + \gamma] \end{aligned} \right\}. \quad (80)\end{aligned}$$

Now, compare (80) against (78), a prudent bank loses on two grounds: first, since a bank with return τ repays more and a prudent bank always has return τ , this hurts the profit of a prudent bank, and this is reflected in the term $\beta\rho(1-q)(\tau - \tau_\ell)a$; second, the prudent bank

has a higher variable cost. The only benefit is higher expected return, reflected in the term $\beta[\tau_h - q\tau - (1 - q)\tau_\ell]a$. Taking ϕ as given, the FOC then gives an optimal asset holding for a prudent bank, A^p . Incentive compatibility then requires

$$\Pi(A^p) \leq \Pi^s(\bar{A}/m).$$

We have the following theorem.

Theorem 5.3. *Let $m \leq m^*$ be given. Then, an equilibrium with all banks gambling exists and is unique if and only if $\omega_1 \leq \rho$, where ω_1 is given by (69). Moreover, optimal gambling equilibrium yields a higher welfare than optimal prudent equilibrium if and only if first-best is not implementable under optimal prudent equilibrium and*

$$\omega_1 < \rho q. \tag{81}$$

Theorem 5.3 shows that a gambling exists whenever the capital requirement is binding. Thus, as long as the regulator does not impose additional capital requirement than ρ , one should expect a gambling equilibrium. It also gives a full characterization for when such gambling equilibrium is better than imposing additional capital requirements to induce prudent behavior. Intuitively, this would be the case when the additional capital requirement is too stringent; inequality (81) gives a precise condition for this.

6 Concluding remarks

In this paper we take the liquidity role of banks seriously and derive optimal banking regulations. We have shown that when banks are subject to limited commitment, an overall leverage ratio requirement with restricted banking licence can be optimal for welfare in a

charter system. In particular, we have shown that under such arrangement, banks have higher profits and higher leverage ratio relative to the laissez-faire economy without banking regulations. This is broadly consistent with the contrast in these two dimensions for the US banking industry entering the Great Depression (an era where no serious regulations) and the industry entering the recent Financial Crisis (an era when more regulations are in place).

Compared to most of the literature, we have shown that considerations for liquidity provision can change many conventional wisdom about banking regulation. First, we show that when moral hazard issue becomes more serious, while it is optimal to increase asset-specific capital requirement, the overall leverage ratio requirement should not be proportionally increased, and it is in fact optimal to allow higher profits for banks to make them more trustworthy. Second, while it is true that in our model under deposit insurance moral hazard would require capital requirement from the regulator, it is not always the case that the regulator should discourage gambling.

Proofs of Lemmas and Theorems

Proof of Lemma 3.1 (a) In equilibrium $A(\iota, \phi) = \bar{A}/m^*$. Taking $\iota = 0$ and $a = \bar{A}/m^*$ into (7), we obtain ϕ given by (4). Finally, (12) ensures that (10) is satisfied with $D(0) = q^*$. (b) Again, in equilibrium $A(\iota, \phi) = \bar{A}/m^*$, and substituting $a = \bar{A}/m^*$ into (7) we obtain ϕ given by (13). Since $D(\iota)$ is strictly decreasing in ι there is a unique solution to (10): when $\iota = 0$, since (12) does not hold, the left-side of (10) is strictly greater than the right-side.

Proof of Lemma 3.2 Since market clearing requires each bank to hold \bar{A}/m units of assets, plugging $a = \bar{A}/m$ into (19) then implies (23). The rest of the argument is similar to Lemma 3.1.

Proof of Theorem 3.1 Note that when (12) holds the economy is at the first-best. So suppose that (12) does not hold. This implies that the unique solution to (27) satisfies $\iota \in (0, r)$, denoted $\tilde{\iota}$. Set $\tilde{\eta} = \tilde{\iota}/r$. We first claim that $\iota(\tilde{\eta})/r = \tilde{\eta}$, or, equivalently,

$$\iota(\tilde{\eta}) = \tilde{\iota}.$$

By (24), it is sufficient to show that

$$D(\tilde{\iota}) = \frac{\rho\tau}{1 - \tilde{\eta}} \bar{A} = \frac{\rho\tau}{1 - \tilde{\iota}/r} \bar{A},$$

which holds since $\tilde{\iota}$ satisfies (27).

Proof of Theorem 3.2 (b) Since (12) does not hold, Lemma 3.1 (b) implies that under $(m, \kappa) = (m^*, 0)$ the equilibrium allocation has $q < q^*$. We show that any optimal policy has $m < m^*$.

Now, define

$$\mathcal{S}(m, \iota) = \max_{\eta, \kappa, \kappa'} \frac{\rho\tau}{1-\eta} \bar{A} + m \frac{1}{1-\eta} (\kappa - \eta\kappa'),$$

subject to (18) and (33). From Theorem 3.1, $\eta = \frac{\iota}{r}$, and from Lemma 3.4, $\kappa = \kappa' = \frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r-\iota}$, which implies that

$$\mathcal{S}(\iota, m) = \max_{\kappa} \frac{r\rho\tau}{r-\iota} \bar{A} + m \frac{(1+r)\left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma\right]}{r-\iota}.$$

It is easy to verify that for any m , the optimal κ and ι is determined by $D(\iota) \leq \mathcal{S}(\iota, m)$ and with equality whenever $\iota > 0$. Let $\iota(m)$ be the unique solution.

Now, for all $\iota < r$,

$$\begin{aligned} \frac{\partial}{\partial m} \mathcal{S}(\iota, m^*) &= \frac{1+r}{r-\iota} \left[\Pi\left(\frac{\bar{A}}{m^*}\right) - \Pi'\left(\frac{\bar{A}}{m^*}\right) \left(\frac{\bar{A}}{m^*}\right) - \gamma \right] \\ &= -\frac{1+r}{r-\iota} \left[\psi''\left(\frac{\bar{A}}{m^*}\right) \frac{\bar{A}^2}{(m^*)^2} \right] < 0. \end{aligned}$$

The second equality is obtained by using the fact that when $m = m^*$, banks have zero profits and $\Pi\left(\frac{\bar{A}}{m^*}\right) - \gamma = 0$. Thus, when $\iota(m^*) > 0$, for $m < m^*$ but sufficiently close to m^* , we have $\iota(m) < \iota(m^*)$ and $D(\iota(m)) > D(\iota(m^*))$.

Before we prove Lemma 4.1, we first prove Claim 4.1.

Proof of Claim 4.1 It is easy to verify that for any given \mathbf{m} , $[A_1(\mathbf{m}), \dots, A_N(\mathbf{m})]$ given by (39) uniquely solves

$$\min_{(A_1, \dots, A_N)} \sum_{n=1}^N [m_n \lambda_n \psi(A_n) + m_n \gamma]$$

s.t. $\sum_{n=1}^N m_n A_n = \bar{A}$. Moreover, these solutions can be characterized as follows: for any \mathbf{m} , define $C(\mathbf{m})$ as the solution to

$$\sum_{n=1}^N m_n (\psi')^{-1} \left(\frac{C}{\lambda_n} \right) = \bar{A}. \quad (82)$$

$C(\mathbf{m})$ is well-defined by strict convexity of ψ . Then,

$$A_n(\mathbf{m}) = (\psi')^{-1} \left(\frac{C(\mathbf{m})}{\lambda_n} \right) \text{ if } m_n > 0, \quad A_n(\mathbf{m}) = 0 \text{ otherwise.}$$

Now, we can compute the derivatives:

$$\frac{\partial}{\partial m_n} C = - \frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}}, \quad (83)$$

$$\frac{\partial}{\partial m_n} A_{n'} = - \frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_{n'} \psi''[A_{n'}(\mathbf{m})]}. \quad (84)$$

Now, define

$$\Psi(\mathbf{m}) \equiv \sum_{n=1}^N [m_n \lambda_n \psi(A_n(\mathbf{m})) + m_n \gamma], \quad (85)$$

and we can rewrite the original problem, (37), as

$$\min_{\mathbf{m}} \Psi(\mathbf{m}) \text{ s.t. } m_n \leq \mu_n, \quad n = 1, \dots, N.$$

By (84), we have

$$\begin{aligned}
\frac{\partial}{\partial m_n} \Psi(\mathbf{m}) &= \lambda_n \psi(A_n(\mathbf{m})) + \gamma - \sum_{k=1}^N m_k \lambda_k \psi'(A_k(\mathbf{m})) \frac{A_n(\mathbf{m})}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_k \psi''[A_k(\mathbf{m})]} \\
&= \lambda_n \psi(A_n(\mathbf{m})) + \gamma - \lambda_n \psi'(A_n(\mathbf{m})) A_n(\mathbf{m}) \frac{\sum_{k=1}^N m_k \frac{1}{\lambda_k \psi''[A_k(\mathbf{m})]}}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \\
&= -\lambda_n [\psi'(A_n(\mathbf{m})) A_n(\mathbf{m}) - \psi(A_n(\mathbf{m}))] + \gamma \\
&= -\lambda_n \Pi(A_n(\mathbf{m})) + \gamma,
\end{aligned} \tag{86}$$

where the second equality follows from (39). Since for any \mathbf{m} , $\lambda_n \Pi(A_n(\mathbf{m}))$ is strictly decreasing in n among which $m_n > 0$. This implies the optimal solution has the form given by (40)-(43). Note that (38) guarantees that $\bar{n} \leq N$.

Proof of Lemma 4.1 We show that the unique equilibrium is given by $\mathbf{m} = \mathbf{m}^*$, and $A_n = A_n(\mathbf{m}^*)$. Notice that there is a unique $\iota > 0$ which is determined by

$$D(\iota) = \frac{\rho\tau}{1-\eta} \bar{A}.$$

It is straightforward to verify that the market clearing and free entry conditions are satisfied. Now, uniqueness of allocation follows from the fact that market-clearing for asset market and the FOC for asset holdings imply (39), and monotonicity of $\lambda_n \Pi(A_n(\mathbf{m}))$.

Proof of Lemma 4.3 Let $\bar{\kappa} = \sum_{n=1}^N m_n (\kappa_n - \eta \kappa'_n)$. Define

$$\mathcal{S}(\iota, \mathbf{m}) = \max_{\eta, \kappa_n, \kappa'_n} \frac{\rho\tau}{1-\eta} \bar{A} + \frac{\bar{\kappa}}{1-\eta},$$

subject to

$$-\frac{r-\iota}{1-\eta} \bar{\kappa} + (1+r) \sum_{n=1}^N m_n [\lambda_n \Pi(A_n(\mathbf{m})) - \gamma] \geq 0. \tag{87}$$

This implies that

$$\begin{aligned}\mathcal{S}(\iota, \mathbf{m}) &= \frac{\rho\tau}{1-\eta}\bar{A} + \frac{(1+r)\sum_{n=1}^N m_n [\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]}{r-\iota} \\ &= \frac{\rho\tau}{1-\eta}\bar{A} + \frac{(1+r)[- \Psi(\mathbf{m}) + \sum_{n=1}^N m_n \lambda_n \psi'(A_n(\mathbf{m}))A_n(\mathbf{m})]}{r-\iota},\end{aligned}$$

where Ψ is given by (85). Note that, for any fixed \mathbf{m} , $\mathcal{S}(\iota, \mathbf{m})$ is strictly increasing in ι .

For each n , let

$$\bar{\kappa}_n(\mathbf{m}, \iota) = \frac{(1-\eta)(1+r) [\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]}{r-\iota}.$$

Fixed some \mathbf{m} , we consider two cases.

(i) If $D(0) \leq \mathcal{S}(0, \mathbf{m})$, then q^* is implementable under $\eta = 0$ with $\kappa_n = \bar{\kappa}_n(\mathbf{m}, 0)$, which is optimal under \mathbf{m} .

(ii) Otherwise, let $\iota(\mathbf{m}) > 0$ be the unique solution to

$$D(\iota) = \mathcal{S}(\iota, \mathbf{m}). \tag{88}$$

Then, $\kappa_n = \bar{\kappa}_n(\mathbf{m}, \iota(\mathbf{m}))$ is optimal under \mathbf{m} .

Proof of Theorem 4.2 (b) Let $\iota(\mathbf{m})$ be defined by (88). Then, $\iota(\mathbf{m}^*) > 0$. Now,

$$\begin{aligned}\frac{\partial}{\partial m_{\bar{n}}}\mathcal{S}(\iota, \mathbf{m}^*) &= \frac{1+r}{r-\iota}\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*) \\ &+ \frac{(1+r)\left[-\frac{\partial\Psi(\mathbf{m}^*)}{\partial m_{\bar{n}}} + \sum_{n=1}^N m_n \lambda_n [\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*) + \psi'(A_n(\mathbf{m}^*))]\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*)\right]}{r-\iota} \\ &= \frac{1+r}{r-\iota}\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*))A_{\bar{n}}(\mathbf{m}^*) \\ &+ \frac{(1+r)\left[\sum_{n=1}^N m_n \lambda_n [\psi''(A_n(\mathbf{m}^*))A_n(\mathbf{m}^*) + \psi'(A_n(\mathbf{m}^*))]\frac{\partial}{\partial m_{\bar{n}}}A_n(\mathbf{m}^*)\right]}{r-\iota},\end{aligned}$$

since by (86) and by definition of \bar{n} ,

$$\frac{\partial}{\partial m_{\bar{n}}} \Psi(\mathbf{m}^*) = -[\lambda_{\bar{n}} \Pi(A_{\bar{n}}(\mathbf{m}^*)) - \gamma] = 0.$$

Now, by (84),

$$\begin{aligned} & \sum_{n=1}^N m_n \lambda_n \psi''(A_n(\mathbf{m}^*)) A_n(\mathbf{m}^*) \frac{\partial}{\partial m_{\bar{n}}} A_n(\mathbf{m}^*) \\ = & - \sum_{n=1}^N m_n \lambda_n \psi''(A_n(\mathbf{m}^*)) A_n(\mathbf{m}^*) \frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_n \psi''[A_n(\mathbf{m}^*)]} \\ = & - \left(\sum_{n=1}^N m_n A_n(\mathbf{m}^*) \right) \frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \\ = & - \frac{\bar{A} A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \end{aligned}$$

and

$$\begin{aligned} & \sum_{n=1}^N m_n \lambda_n \psi'(A_n(\mathbf{m}^*)) \frac{\partial}{\partial m_{\bar{n}}} A_n(\mathbf{m}^*) \\ = & - \sum_{n=1}^N m_n \lambda_n \psi'(A_n(\mathbf{m}^*)) \frac{A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \frac{1}{\lambda_n \psi''[A_n(\mathbf{m}^*)]} \\ = & - \lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) A_{\bar{n}}(\mathbf{m}^*) \frac{\sum_{n=1}^N \frac{m_n}{\lambda_n \psi''[A_n(\mathbf{m}^*)]}}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \\ = & - \lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) A_{\bar{n}}(\mathbf{m}^*), \end{aligned}$$

where the second last equality follows from the fact that $\lambda_{\bar{n}}\psi'(A_{\bar{n}}(\mathbf{m}^*)) = \lambda_n\psi'(A_n(\mathbf{m}^*))$ for all n with $\mathbf{m}_n^* > 0$. Combining the terms, we obtain

$$\begin{aligned} \frac{\partial}{\partial m_{\bar{n}}} \mathcal{S}(\iota, \mathbf{m}^*) &= \frac{1+r}{r-\iota} \lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) A_{\bar{n}}(\mathbf{m}^*) \\ &\quad - \frac{1+r}{r-\iota} \left[\lambda_{\bar{n}} \psi'(A_{\bar{n}}(\mathbf{m}^*)) A_{\bar{n}}(\mathbf{m}^*) + \frac{\bar{A} A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \right] \\ &= -\frac{(1+r)}{r-\iota} \left[\frac{\bar{A} A_{\bar{n}}(\mathbf{m}^*)}{\sum_{j=1}^N \frac{m_j}{\lambda_j \psi''[A_j(\mathbf{m})]}} \right] < 0. \end{aligned}$$

For $\iota > 0$, $\kappa_n = \bar{\kappa}_n(\mathbf{m}, \iota(\mathbf{m})) = \frac{(1+r)[\lambda_n \Pi(A_n(\mathbf{m})) - \gamma]}{r-\iota}$ is optimal under \mathbf{m} . We show that $\lambda_n \Pi(A_n(\mathbf{m}))$ decreases in n , so does $\kappa_n(\mathbf{m})$. Suppose that $x < y$. Then, equilibrium requires

$$\lambda_x \psi'(A_x) = \lambda_y \psi'(A_y).$$

Hence,

$$\lambda_x [\psi'(A_x) A_x - \psi(A_x)] > \lambda_y [\psi'(A_y) A_y - \psi(A_y)]$$

iff

$$A_x - \frac{\psi(A_x)}{\psi'(A_x)} > A_y - \frac{\psi(A_y)}{\psi'(A_y)}.$$

Now,

$$\frac{d}{dA} \left[A - \frac{\psi(A)}{\psi'(A)} \right] = 1 - \frac{[\psi'(A)]^2 - \psi(A)\psi''(A)}{[\psi'(A)]^2} = \frac{\psi(A)\psi''(A)}{[\psi'(A)]^2} > 0.$$

Proof of Lemma 5.1 We show that banks have no incentive to gamble if and only if $\omega \leq \omega_1$. Note that we only need to show that $A^s \leq \bar{A}/m$. This can be done by comparing the first-order conditions for the profits given by (64) and (67), and this would be the case

if and only if

$$\begin{aligned}
& \frac{\iota - r\eta}{1 - \eta} [\rho\tau_\ell + \omega(\tau - \tau_\ell)] + \tau - (1 + r)\phi \\
\geq & \frac{\iota - r\eta}{1 - \eta} [\rho\tau_\ell + \omega(\tau - \tau_\ell)] + \tau - (1 + r)\phi \\
& + q\tau_h + (1 - q)\tau_\ell - \tau + (1 - q)\omega(\tau - \tau_\ell) + (1 + r)e,
\end{aligned}$$

which holds if and only if $\omega \leq \omega_1$.

Proof of Theorem 5.1 First we show that for any given m , it is optimal to set $\omega = \omega_1$. Note that since m determines asset-management efficiency, the regulator's goal is only to increase liquidity, or, equivalently, to have the lowest equilibrium ι among all (κ, ω) that are incentive compatible. To do this, we consider a relaxed problem. Instead of working with the constraint (75), we consider a relaxed constraint: we assume that the shirking bank also chooses \bar{A}/m . In this case, the gain from gambling is the difference between two expressions (71) and (74) with $a = \bar{A}/m$, which is given by

$$\Phi \equiv \beta\{[\omega(1 - q)(\tau - \tau_\ell) + q(\tau_h - \tau) - (\tau - q\tau - (1 - q)\tau_\ell)]\bar{A}/m + (1 + r)e\bar{A}/m + (1 - q)\kappa\}. \quad (89)$$

Thus, for banks holding \bar{A}/m units of trees not to shirk it requires

$$-\Phi + (1 - q)\frac{\beta}{1 - \beta} \left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma + \frac{\iota \cdot \kappa}{1 + r} \right] \geq 0,$$

which can be simplified to

$$\begin{aligned}
& -r [(\omega - 1)(1 - q)(\tau - \tau_\ell) + (1 + r)e + q(\tau_h - \tau)] \frac{\bar{A}}{m} \\
+ & (1 - q) \left\{ -(r - \iota)\kappa + (1 + r) \left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma \right] \right\} \geq 0. \quad (90)
\end{aligned}$$

Note that we could rewrite (75) as

$$-r(1+r) \left[\Pi(A^s) - (1-q)\Pi\left(\frac{\bar{A}}{m}\right) \right] + (1-q) \left\{ -(r-\iota)\kappa + (1+r) \left[\Pi\left(\frac{\bar{A}}{m}\right) - \gamma \right] \right\} \geq 0.$$

Moreover, $\Pi(A^s) - \Pi\left(\frac{\bar{A}}{m}\right)$ and $[(\omega-1)(1-q)(\tau-\tau_\ell) + (1+r)e + q(\tau_h - \tau)]$ have the same sign for all ω (it is positive for $\omega > \omega_1$, negative for $\omega < \omega_1$, zero for $\omega = \omega_1$), and

$$\Pi(A^s) - \Pi\left(\frac{\bar{A}}{m}\right) > [(\omega-1)(1-q)(\tau-\tau_\ell) + (1+r)e + q(\tau_h - \tau)] \left(\frac{\bar{A}}{m}\right)$$

for all $\omega > \omega_1$. Since when both terms are negative the corresponding constraints are weaker than (33), (90), combined with (33), is indeed weaker than (75) combined with (33). When $\omega = \omega_1$, they are equivalent.

Now, define

$$\mathcal{S}(\iota) = \max_{\omega, \kappa} \frac{r\rho\tau_\ell + \omega r(\tau - \tau_\ell)}{r - \iota} \bar{A} + m\kappa$$

subject to (90) and (33). We claim that the minimum equilibrium ι subject to (90) and (33) is determined by $D(\iota) \leq \mathcal{S}(\iota)$ (at equality whenever $\iota > 0$). Note that $\mathcal{S}(\iota)$ is strictly increasing in ι : as ι increases both constraints (90) and (33) are more relaxed, and the objective function is strictly increasing in ι .

For any fixed ι , the maximization problem in $\mathcal{S}(\iota)$ is a linear programming problem in (κ, ω) and can be reduced to

$$\begin{aligned} & \max_{\kappa, \omega} \frac{\omega r(\tau - \tau_\ell)}{r - \iota} \bar{A} + m\kappa, \\ \text{s.t.} \quad & -r\omega(\tau - \tau_\ell) \frac{\bar{A}}{m} - (r - \iota)\kappa + C \geq 0, \\ & -(r - \iota)\kappa + D \geq 0. \end{aligned}$$

where by (62),

$$\begin{aligned} C &= r \left[(\tau - \tau_\ell) \frac{\bar{A}}{m} - \frac{(1+r)e + q(\tau_h - \tau)}{1-q} \frac{\bar{A}}{m} \right] + (1+r) \left[\Pi \left(\frac{\bar{A}}{m} \right) - \gamma \right] \\ &> (1+r) \left[\Pi \left(\frac{\bar{A}}{m} \right) - \gamma \right] = D. \end{aligned}$$

Since $\rho < 1$, the optimal choice is given by

$$\kappa = \frac{D}{r - \iota}, \quad \omega = \frac{C - D}{r(\tau - \tau_\ell) \frac{\bar{A}}{m}} = \omega_1.$$

Proof of Theorem 5.2 The social planner's problem is given by

$$\begin{aligned} &\max_{m \geq 0, \iota \geq 0} \sigma[u(q) - c(q)] - \left[m\gamma + m\psi \left(\frac{\bar{A}}{m} \right) \right], \\ \text{s.t. } &D(\iota) \leq \frac{r}{r - \iota} [\rho\tau_\ell + \omega_1(\tau - \tau_\ell)] \bar{A} + m \frac{(1+r)[\psi'(\bar{A}/m)\bar{A}/m - \psi(\bar{A}/m) - \gamma]}{r - \iota}, \\ &q = D(\iota), \end{aligned}$$

where

$$\omega_1 = \min \left\{ \rho, 1 - \frac{(1+r)e + q(\tau_h - \tau)}{(1-q)(\tau - \tau_\ell)} \right\}.$$

Now, let

$$F(\iota, m) = -D(\iota) + \frac{r}{r - \iota} [\rho\tau_\ell + \omega_1(\tau - \tau_\ell)] \bar{A} + m \frac{(1+r)[\psi'(\bar{A}/m)\bar{A}/m - \psi(\bar{A}/m) - \gamma]}{r - \iota}.$$

Then,

$$\begin{aligned} \frac{\partial}{\partial m} F &= \frac{(1+r)}{r - \iota} \left\{ \left[\psi' \left(\frac{\bar{A}}{m} \right) \frac{\bar{A}}{m} - \psi \left(\frac{\bar{A}}{m} \right) - \gamma \right] - \psi'' \left(\frac{\bar{A}}{m} \right) \left(\frac{\bar{A}}{m} \right)^2 \right\}, \\ \frac{\partial}{\partial \iota} F &= -D'(\iota) + \frac{r[\rho\tau_\ell + \omega_1(\tau - \tau_\ell)] \bar{A} + m(1+r) \left[\psi' \left(\frac{\bar{A}}{m} \right) \frac{\bar{A}}{m} - \psi \left(\frac{\bar{A}}{m} \right) - \gamma \right]}{(r - \iota)^2}. \end{aligned}$$

Let $\iota(m)$ be the implicit function defined by $F(\iota, m) = 0$, $m \leq m^*$. Then,

$$\begin{aligned} \iota'(m) &= -\frac{(1+r)(r-\iota) \left\{ \left[\psi' \left(\frac{\bar{A}}{m} \right) \frac{\bar{A}}{m} - \psi \left(\frac{\bar{A}}{m} \right) - \gamma \right] - \psi'' \left(\frac{\bar{A}}{m} \right) \left(\frac{\bar{A}}{m} \right)^2 \right\}}{-D'(i)(r-\iota)^2 + \left\{ r[\rho\tau_\ell + \omega_1(\tau - \tau_\ell)]\bar{A} + m(1+r) \left[\psi' \left(\frac{\bar{A}}{m} \right) \frac{\bar{A}}{m} - \psi \left(\frac{\bar{A}}{m} \right) - \gamma \right] \right\}} \\ &= -\frac{(1+r) \left\{ \left[\psi' \left(\frac{\bar{A}}{m} \right) \frac{\bar{A}}{m} - \psi \left(\frac{\bar{A}}{m} \right) - \gamma \right] - \psi'' \left(\frac{\bar{A}}{m} \right) \left(\frac{\bar{A}}{m} \right)^2 \right\}}{-D'(\iota)(r-\iota) + D(\iota)} > 0. \end{aligned}$$

Thus, the FOC for the social planner's problem is given by

$$\sigma \left[\frac{u'(q)}{c'(q)} - 1 \right] D'(\iota)\iota'(m) + \left[\Pi \left(\frac{\bar{A}}{m} \right) - \gamma \right] = 0,$$

that is,

$$\sigma \left[\frac{u'(q)}{c'(q)} - 1 \right] \frac{(1+r) \left\{ \left[\Pi \left(\frac{\bar{A}}{m} \right) - \gamma \right] - \psi'' \left(\frac{\bar{A}}{m} \right) \left(\frac{\bar{A}}{m} \right)^2 \right\}}{(r-\iota) + \frac{D(\iota)}{-D'(\iota)}} + \left[\Pi \left(\frac{\bar{A}}{m} \right) - \gamma \right] = 0. \quad (91)$$

Now, if we plug in the functional form $\psi(A) = \lambda A^x/x$ and $c(q) = q$, then We have the following FOC:

$$\sigma \{u'[D(\iota)] - 1\} \frac{-(1+r) \left[\lambda \frac{(x-1)^2}{x} \left(\frac{\bar{A}}{m} \right)^x + \gamma \right]}{(r-\iota) + \frac{D(\iota)}{-D'(\iota)}} + \left[\lambda \frac{x-1}{x} \left(\frac{\bar{A}}{m} \right)^x - \gamma \right] = 0,$$

that is,

$$\frac{\sigma \{u'[D(\iota)] - 1\}}{(r-\iota) + \frac{D(\iota)}{-D'(\iota)}} = \frac{\left[\lambda \frac{x-1}{x} \left(\frac{\bar{A}}{m} \right)^x - \gamma \right]}{(1+r) \left[\lambda \frac{(x-1)^2}{x} \left(\frac{\bar{A}}{m} \right)^x + \gamma \right]}, \quad (92)$$

where ι is an implicit function of m with $\iota'(m) > 0$. Now, by (76), the left-side of (92) is strictly increasing in m , while the right-side is strictly decreasing. Since the first-best is not implementable, there exists a unique \tilde{m} that solves (92) which is the optimal m . As optimal

ω_1 decreases, $i(m)$ increases for any m , and decreases the optimal \tilde{m} .

Proof of Theorem 5.3 First we show that banks have no incentive to be prudent if and only if $\rho \leq \omega_1$. Note that we only need to show that $A^p \leq \bar{A}/m$. This can be done by comparing the first-order conditions for the profits given by (78) and (80), and this would be the case if and only if

$$\begin{aligned} & \frac{\iota - r\eta}{1 - \eta} \rho [q\tau + (1 - q)\tau_\ell] + [q\tau_h + (1 - q)\tau_\ell - (1 + r)\phi] + (1 + r)e \\ \geq & \frac{\iota - r\eta}{1 - \eta} \rho [q\tau + (1 - q)\tau_\ell] + [q\tau_h + (1 - q)\tau_\ell - (1 + r)\phi] \\ & + [\tau - q\tau_h - (1 - q)\tau_\ell]a - \rho(1 - q)(\tau - \tau_\ell), \end{aligned}$$

which holds if and only if $\omega_1 \leq \rho$.

Now for any fixed m , it is easy to show that the optimal gambling equilibrium is characterized by the following

$$D(\iota) \leq \mathcal{S}^g(\iota),$$

where

$$\mathcal{S}^g(\iota) = \max_{\kappa} \frac{\rho r [q\tau + (1 - q)\tau_\ell]}{r - \iota} \bar{A} + m\kappa, \quad (93)$$

subject to (33). Note that under this equilibrium banks profits are still given by $\Pi(\bar{A}/m) + \iota\kappa/(1 + r) - \gamma$.

However, as shown in Theorem 5.1, the optimal prudent equilibrium is characterized by

$$D(\iota) \leq \mathcal{S}^p(\iota),$$

where

$$\mathcal{S}^p(\iota) = \max_{\kappa} \frac{r[\rho\tau_{\ell} + \omega(\tau - \tau_{\ell})]}{r - \iota} \bar{A} + m\kappa, \quad (94)$$

subject to (33).

Since the κ term will be replaced by the same expression, $\mathcal{S}^g(\iota) > \mathcal{S}^p(\iota)$ if and only if

$$\begin{aligned} & \rho r[q\tau + (1 - q)\tau_{\ell}] \\ > & r[\rho\tau_{\ell} + \omega(\tau - \tau_{\ell})], \end{aligned}$$

which is equivalent to (81).

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Appendix

A1. General linear pledgeability constraint

In this section we show that the pledgeability constraint (28) is optimal among all other linear constraints of the form:

$$d \leq (\eta_0\phi + \eta_1\tau)a + \kappa. \quad (95)$$

For the given η_0 and η_1 , one can show that in equilibrium we have

$$\phi = \frac{(1 + \nu\eta_1)\tau - (1 + r) \left[\psi' \left(\frac{\bar{A}}{m} \right) + \gamma \right]}{r - \nu\eta_0}.$$

Hence, the total liquidity banks provide is given by

$$\begin{aligned} \mathcal{S} &= \frac{(r\eta_1 + \eta_0)\tau - \eta_0(1 + r) \left[\psi' \left(\frac{\bar{A}}{m} \right) + \gamma \right]}{r - \nu\eta_0} \bar{A} + m\kappa \\ &= \frac{[r(\eta_1 - \rho) + (\eta_0 - \rho) + \rho(\eta_1 - \eta_0)\iota]\tau - (\eta_0 - \rho)(1 + r) \left[\psi' \left(\frac{\bar{A}}{m} \right) + \gamma \right]}{r - \nu\eta_0} \bar{A} + m\kappa + \rho(\phi + \tau)\bar{A}. \end{aligned}$$