# A Model of Collateral*

Yu Awaya[†] Hiroki Fukai[‡] Makoto Watanabe[§]

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**Abstract**

This paper presents a simple equilibrium model in which collateralized credit emerges endogenously. Individuals cannot commit to the use of collateral as a guarantee of repayment, and both lenders and borrowers have incentives to renege. Our theory provides a micro-foundation to justify the borrowing constraints that are widely used in the existing macroeconomic models. We explain why assets are often used as collateral, rather than as a means of payment, why there is a tradeoff in assets between return and liquidity, and what kinds of assets are useful as collateral.

**Keywords:** collateral; payment puzzle; search; medium of exchange

**JEL Classification Numbers:** C73, E30, E50

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[†]University of Rochester. Address: 208 Harkness Hall, Rochester, NY 14627, US. Email: Yu-Awaya@gmail.com

[‡]Kyushu University. Address: 744 Motooka W2-1023, Nishi-ku, Fukuoka-shi, Fukuoka 819-0395, Japan. E-mail: fukai.hiroki@gmail.com

[§]Vrije Universiteit Amsterdam. Address: DeBoelelaan 1105, 1081 HV Amsterdam, The Netherlands. Email: makoto.wtnb@gmail.com

# 1 Introduction

*If you repay me not on such a day,*
*In such a place, such sum or sums as are*
*Express'd in the condition, let the forfeit*
*Be nominated for an equal pound*
*Of your fair flesh, to be cut off and taken*
*In what part of your body pleaseth me.* *(1.3.156-163, The Merchant of Venice.)*

Collateral is the linchpin of credit and intertemporal resource allocations. It is commonly believed that pledgable assets, typically, houses or lands, need to have sufficiently high monetary value. However, secured credit can occur even with an object intrinsically worthless to lenders, e.g., a pound of Antonio's flesh to Shylock. Why do lenders agree to take such an object as collateral?

This paper proposes a micro-founded model where secured credit emerges endogenously. Even in a frictional world, where commitment is limited, we show that collateral can serve as a credible device that prevents the participating parties from reneging. Our starting point is to realize that the borrower's value of a pledgeable asset may be above its return. Indeed, in many real-life settings, the borrower may not wish to lose his collateralizable asset even if there is a chance to get it back on the market, e.g., a suitable land for a restaurant owner, the ownership of a company for an entrepreneur, and even liquid securities for an investor. In our theory, if a borrower does not have pledgeable assets, lenders may not want to lend to him, because it is indicated that he has reneged and his collateral has been confiscated in the past. Thus, a defaulter would lose not only the returns of the pledged asset but also future transactions. The maximum payment a borrower can promise in equilibrium is determined not only by the return of the pledged asset, just like in the seminal work by Kiyotaki and Moore [16], but also by the value of future transactions. This is the reason why the lender is willing to accept even an intrinsically worthless object as collateral.[1]

Using this framework, we address the "payment puzzle" raised by Lagos [21]. Namely, why even to this day assets are used as collateral rather than simply as a means of pay-

[1]In old days, when lending to new college graduates, a moneylender accepts the original copy of their graduation certificate as collateral, which is intrinsically worthless. According to the critics of that time (Wagatsuma [29]), this is because of the earlier practice that new hirees need to show the original copy of their college certificate directly to their employer when starting a new job, and so losing it implies the loss of future potential productivity.

ment? Our answer is based on the nature of collateralized credit. In our equilibrium, a lender knows that a borrower will redeem his debt and get back his pledged asset because it is a valuable pass for him to enter future transactions and he does not want to lose it. Hence, even with a poor means of payment, the borrower is able to make a credible promise that he will never renege and so the lender agrees to trade with him. This result survives even if there is a market for the pledgable asset. We show that if the market is frictionless, then the parties involved can buy or sell it at a competitive price whenever needed whenever they need to, and so the asset can be used as either collateral or a means of payment, or as both. If the market is frictional, e.g., if a chance exists—however tiny it is—not to be able to find his counterparty, the borrower is strictly better-off by keeping his pledged asset, rather than buying a new one on the market. Hence, with market frictions, collateral is *essential*, in the sense that the equilibrium with collateral is more efficient than the equilibrium without collateral (even if the latter equilibrium exists). Further, if the market is frictional enough, assets will be used only as collateral, but never as a means of payment. Notice that this holds true even for financial assets, e.g., government bond, whose intrinsic value is independent of the identity of the assets' holders.

While the above logic is driven solely by the borrower's incentive, we find that the lender's lack of commitment is also important for generalizing our insight. What is a good asset used as collateral? Consider a repurchase agreement (repo), where a dealer (borrower) sells government securities to an investor (lender) and receives money, usually on an overnight basis, and buys the securities back the following day (see, e.g., Gorton and Metrick [10] for the recent empirical findings on repo markets). Despite the similarities to collaterialized loans, repos are actual purchases. During the life of a repo, the investor holds legal title to the securities. The investor can renege and keep owning the securities, rather than returning them back to the original owner and getting her money back with some interests. Our theory captures this feature of repos with the setup that no individual can commit to the use of collateral as a guarantee of repayment, and so both dealers and investors have incentives to renege. Then, the payment puzzle still applies: why not settle the payment by using the securities, i.e., selling off the securities, rather than buying them back? In contrast to the borrower (dealer)'s deviation described above, which works in favor of repos over spot trade, the lender (investor)'s incentive makes repos harder to sustain, unless her action is observable to the future market, especially when the borrower's pledged asset has high returns. Hence, an asset suitable as collateral should have relatively high value for the borrower and low value for the lender.

Our model delivers novel implications to other related macroeconomic issues. For instance, many macroeconomic models consider cases in which some assets have high returns but low liquidity, and it is shown that such a case delivers interesting macroeconomic implications regarding market liquidity (see e.g., Matsuyama [25, 26], and Lagos and Rocheteau [22]). As a complementary effort to this line of works, we explore the very reason why this phenomenon can occur in the first place—why does an asset with high return, which should be attractive to many market participants, fail to deliver high liquidity? Our answer is that an asset with very high return is problematic because the lender has a very strong incentive to default and run away with it. Hence, an asset with high return can poorly back credit trades, only generating low liquidity.

We can also study situations where assets are better described as storable goods rather than as durable goods. While durable goods yield utility every period, e.g., houses, lands, and financial assets, the owner of a storable good has to "liquidate" it to obtain utility, and once it is liquidated, the good disappears, e.g., production inputs, inventories, and wine. Because of this liquidation concern, the durable good is more suitable as collateral than the storable good from both the efficiency and the incentive point of view. This result may justify the wide use of durable goods as collateral in real-life markets.

Before closing this introductory section, it is worth comparing our paper with the existing literature. In their influential work, Kiyotaki and Moore [16] emphasize the importance of the borrowing constraint associated with collateralized credit. The debt limit is determined by the maximum payment a borrower can promise. In their model, given that the borrower cannot sell off his future labor to guarantee his debts, it is bounded by the value of the asset pledged as collateral, which will be confiscated in case of default. We generalize their insight with an endogenous market institution in which the punishment is allowed to involve not only seizing assets pledged as collateral but also taking away defaulters' future credit.

The role of collateral to back credit trade is related to the role of money in monetary models (see Kiyotaki and Wright [18]), in which an intrinsically worthless object—fiat money—has a positive equilibrium value because it provides partial information on whether an individual has worked in the past or not (Kocherlakota [19]). Other related papers include Kiyotaki and Wright [17] on commodity money, Kehoe and Levine [15] and Gu, Mattesini, Monnet and Wright [12, 13] on the issue of commitment and credit constraints, Albuquerque and Hopenhayn [1] on some related issues using a dynamic contract approach, Kocherlakota [20] on risky collateral as a mechanism to enforce contracts, and Ferraris and Watanabe [5, 6] on a monetary equilibrium with collateralized credit. Recent related papers on repo contracts are, among others, Parlatore [27], who studies repos as the borrower's financing choice problem, Dang, Gorton, and Holmstrom [2], who study overcollateralization and repo runs as the adverse selection problem with a borrower's default risk, Gottardi, Maurin, and Monnet [11], who study the role of repos as insuring against price fluctuations, and Infante [14] who studies the intermediary's problem in repo markets to bring together lenders and borrowers.

Finally, while we assume in the current paper that matched agents separate exogenously each period, our insight will go through with an alternative setup where agents can choose whether or not to continue their relationship each period.[2] That is, limited commitment applies not only to actions within matches but also to partnerships themselves. In this alternative setup, credit should be based on a long-term relationship, which may be a good description of some real-life credit relationships.[3]

The rest of the paper is organized as follows. Section 2 presents the basic setup and studies the equilibrium without collateral. Section 3 describes the equilibrium with collateralized credit. Section 4 derives the macroeconomic implications. Section 5 considers extensions of the model. Section 6 concludes.

[2]This version of the model is available upon request.

[3]This line of work would be related to a version of voluntary separable repeated games, originated in Datta [3] and Ghosh and Ray [9]. Eeckhout [4], Fujiwara-Greve and Okuno-Fujiwara [7], and Fujiwara-Greve, Okuno-Fujiwara and Suzuki [8] consider institutions that facilitate cooperation under such frictions: Eeckhout [4] considers color–a payoff irrelevant and history independent characteristic, and Fujiwara- Greve, Okuno-Fujiwara and Suzuki [8] consider referral letters as methods to convey partial information on past actions. In our paper, another institution—collateral—works in the presence of such frictions.

## 2 Baseline environment

We first examine a benchmark environment *without* any collateral object.

Time is discrete and lasts forever. It is indexed by $t = 1, 2, \cdots$. There is a continuum of individuals. Each individual is either a *borrower* or a *lender*. The measures of both parties are unity. All individuals are long-lived and have a common discount factor $\delta \in (0, 1)$.

In this benchmark model, there are two kinds of goods—durable *production* and perishable *consumption* goods. In Section 3, we will introduce another kind of good into the economy, a good used as collateral.[4] Both production and consumption goods are perfectly divisible. Each lender owns one unit of the production good. In each period, a lender (resp. a borrower) can costlessly produce one unit (resp. $a$ units) of the consumption good by using one unit of the production good. Assume $a > 1$ so that borrowers have a better production technology. Individuals can only consume the consumption good. It is impossible to produce the production good.

In each period, a borrower and a lender engage in a pairwise trade. At the start of each period, no individual has a partner. Then, a borrower (resp. lender) finds a lender (resp. borrower) from the set of all lenders (resp. borrowers) at random. For simplicity, we assume that every individual can find his or her counterpart with probability one.[5]

Each period is divided into two subperiods in the following manner.

**Subperiod 1** In a pair, the lender can lend the right to use of the production good to the borrower. She chooses what portion of it to lend. Let $q \in [0, 1]$ denote the portion lent to borrower. The lender uses the rest $1 - q$ by herself.

**Subperiod 2** The borrower then produces $aq$ units of the consumption good by using $q$ units of the production good and chooses how much consumption good to give to the lender. Let $r \in [0, aq]$ denote the amount of the consumption good given to lender. After this trade, all matches separate.

The per-period payoff is linear in consumption. Given $(q, r)$, the per-period payoff

[4] In a repo, borrowers (resp. lenders) are referred to as sellers (resp. buyers) who sell (resp. buy) government securities in exchange for cash, instead of production good.

[5] Our results survive when there is a small chance that an unmatched individual cannot find a partner.

of a lender is $r + 1 - q$, while that of a borrower is $aq - r$. Obviously, the unique Nash equilibrium in the corresponding one-shot game is $(q, r) = (0, 0)$, i.e., no trade, while any efficient allocation must satisfy $q = 1$ (i.e., full lending) in (almost) every match.[6] We assume that pairwise trades are the only possible opportunity to trade goods. In particular, there is no centralized market for the consumption good.

Importantly, the history of past actions is not public. Each individual only observes her partners' actions, but does not observe the past actions of any other individual. The equilibrium notion is *sequential equilibrium* (simply, *equilibrium* henceforth). The lifetime payoff is

$$(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}c_t$$

where $c_t$ is the consumption in period $t$.

The following proposition establishes inefficiency in our environment without collateral.

**Proposition 1.** *In any equilibrium, all lenders choose $q = 0$ on the equilibrium path.*

*Proof.* Suppose, by way of contradiction, that there were an equilibrium in which a positive measure of lenders choose $q > 0$. Then a positive fraction of borrowers must choose $r > 0$ with a positive probability because, otherwise, $q > 0$ could never be optimal for lenders. It would be profitable for such a borrower to choose $r = 0$—future lenders would not know that the borrower had deviated. □

# 3 Collateral

## 3.1 Durable good

We now introduce another kind of good into the economy. We will show that individuals can use it as collateral—in particular, they can arrange repurchase agreement using it—and that it improves efficiency. The good—called the *durable* good—is durable and indivisible. At start of the first period, each borrower has one unit of such a durable good. We assume that the durable good cannot be produced.[7] We also assume that paired individuals observe each other's durable good holdings.

[6] The word *almost* simply comes from the fact that each individual has zero measure, and will be omitted hereafter.

[7] Our results survive when the durable good can be costly produced and the cost of production is sufficiently large. On the other hand, if the cost of production is too small, our results fail to hold.

We keep the assumption that all trades must be made within each pair. In particular, it is impossible to transfer the durable good to someone outside (in Section 5.4, we relax this assumption and analyze the case where the durable good is traded in a market). Specifically, a pair can trade the durable good in the first and second subperiods. Let $g_b \in \{\textit{not give}, \textit{give}\}$ denote the borrower's action, where $g_b = \textit{give}$ (resp. $g_b = \textit{not give}$) denotes giving (resp. not giving) the durable good to the lender. Let $g_\ell \in \{\textit{not give}, \textit{give}\}$ be defined likewise as the lender's action.

At any moment, an owner of the durable good can freely destroy it. If he or she does not destroy it, he or she obtains "utility" flow $y \in \mathbb{R}$ at the end of each period from holding it. We emphasize that we do *not* restrict $y$ to be positive. If $y = 0$, the good is intrinsically useless, and if $y < 0$, the good is costly to hold. We call $y$ the *value* of a durable good, because if an individual holds the durable good forever, he or she obtains lifetime utility $y = (1 - \delta) \sum_{t=1}^{\infty} \delta^{t-1} y$.

With such a durable good, consider the following *repurchase agreement (repo) strategy*: In each period, in the first subperiod the borrower "sells" the durable good in exchange for the right to use of one unit of the production good, and in the second subperiod, he "buys back" the durable good by paying the consumption good.

## 3.2 Repurchase agreement strategy

Formally, define the repo strategy as follows:

**Subperiod 1** If the borrower has the durable good, then trade it for the right to use of the production good, that is, $(q, g_b) = (1, \textit{give})$.

Otherwise, make no trade, that is, $(q, g_b) = (0, \textit{not give})$.

**Subperiod 2** If the trade took place in the first subperiod, then trade the durable good for the consumption good, that is, $(r, g_\ell) = (r^*, \textit{give})$, where $r^* \in [0, a]$ will be specified later.

Otherwise, make no trade, that is, $(r, g_\ell) = (0, \textit{not give})$.

After this trade, all matches separate.

**Subperiod 3** If the borrower has the durable good, he keeps it. If the lender has it, she keeps it if and only if $y > 0$ and destroy it otherwise. If one keeps the durable good, he or she receives utility or disutility of $y$.

Following the convention of exchange, the transfer of goods within each subperiod occurs at the same time. For example, in the first subperiod, once a borrower and a lender agree with the trade $(q, g_b) = (1, give)$, the lender cannot escape without giving the right to use of the production good after receiving the durable good from the borrower. The same in the second subperiod.

Notice, however, that a lender can, if she wants to, escape with collateral. In Section 5.1, we consider the case where lenders cannot, and we compare results and implications of the two cases.

By construction, clearly, the outcome of the repo strategy is efficient. In the following proposition, we provide a necessary and sufficient condition for which the repo strategy constitutes an equilibrium. In particular, we show that for any $(\delta, a)$, there is a non-empty open set of the value $y$ of the durable good in which the repo strategy constitutes an equilibrium.

**Theorem 1.** *The repo strategy constitutes an equilibrium if and only if*

$$y \in [1 - \delta a, (1 - \delta)a] \tag{1}$$

Notice that $(1-\delta)a-(1-\delta a) = a-1 > 0$. Thus, for any $(\delta, a)$, there is a non-empty open set of $y$ in which the repo strategy constitutes an equilibrium. Notice also that it does not have to be the case that $1-\delta a > 0$. When $\delta a > 1$, the repo strategy constitutes an equilibrium even when $y < 0$.

*Proof.* First, consider a borrower's incentives. Observe that the most profitable deviation is to refuse the trade in the second subperiod. In the first subperiod, it is not optimal for the borrower to refuse the trade. If he refuses, then there will be no trade and he will simply lose one period.

Here, we will check the deviation in the second subperiod. If he refuses the trade, he can go with all the consumption good he produced. However, he will lose the durable good. This has two consequences—he not only loses utility $y$ from the durable good, but also loses all the possible future trades because all future lenders will refuse to trade with him. His continuation payoff is 0 in such a circumstance. On the other hand, the equilibrium payoff of a borrower is $a - r^* + y$. So, this deviation is not profitable if and only if $a - r^* + y \geq (1 - \delta)a$, or

$$r^* \leq y + \delta a \tag{2}$$

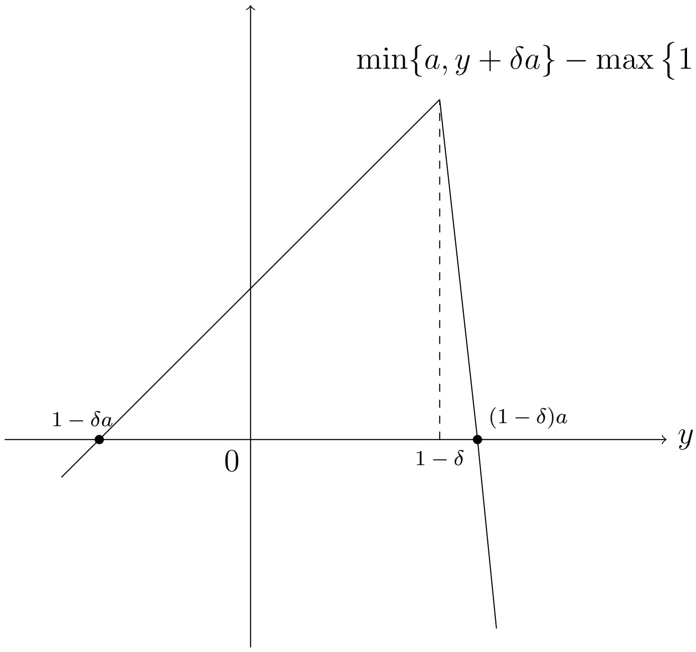


Figure 1: This figure depicts $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\}$ as a function of $y$. The repo strategy constitutes an equilibrium if and only if $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\} \geq 0$, or $y \in [1-\delta a, (1-\delta)a]$. Notice that $y$ does not have to be positive for the repo strategy to constitute an equilibrium.

Next, consider a lender's incentives. Obviously, it must be the case that $r^* \geq 1$, otherwise, she will produce the consumption good by herself. Also, consider a deviation to refuse the trade and escape with the durable good in the second subperiod. If she refuses and takes the durable good, she will receive the value of collateral from the period onward. However, she will lose the return $r^*$ for this period. Hence, the payoff from such a deviation is $y + \delta r^*$. This deviation is not profitable if and only if $r^* \geq y + \delta r^*$ or

$$r^* \geq \frac{y}{1-\delta} \tag{3}$$

In short, if

$$r^* \in \left[\max\left\{1, \frac{y}{1-\delta}\right\}, y+\delta a\right] \tag{4}$$

then the repo strategy constitutes an equilibrium. Now, such $r^* \in [0, a]$ exists if and only if $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\} \geq 0$. This inequality is satisfied if and only if (1) is satisfied. □

Figure 1 depicts $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\}$ as a function of $y$. The repo strategy constitutes an equilibrium if and only if $\min\{a, y+\delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\} \geq 0$.

### 3.3 Borrowing constraint

Equation (2) says that the maximum repayment that a borrower can credibly promise is $y + \delta a$. Notice that this is increasing in $y$, the value of the good used as collateral. This result is consistent with the argument by Kiyotaki and Moore [16].

Notice, also, that $y+\delta a$ exceeds the intrinsic value $y$ of the durable good, and can be positive even when $y \leq 0$. The reason is somewhat similar to the one considered in the literature of new monetary economics (Kiyotaki and Wright [17]). There, an intrinsically useless object—fiat money—has a positive equilibrium value because it provides partial information on whether an individual has worked in the past or not. In our equilibrium, if a borrower does not have the durable good, it is indicated that the borrower has escaped without giving the lender the consumption good, and so, future lenders can punish him.

## 4 Macroeconomic implications

### 4.1 Payment puzzle

Lagos [21] raised the "payment puzzle": Why even to this day assets are used as collateral rather than simply as a means of payment. Our result provides an explanation for it. To see this, consider a spot transaction using the durable good as a means of payment: in the first match, a borrower and a lender exchange the durable and production goods. From then on, no trade occurs.[8] Now, notice that in autarky, the lender can produce one unit of the consumption good every period. Thus, if $y < 1$, the lender never agrees with such a transaction.

What about repo? Proposition 1 says that the repo strategy constitutes an equilibrium when

$$y \in [1 - \delta a, (1 - \delta)a]$$

Because $1 - \delta a < 1$, we have

**Proposition 2.** *For any $(\delta, a)$, there is a non-empty open set of $y$ in which the durable good cannot be used as a method of payment while it can be used as collateral.*

[8] In Section 5.4, we introduce a market of collateral and show that there will still be a price for which a borrower has no incentive to default.

This result is quite intuitive. If a borrower does not pay back, he will lose collateral, and so, he will lose future lenders. Thus, borrowers are willing to pay back even when the intrinsic value of collateral is small. Given this, lenders are willing to accept the repo arrangement even when the intrinsic value of collateral is small.

We emphasize that the argument here does *not* depend on the assumption that lenders can escape with collateral. Section 5.1 provides further arguments for it.

## 4.2 Return vs. liquidity trade-off

Matsuyama [25, 26] and Lagos and Rocheteau [22]—among many others—consider cases in which some assets have high returns but low liquidity.

Our result explains why and when this can happen. Immediately from (1), we have

**Proposition 3.** *For any $(\delta, a)$, there exist $y$ and $y' > y$ such that the repo strategy constitutes an equilibrium for $y$ but not for $y'$.*

This is because when $y$ is too big, the lender has an incentive to escape with the durable good (see (3)). In such a case, the asset always stays at the borrower and is hence not circulated.

In Sections 5.1 and 5.3, we will argue further about this return vs. liquidity trade-off.

# 5 Extensions

## 5.1 Inescapable lenders

There are some collateralized loans that do not look like repos. In the contract between Shylock and Antonio, Antonio did *not* give a pound of his flesh to Shylock at the moment when they agreed with a collateralized loan. Likewise, with a home equity loan, a lender gives money to a borrower, and the borrower promises to pay the loan in the future; if the borrower reneges, then at a promised time, the lender takes the house. But, the lender does *not* live in it between the periods. With this type of collateralized loans, lenders cannot escape with collateral goods. In this subsection, we compare this type of collateralized loans with repos.

For this purpose, we consider the case in which a pair can write the following *collateral contract* in the first subperiod: In the second subperiod, if the borrower pays some $r^*$, then he can keep the durable good. If he chooses not to pay, then the lender takes the durable good. If a pair agrees with such a collateral contract in the first subperiod, the lender does not make any decision in the second subperiod—in particular, she cannot escape with the durable good. All the other assumptions are the same as those in Section 3. Obviously, we are now considering an environment with less frictions than the one we considered in the previous section.

In this environment, consider the following strategy (we call it the *collateral strategy with inescapable lenders*):

**Subperiod 1** If the borrower has the durable good, then the lender chooses $q = 1$, and the lender and the borrower sign on a collateral contract.

Otherwise, make no trade.

**Subperiod 2** If they signed on a collateral contract, the borrower pays $r^*$ and keeps the durable good.

Otherwise, make no trade, i.e., the borrower chooses $r^* = 0$.

After this trade, all matches separate.

**Subperiod 3** If the borrower has the durable good, he keeps it. If the lender has it, she keeps it if and only if $y > 0$ and destroy it otherwise. If one keeps the durable good, he or she receives utility or disutility of $y$.

We examine the condition for which this strategy constitutes an equilibrium. First, observe that because lenders' behavior is the same as in the repo strategy considered in Section 3.2, borrowers' incentives are also the same. Thus, (2) still holds. The maximum repayment that a borrower can credibly promise $y + \delta a$ exceeds the intrinsic value $y$ of collateral.

From the lender side, borrowers' behavior is also the same as in the repo strategy considered in Section 3.2. Now, however, lenders cannot escape with collateral, and so (3) is not binding. The only binding constraint for a lender is now $r^* \geq 1$.

Thus, the collateral strategy with inescapable lenders constitutes an equilibrium if and only if $\min\{a, y + \delta a\} - 1 \geq 0$. Hence, we have

**Proposition 4.** *The collateral strategy with inescapable lenders constitutes an equilibrium if and only if*

$$y \in [1 - \delta a, \infty) \tag{5}$$

Compare (5) with (1). The range of value $y$ expands in which the durable good can work as collateral. This is intuitive, because a pair can write a better contract. However, notice that the lower bound of the value $y$ is the same as that in the case with repo arrangement, $1 - \delta a$. That is, the fact that lenders are impossible to escape with collateral does not allow individuals to use a less valuable good as collateral.

Because we have the same lower bound as in (1), Proposition 2 still holds. That is, even in this environment, there is a non-empty open set of $y$ in which the durable good cannot be used as a method of payment while it can be used as collateral.

On the other hand, Proposition 3 fails. In contrast to the precious section, where the tighter incentive constraint is what makes the asset less liquid, high return and high liquidity always come together in the environment with inescapable lenders.

## 5.2 Observable lenders

Lending activities through repo contracts are in some cases operated by large financial institutions such as investment banks and commercial banks. Activities of such large institutions have often been paid careful attention to in the market, especially since the financial crisis of 2007-2008 (see, for example, Gorton and Metrick [10]). Motivated by this, we now consider the case where lenders can escape with the durable good, but it is observable by *all* (future) borrowers. All the other assumptions are the same as those in Section 3. In particular, we maintain the assumption that borrowers' histories are not observable by non-partner lenders.

In this case, a slightly modified version of the repo strategy is feasible and we call it the repo strategy with observable lenders. The strategy on the equilibrium path is the same as the one defined in Section 3.2. However, if a lender has escaped with the durable good, all future borrowers refuse to trade with her.

Under this strategy, if a lender escapes with the durable good, her lifetime payoff is $y$. The lender's incentive constraint is now

$$r^* \geq \max\{1, y\}$$

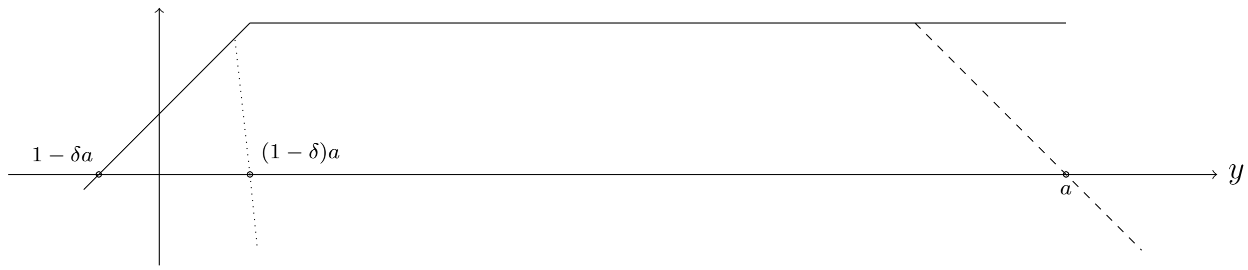


Figure 2: This figure depicts the three different cases—repo, inescapable lenders, and observable lenders. The real (resp. dotted, dashed) line is $\min\{a, y+\delta a\}-1$ (res. $\min\{a, y+\delta a\}-\max\{1, \frac{y}{1-\delta}\}$, $\min\{a, y+\delta a\}-\max\{1, y\}$). They give the regions of the value $y$ in which the durable good works as collateral.

The borrowers' incentives remain unchanged. Thus, the repo strategy with observable lenders constitutes an equilibrium if and only if $\min\{a, y+\delta a\}-\max\{1, y\} \geq 0$.

This gives us the following proposition.

**Proposition 5.** *The repo strategy with observable lenders constitutes an equilibrium if and only if*

$$y \in [1-\delta a, a] \tag{6}$$

Comparing the upper bounds in (6) and (1), we have $a > (1-\delta)a$, and so the range of value $y$ expands in which the durable good can work as collateral. However, as in the case with inescapable lenders, the lower bound of the value $y$ is unchanged, $1-\delta a$. Like in the case with inescapable lenders, the fact that lenders are observable does not allow individuals to use a less valuable good as collateral.

Also, the counterpart of Proposition 2 holds because we have the same lower bound for $y$. The counterpart of Proposition 3 holds because the range of $y$ is bounded from above.

Figure 2 depicts the cases with inescapable (Section 5.1) and observable lenders (this section).

## 5.3 Storable good

So far, we have assumed that an owner of the durable good obtains utility every period. Houses, lands and brand bags are typical examples of such goods. Financial assets are another example, because it yields dividend every period. Production inputs, inventories, and wine, on the other hand, disappear once an owner of the good obtains utility. In this section, we examine whether this difference matters for the appropriateness of asset as collateral.[9]

Formally, let us introduce another kind of good—a *storable* good. The storable good is, like the durable good, durable and indivisible. However, one has to "liquidate" it to obtain utility, and once it is liquidated, the good disappears. This is in contrast to the durable good from which, *in each period*, its owner can enjoy utility. More precisely, in the third subperiod, an owner of the storable good determines whether to keep, trash, or *liquidate* it. If he or she liquidates it, he or she gets *lifetime* utility $z \in \mathbb{R}$.[10] Again, we do *not* restrict $z$ to be positive.

Now consider an economy with three goods—production, consumption and storable goods. Except that the durable good is replaced by the storable good, everything else is the same as in Section 3.

With such a storable good, consider the repo strategy defined in Section 3.2. First, notice that, unlike the durable good, when $z > 0$, *the repo strategy does not achieve the first best.* In any first best allocation, the storable good must be liquidated in the first period, but then it cannot be used as collateral. In the case of the durable good, one could use it as collateral and for its own purpose at the same time. This does not happen in the case of the storable good which one must liquidate to enjoy its value. Hence, from the efficiency point of view, the durable good is more suitable as collateral than the storable good is. Hereafter, we will show that when $y, z > 0$, the durable good is more suitable as collateral than the storable good is from the incentive point of view as well.

To see this, first, derive the region of $z$ in which the repo strategy constitutes an equilibrium. The lender's incentives are the same as in the durable good case—she follows the strategy if and only if $r^* \geq \max\{1, \frac{z}{1-\delta}\}$.[11]

The borrower's incentives are, however, different. First, his lifetime payoff is $a - r^*$,

[9]During the 1994-1995 financial crisis, Mexico used its oil as collateral.

[10]That is, the per-period utility is $z/(1-\delta)$. Later, we will compare the durable good and the storable good, and this formulation makes the comparison easier.

[11]If she liquidates, her lifetime utility is $(1-\delta)\frac{z}{1-\delta} + \delta r^* = z + \delta r^*$. This value must be smaller than $r^*$.

because he cannot enjoy the value of the good. His payoff, when he deviates in the second subperiod, is the same as in the case of the durable good, and is simply $(1-\delta)a$. This gives us one incentive constraint $r^* \leq \delta a$. This says that the maximum repayment that a borrower can credibly promise is $\delta a$, and it is independent of the intrinsic value $z$ of the storable good. This is in contrast to the durable good case in which the maximum repayment that a borrower can credibly promise is increasing in its intrinsic value (see (2)).

Notice also that the borrower now has another possible deviation in the third subperiod, where he liquidates the storable good. In this case, he can enjoy the value of the storable good but will lose all future trades. His lifetime payoff from this deviation is $z$. This gives us another incentive constraint

$$r^* \leq a - \frac{z}{\delta} \tag{7}$$

For the repo strategy to constitute an equilibrium, there must exist an $r^* \in [0, a]$ where all the conditions above are satisfied. Thus, for the repo strategy to constitute an equilibrium, it must follow that

$$\min\left\{\delta a, a - \frac{z}{\delta}\right\} - \max\left\{1, \frac{z}{1-\delta}\right\} \geq 0$$

From this, we have

**Proposition 6.** *If $\delta a \geq 1$, then the repo strategy constitutes an equilibrium if and only if*

$$z \in (-\infty, \delta(1-\delta)a] \tag{8}$$

*If $\delta a < 1$, then the repo strategy does not constitute an equilibrium for any $z \in \mathbb{R}$.*

Before comparing this case with the durable good case, first let us revisit the macro implications—payment puzzle in Section 4.1 and return vs. liquidity trade-off in Section 4.2—in this storable good case. First, consider the payment puzzle. As in the case with the durable good (see Section 4.1), lenders will refuse to exchange the production good for the storable good when $z < 1$. This gives us the counterpart of Proposition 2 as follows:

**Proposition 7.** *If $\delta a \geq 1$, there is a non-empty open set of $z$ in which the storable good cannot be used as a method of payment while it can be used as collateral.*

Also, there is return vs. liquidity trade-off with the storable good. Indeed, from (8), the trade-off is even stronger than that with the durable good is, in the sense that

**Proposition 8.** *Suppose the repo strategy does not constitute an equilibrium for $z$. Then it does not for any $z' > z$.*

It is also worth mentioning that the trade-off survives with the form of collaterized contracts considered in Section 5.1. This is a consequence of (7), one of the borrower's incentive constraints. This is in contrast to the durable good case (see Proposition 4).

Finally, compare the storable good with the durable good. To do so, let

$$y = z$$

so that the value that one gets from holding the durable good forever is the same as the value that one gets from liquidating the storable good today.

When $\delta a < 1$, from Proposition 6, the durable good is more suitable as collateral than the storable good is. Suppose $\delta a \geq 1$. Comparing the upper bounds in (8) and (1), we have $(1 - \delta)a - \delta(1 - \delta)a > 0$ (see Figure 3). This gives us that

**Proposition 9.** *Suppose that $y = z > 0$. Then, the repo strategy constitutes an equilibrium with the durable good if it does with the storable good, but not vice versa.*

Notice also that when $y = z < 0$, the result is reversed—when $\delta a > 1$, the storable good works as collateral no matter how small $z < 0$ is.

## 5.4 Trade of the durable good

So far, we have *assumed* that the durable good cannot (not only be produced but also) be traded. If it can be traded, it affects incentives of individuals—a borrower, for example, can now escape with the consumption good and then buy another durable good some time later.

In this section, we relax the assumption by allowing individuals to trade the durable good on a market. We will first show that such trading opportunity does not change our results—the repo strategy still constitutes an equilibrium. We will then show how our answer to the payment puzzle can be generalized.

More precisely, we modify the model in a way that is reminiscent of Lagos and Wright [23]. Now there are four subperiods in each period. The first two subperiods are the

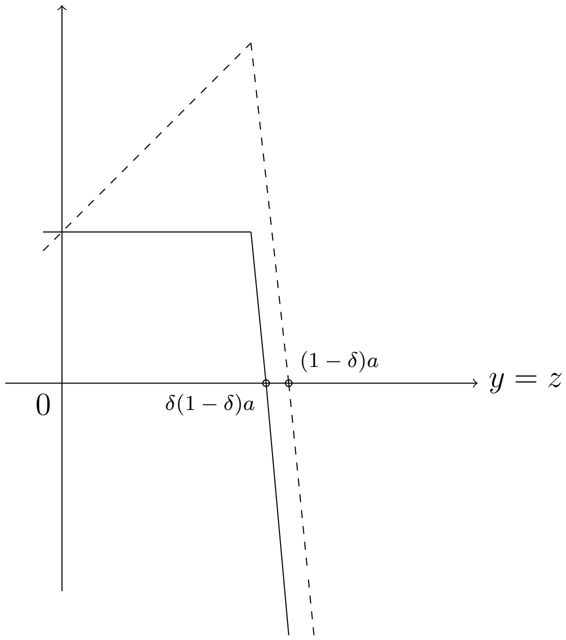


Figure 3: This figure compares the durable good and the storable good. The real (resp. dashed) line depicts $\min\{a, y + \delta a\} - \max\left\{1, \frac{y}{1-\delta}\right\}$ (resp. $\min\left\{\delta a, a - \frac{y}{\delta}\right\} - \max\left\{1, \frac{y}{1-\delta}\right\}$) as a function of $y$ (normalized so that $y = z$). The range of $y$ in which the repo strategy constitutes an equilibrium is wider with the durable good.

same as in Section 3. In the third subperiod, there is a market for the durable good. The market is competitive so that everyone is price taker. The fourth subperiod is the same as that of third subperiod in Section 3.

In the market, individuals can trade the durable good in exchange for labor. The utility from consuming $h$ units of labor is the same as the cost of supplying the same units of labor. Labor cannot be carried over periods. The lifetime payoff of an individual is now $(1-\delta)\sum_{t=1}^{\infty}\delta^{t-1}(c_t + h_t)$. Recall that utility from the durable good $y$ is obtained at the end of each period (i.e., after the durable good market transaction).

Now, we modify the repo strategy as follows. In the first two subperiods, individuals play the repo strategy, described in the Section 3.2. If a borrower has multiple durable goods, then the lender refuses to trade with the borrower. Any lender (resp. any borrower) who has (resp. does not have) the durable good can go to the market,[12] and provide (resp. get) the durable good in exchange for $r^*$ units of labor, where $r^*$ is the

[12]On the equilibrium path, there are no such agents. One way to avoid this is to assume that some pairs exogenously separate right after production of the consumption good. A detailed argument is available upon request.

one defined in the Section 3.2. In other words, letting the competitive equilibrium price of the durable good be $h^*$, we have

$$h^* = r^* \tag{9}$$

In the following, we will show that an opportunity of trading the durable good does not affect our analysis, in the sense that if a repo strategy constitutes an equilibrium for an $r^*$ satisfying (4), then the modified repo strategy also constitutes an equilibrium for the same $r^*$ and equilibrium price $h^*$. To see this, first observe that, when (9) holds, the payoff to a borrower when he escapes from the current lender and buys back the durable good in the frictionless market (where all the participants can trade for sure) is exactly the same as that when he pays to the current lender and gets back the durable good—the borrower is paying the same amount $h^* = r^*$ in any case. By the same token, the payoff to a lender when she escapes from the current borrower and sells the durable good in the frictionless market is exactly the same as that when she gives the durable good to the current borrower and gets the consumption good. Of course, if the market is frictional so that there is an arbitrarily small chance of not being able to participate in the durable good market, then trading with the current partner is strictly better than trading in the market. Thus, as long as the price of the durable good is given by (4), individuals do not utilize the opportunity of trading the durable good.

Next, it can be easily seen that, in the durable good market, individuals trade the durable good at a price in the range (4). In this situation, the demand from borrowers and the supply from lenders are the same. This of course implies that the price of the durable good in the market must be in between the value of the durable good to the lenders and that to the borrowers. The value of the durable good to the lenders is simply its intrinsic value. Its discounted value is $y$. The value to the borrowers is its intrinsic value plus the value from future trades. Provided that $a > r^*$, its discounted value (including the value from the durable good) is $(1-\delta)y + \delta(a - r^* + y)$. Thus, $h^*$ is a competitive equilibrium price of the durable good (when the price of labor is normalized to 1) if and only if

$$h^* \in \left[\frac{y}{1-\delta}, \frac{(1-\delta)y + \delta(a - r^* + y)\}}{1-\delta}\right] \tag{10}$$

Observe that (10) is non-empty as long as (4) holds true—that is, the repo strategy constitutes an equilibrium when it is assumed that individuals cannot trade the durable good—so that we have the competitive equilibrium price $h^* = r^* \in \left[\frac{y}{1-\delta}, \frac{(1-\delta)y+\delta(a-r^*+y)\}}{1-\delta}\right]$. This can be checked by noting that if $r^* \geq \max\{1, \frac{y}{1-\delta}\}$, then $r^* \geq \frac{y}{1-\delta}$. To see the other inequality, notice that the condition $r^* \leq \frac{(1-\delta)y+\delta(a-r^*+y)\}}{1-\delta}$ is equivalent to $r^* \leq y + \delta a$. Hence, as shown in the proof of Theorem 1, this is satisfied within the range of the parameter given by (1).

**Collateral and money**

When there is a competitive equilibrium market of the durable good and it is frictionless, there is another equilibrium that achieves the first best. The strategy is

**Subperiod 1** If the borrower has the durable good, then trade it for the right to use of one unit of the production good, that is, $(q, g_b) = (1, give)$.

Otherwise, make no trade, that is, $(q, g_b) = (0, not\ give)$.

**Subperiod 2** No trade occurs, that is, $(r, g_\ell) = (0, not\ give)$.

After this trade, all matches separate.

**Subperiod 3** The borrowers and lenders exchange the durable good and labor in the durable good market. The price of the durable good (normalizing that of labor to be 1) is $r^*$, where $r^*$ satisfies (4).

**Subperiod 4** If the borrower has the durable good, he keeps it. If the lender has it, she keeps it if and only if $y > 0$ and destroys it otherwise. If one keeps the durable good, he or she receives utility or disutility of $y$.

From the discussion above, it is clear that the strategy constitutes an equilibrium, and achieves the first best. It is also clear that any convex combination of this strategy and the repo strategy—a fraction of pairs play this strategy and the rest play the repo strategy—also constitutes an equilibrium and achieves the first best.

Notice that, in this strategy, the durable good is used as *money*.[13] That is, a "borrower" gives the durable good to a "lender," as evidence that the "lender" gave the right to use of the production good in subperiod 1. In this strategy, this is the *only* reward that the "borrower" gives to the "lender." But then the "lender" can use the durable

[13]In particular, the model is now completely isomorphic to a version of Lagos and Wright [23].

good and get utility in the future. Our results shows a kind of *neutrality*—from the social point of view, the durable good can be used as collateral or money, and they give the same social welfare.

It is also obvious that when there is a friction in the market of the durable good—an extreme case is the one considered in Section 3 and a not so extreme case is the costly participation or with a small chance of not being able to find a counterparty—from the social point of view, the durable good should be used as collateral rather than money.[14]

In relation to the payment puzzle, our analysis above can be summarized as follows.

**Proposition 10.** *Consider a competitive market for the pledgable asset. If the market is frictionless, then the asset can be used as either collateral or a means of payment (money), or as both. If the market is frictional, then collateral is essential.*

Here, collateral is essential in the sense that the strategy in which the durable good is used as collateral is more efficient than that in which that is used as a means of payment (even if the latter constitutes an equilibrium). Note, if the market is frictional enough, assets will be used only as collateral, never as a means of payment.

# 6 Conclusion

This paper presented a simple equilibrium model in which collateralized credit emerges endogenously. Even in a frictional world, where commitment is limited, we show that collateral can serve as a credible device that prevents the participating parties from reneging. Our theory provides a micro-foundation to justify the borrowing constraints that are widely used in the existing macroeconomic models. We provide an answer to the "payment puzzle" raised in the macroeconomic literature. We also explain why there is a tradeoff in assets between return and liquidity, and what kinds of assets are useful as collateral.

While our model captures the features of repos well, it would be interesting to investigate the dynamic implication of the model. We believe such an extension will offer a novel insight into the observed phenomena of run on repos.

[14] In the literature of new monetarists economics, the cost of using money as a means of payment may occur due to inflation or the cost of fraud (Li, Rocheteau and Weill [24])), or because money is susceptible to theft (Sanches and Williamson [28]).

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